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Abstract

In 2007, in the *IV Actas de las Jornadas Internacionales sobre Textiles Precolombinos,* Kylie Quave noted the existence of various structural anomalies in khipus presumed to date to the Late Horizon. These anomalies included the use of non-cotton vegetal fibres, the inclusion of single red strings, subsidiary cords that are plied through rather than half hitched, and the placement of long knots and figure-8 knots "in a way that precludes a numerical reading". Since Quave's article, there has been little examination of such anomalies, nor have scholars known whether such anomalies were to be found in khipus that had been radiocarbon dated to the Late Horizon, or whether they occurred only in post-Inka khipus.

This chapter examines anomalies in a khipu radiocarbon dated to the Late Horizon (475+-26 cal BP), focusing on a specific type of anomalous knot, referred to as a "nether knot", which occurs below the unit position on a khipu pendant in a zone where, according to Locke's knot typology, no knot should be present. Nether knots, which are found on one or more pendants in over 20% of the khipus in the Online Khipu Repository, the world's largest khipu database, form a significant feature of the khipu corpus. This article proposes a reading of nether knots based on ethnographic analogy with nether knots on 20th century khipus. A better understanding of nether knots allows us to provide more precise readings for the khipus that contain them, necessitating a revision to Leland Locke's influential knot typology.

Key words: Khipu, Peru, Andean, ethnomathematics

Resumen

En 2007, en las *Actas de las Jornadas Internacionales sobre Textiles Precolombinos IV,* Kylie Quave señaló la existencia de varias anomalías estructurales en khipus que se presume datan del Horizonte Tardío. Estas anomalías incluyeron el uso de fibras vegetales distintas del algodón, la inclusion de hilos rotos simples, cordones subsidiarios que están trenzados en lugar de medio entrelazados y la colocación de nudos largos y nudos en forma de 8 "de una manera que impide una lectura numérica". Después del artículo de Quave, ha habido poco examen de tales anomalías, y los estudiosos tampoco han sabido si tales anomalías se encontraron en khipus que habían sido fechados por radiocarbon en el Horizonte Tardío, o si ocurrieron solo en khipus post-Inka.

Este capítulo examina anomalías en un khipu que data del Horizonte Tarde (475+-26 cal BP), centrándose en un tipo específico de nudo anómalo, denominado "nudo inferior", que ocurre debajo de la posición de las unidades en un colgante en un zona donde, según la tipología de Leland Locke, no debería haber ningún nudo. Los nudos inferiors, que se encuentran en uno o más colgantes en más de 20% de los khipus en el OKR (Open Khipu Repository), la base de khipus más grande del mundo, forman una característica importante del corpus de khipu. Este artículo propone una lectura de los nudos inferiores basada en una analogia etnográfica con los nudos inferiores en los khipus del siglo XX. Una mejor comprensión de los nudos inferiores nos permite proporcionar lecturas más precisas de los khipus que los contienen, lo que require una revision de la influyente tipología de nudos de Locke.

Palabras claves: Khipu, Peru, Andino, etnomatemática

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Introduction

Archaeologist Kyle Quave's 2009 article, "Confronting Anomaly in the Khipu Structure", brought attention to our current limitations in understanding Andean khipus by focusing on irregular khipu features whose significance is unknown. She identified numerous structural anomalies in khipus presumed to date to the Late Horizon, including the use of non-cotton vegetal fibres, the inclusion of single red strings, subsidiary cords that are plied through rather than half hitched, and the placement of long knots and figure-8 knots "in a way that precludes a numerical reading". Since Quave's article, there has been little examination of such anomalies, nor have scholars known whether these anomalies were to be found in khipus that had been radiocarbon dated to the Late Horizon, or whether they occurred only in post-Inka khipus.

This article examines anomalies in the knot structure of Andean khipus, proposing a revision to Leland Locke's decipherment of khipus' numerical knot system. It begins by describing Locke's revolutionary insights into how khipus encoded numbers, and then surveys the different types of "anomalous" knots found on Late Horizon khipus which fall outside of Locke's knot typology. Such non-Lockean knots include long knots with more than nine turns, knots located on the primary cord, long knots that shift colour within the knot, and so forth. Finally, it will focus on a specific type of anomalous knot, referred to as a "nether knot", which occurs below the unit position on a khipu pendant in a zone where, according to Locke's knot typology, no knot should be present. Nether knots, which are found on one or more pendants in over 20% of the khipus in the Online Khipu Repository, the world's largest khipu database, form a significant feature of the khipu corpus. Nonetheless, scholars have not analysed nether knots systematically, nor have they achieved any consensus for how nether knots should be interpreted. This article proposes a reading of nether knots based on ethnographic analogy with nether knots on 20th century khipus from the Central Andes. The applicability of the post-Inka understanding of these anomalous knots to Inka khipus is confirmed by a Late Horizon khipu, KH0424 (B/8707), whose nether knots are shown to express the same numerical principles as those found in khipus from 20th century Huarochirí. A better understanding of nether knots allow us to provide more precise readings for the khipus that contain them. It also demonstrates that pendants on Inka khipus could encode more than one numerical quantity, illustrating the conceptual continuities in the Andean khipu tradition from ancient times to the 20th century.

In 1923, Leland Locke's landmark book, *The Ancient Quipu*, transformed our ability to understand khipus, explaining for the first time how to read the numbers encoded by knots in Inka khipus. All subsequent khipu research rests upon Locke's foundational insights into the number system. Yet his decipherment of the numerical system, while crucial, is insufficient to understand khipu knots in their entirety. A century after the publication of Locke's influential book, it is time to revise some aspects of his knot typology, acknowledging that Inka khipu knots are more complex than has been appreciated until now.

Leland Locke (1875 – 1943) and the decipherment of the khipu numeracy

Leslie Leland Locke was born in 1875 in the sleepy Pennsylvania mill town of Grove City, just sixty miles north of Pittsburgh. He attended college and graduate school at Grove City College, specialising in Mathematics. After a brief stint of teaching at Michigan State University, he moved to New York City in 1908, where he joined the Maxwell School for Teacher Training and undertook graduate studies at Columbia University with David Eugene Smith (Kidwell 2016, 208) who introduced him to the topic of khipus.

Locke's graduate studies at Columbia with Professor David Eugene Smith paved the way for his decipherment of the khipu number system. Smith had gained renown for his publications in the history of mathematics; he founded the journal *Scripta Mathematica* and would serve as President of both the History of Science Society and the Mathematical Association of America. Smith travelled extensively throughout Europe, Asia, Africa, and Latin America to collect manuscripts and books on mathematical topics. His first trip to South America took place in the 1880s, and he would return repeatedly to purchase rare volumes and manuscripts (O'Connor and Robertson 2015). He was proud to have demonstrated that the earliest mathematical treatise published in the Americas was from colonial Mexico, rather than colonial New England as had been previously supposed (Smith

1921). Eventually his library grew to contain over 11,000 volumes, many of them rare (Simons 1945: 43). Among the library's 1500 manuscripts were treasures from India, Japan, China, Persia, and Latin America, as well as early European manuscripts dating from between 1118 and 1650.

Smith filled his Edwardian house not only with rare books and manuscripts, but also with hundreds of curious mechanical objects related to science and math. The following account provides a partial view of the variety and uniqueness of the rare antiques he collected:

> "Outstanding items include several celestial spheres of the seventeenth century. One of these is from Japan, made of *papier-mâché*; another is of Hindu origin, of bronze with realistic stars; while a third is from Persia, made by the grandson of Haddad, the emperor Humayun's chief astronomer. A telescope made by the famous instrument maker, Jesse Ramsden, about 1775, is still in working order. A rare group of old English tally sticks dating from about 1296 [found in] the Chapel of the Pyx, Westminster, in 1906, where they had lain undisturbed for six hundred years... Astrolabes of intricate and delicate workmanship and from many lands show the development of this ancient scientific instrument... Compasses dating from the beginning of the Christian era, quadrants, protractors and sextants further enlarge the scope of the collection" (Simons 1945: 41).

At the end of every semester, Smith held a party for twenty or thirty select students in his "beautiful, treasure-filled home". For each occasion he prepared a display of special objects, books, letters, and manuscripts. As one former student recalled, "tea was served and was followed by a talk that was pure magic on the books and other objects assembled on tables at the front... [It was] a glimpse into an unknown world" (Simons 1945: 49). Most importantly, Smith granted his students access to his library for their research. Another former student reminisced, "The students in the seminary in the History of Mathematics under Dr David Eugene Smith have been privileged at various times to visit the unique private library of their instructor and to use such manuscripts as were necessary in their field of research... Books of all periods of history and of all countries are found there. Early historic tablets, original manuscripts, autographed letters, presentation copies and first editions as well as rare translations are included in this excellent library. Dr Smith has personally collected these rarities during his numerous travels in Europe, Asia, Africa, and South America ... he has ingeniously gathered them from remote and obscure places, with the view in mind of preserving them for his students" (quoted in O'Connor and Robertson 2015).

When Locke entered this milieu as a graduate student, he joined a circle of scholars around Smith who strove, in part, to bring the "genius" of non-Western mathematics to the appreciation of European and North American audiences. According to Smith, Locke began his research on khipus as "a chapter in the extensive history of the abacus, a topic that has never yet been worthily treated but one that Mr Locke is beginning to investigate" (Locke 1912: 325). Locke had arrived in Brooklyn not long after Adolph Bandelier – with whom Locke corresponded -- presented a collection of khipus to the American Museum of Natural History in New York. According to the American Museum of Natural History's records, Smith borrowed a khipu in the collection in November 1911, presumably with the intention of showing it to Locke. (Medrano 2022). The physical khipus in the AMNH together with the written khipu descriptions in Smith's rare South American books would provide the foundation for Locke's decipherment. Ayalos y Figueroa's *Miscelanea Austral* (1602), Calancha's *Coronica Moralizada* (1638), and Boturini's *Nueva Historia General de la América Septentrional* (1746) are just a few of the rare volumes in Smith's library that Locke would cite in *The Ancient Quipu*.

While Locke pursued his khipu research, however, he also worked as an assistant for the book that Smith was preparing with his co-author, Yoshio Mikama, a prominent Japanese historian of mathematics. *The History of Japanese Mathematics*, published in 1914, was one of the first books to show Westerners the nature of Japanese mathematics, known as *wasan*. A "euphoric cult of Japan" had emerged in Britain and the US during the Edwardian era, when Japan became idealised as a model of efficiency and culture (Tonooka 2015). Realising that virtually no information on Japanese mathematics was available in the US, Smith had begun to acquire material on *wasan* during his first visit to Japan in 1907. He eventually collected more than a hundred Japanese mathematical manuscripts and over two hundred printed books, many from the 17th and 18th centuries (Simons 1945: 45). The *History of Japanese Mathematics* begins with a description of the indigenous Japanese arithmetic in the earliest known period, followed by the era of Chinese influence (552 and 1600 AD), continuing through the "Renaissance" of Japanese mathematics under the aegis of the great Seki Kowa in the 17th century, and then through subsequent phases of Japanese history. Separate chapters treat the development of the Japanese abacus, the *soroban*, and the calculating rods known as *sangi* or *chou*.

Locke created the 74 illustrations for the volume by photographing Japanese bronzes, wood-block prints, and books in Smith's library. His mentor praised Locke's assistance: "…[Locke's] intelligent and painstaking efforts to carry out the wishes of the authors have resulted in a series of illustrations that not merely elucidate the text but give a visual idea of the genius of the Japanese mathematics that words along cannot give. To him I take pleasure in ascribing the credit for this arduous labor, and in expressing the thanks of the authors" (Smith and Mikama 1914). Locke's photographs of Chinese and Japanese abacuses provide detailed information about how abacuses represent numbers and are used for calculation. His illustrations also include numerous depictions of Japanese counting rods, explaining how they functioned by themselves and in relation to a counting grid. The counting rods provided Locke with direct, hands-on experience of a non-western numerical system in which, unlike Arabic numerals, digits are represented in an iconic manner, through the repetition of sticks. Zero is shown by the absence of any rods. Moreover, the place value of the rods is determined by their relative position on the grid of a counting board. In a very general sense, this is similar to Locke's discoveries about khipu numeracy: digits are shown by the repetition of either single knots or by nodes on a long knot; zero is the absence of knots; and the relative place of knots within the khipu grid determines the digits' place value.

While Locke worked on Smith and Mikama's book, he also initiated his research on Andean khipus, publishing his first article, "The Ancient Quipu, A Peruvian Knot Record" in 1912 (Locke 1912). In this piece he suggested the basic principles of his decipherment of khipu knots and argued that khipus only recorded numerical information without performing calculations. His book, *The Ancient Quipu or Peruvian Knot Record* (1923), provided a full explanation of his decipherment of the khipu numerical system, accompanied by photographs and descriptions of khipus in the American Museum of Natural History, along with several other khipus from museums and private collections around the world. Finally, in 1928 he published a 73-page volume, *Supplementary Notes on the Quipus in the American Museum of Natural History*, which presented additional information about some of the khipus described in the 1923 work (Locke 1928).

Before his last book on khipus came out, Locke already had turned his attention to a more recent device, the calculating machine (Kidwell 2016: 208). He was particularly interested in machines that could multiply numbers together directly, rather than merely repeat addition. Inspired, perhaps, by the mechanical antiques on display in his mentor's home, Locke began collecting early calculating machines, including the first direct multiplication machine designed by Ramon Verea in 1878, the lever-set barrel calculating machine patented by George Grant in 1887, and a cylindrical slide rule invented by George Fuller in 1878. He published occasional articles and book chapters on math pedagogy and calculating machines, such as his 1924 essay, "Mathematics of the Calculating Machine" (Locke 1924), and his 1926 piece about Verea's ground-breaking prototype (Locke 1926). He was an active member of multiple academic societies: the American Mathematics Society, the National Council of Teachers of Mathematics, the Mathematics Association of America, and the History of Science Society. He taught mathematics at the Maxwell School for Teacher Training and was also a Professor of Mathematics in the Evening Session at Brooklyn College (Kidwell 2016: 208). His invaluable collection of more than one hundred calculating machines was donated to the Smithsonian Museum. Lauded as "a noted mathematician" and "an outstanding authority on the history of mathematics" (Anon 1943), Locke passed away on August 28, 1943, at the age of 63. Louis Karpinski eulogised him as "an able teacher and a learned investigator and writer on the history of mathematics" (Karpinski 1943). Despite the impact of his khipu decipherments on the anthropology and archaeology of the Andes, Locke was first and foremost a mathematician who viewed khipus primarily as a stage in the history of counting machines.

The Ancient Quipu or Peruvian Knot Record (1923)

"It is hard to imagine", Philip Ainsworth Means proclaimed in his review of *The Ancient Quipu*, "how anyone not frankly a 'bad boy' of science will be able to dissent from the conclusions reached by Professor Locke" (Means 1924: 271). This outspoken praise from a leading Andean scholar illustrates the contemporary reaction to Locke's work, which was viewed as the definitive final word on the topic of khipus. P. A. Means stated that the only other worthwhile addition to our knowledge of khipus would come from the research of the great Peruvian anthropologist, Julio C. Tello: "When the work of Dr. Tello... comes out, we shall know all there is to be known about the quipu, for Locke and Tello between them will have covered the whole field" (Means 1924: 271). Tello's archives in Lima preserve his unpublished notes about both historical and modern khipus (Hyland 2016). However, for reasons that remain unclear, Tello published very little about khipus, leaving Locke the undisputed master of the subject for most of the 20th century.

In the early stages of preparing his study of khipus, Locke benefitted from the advice and guidance of Adolph Bandelier, whom he thanked "for much help and suggestions generously given" (Locke 1912: 326, n. 5). Bandelier, after whom Bandelier National Monument in New Mexico is named, was a distinguished expert on the anthropology of indigenous North and South America. During Bandelier's ethnographic research in Peru and Bolivia between 1892 and 1903, he had seen Andean men utilizing knotted cords for record-keeping. Bandelier's perspectives on the intellectual and moral character of the Aymara people would deeply influence Locke's outlook on the khipus still used by Andean highlanders in the early 20th century.

The Ancient Quipu commences with a brief consideration of the textile arts in Inka Peru and the geographical conditions which favour the preservation of specimens. The heart of the book follows: a description of the material characteristics of khipus, along with the decipherment of how the knots encoded numbers. As further support of his conclusions, Locke then presents lengthy excerpts from the Spanish chroniclers who witnessed khipu use soon after the fall of the Inka Empire, as well as from more recent observers of the survival of khipu traditions in the highlands of Peru and Bolivia. In the Foreword, Charles Means, a curator at the American Museum of Natural History, noted that, "Several times during the progress of this work Professor Locke has expressed to me his regret that he felt obliged to include so much of "this bizarre stuff" in the different quotations he made" (quoted in Locke 1923). By "bizarre stuff", Locke was referring to passages by Spanish observers that attributed "history, folklore, and poems" to khipus, a view which Locke rejected in favour of the belief that khipus were merely a "*memoria technica*" for "memorizing historical items, poems, lists of kings, etc" (Locke 1923: 31). The final section of the book includes 52 beautiful photographic plates of the khipus discussed in the work. The clarity and detail of Locke's photographs were exceptional; P A Means called them "superb" (Means 1924: 271).

The Inka khipu that Locke decoded consists of a horizontal primary cord from which hang pendant cords; these pendant cords may have strings tied onto them known as "subsidiaries". Subsidiary knots usually are subtracted from the number on the pendant. Knots on pendants follow a decimal pattern in which the power of ten is denoted by the knots' position along the pendant. Knots closer to the primary cord indicate a higher decimal value, while knots at the "bottom" of the pendant are in the "one's" position.

Digits in positions for 10 and higher powers are shown by clusters of simple overhand knots. Digits in the "one's" position are indicated by the number of turns on a long knot or, in the case of "1", by a "figure-8" knot. For example, a pendant with one single overhand knot in the hundreds position ("100"), no knots in the tens position ("0"), and a long knot with 5 twists in the one's position ("5") would read "105". A pendant with one overhand knot in the hundreds position ("100"), six overhand knots in the ten's position ("60"), and a long knot with 7 twists in the one's position ("7") equals 167 (see Figure 1.1). Zero is represented by the absence of knots in the appropriate position. In general, the positions for the powers of ten are aligned among successive pendants, so that a numerical khipu appears to have horizontal rows of knots within an imaginary grid.

An Inka period khipu from an archaeological site in Huando, located in the coastal valley of Chancay, provided the confirmation of Locke's decipherment. The pendants on this khipu were distributed along the primary cord in six groups, each comprised of four cords. For each of the six groups, a "top cord" had been strung through the top loops of the pendants in the group; there was, in other words, a "top cord" for each group of pendants. Locke discovered that these "top cords" provided the sum of the numbers on their respective pendants, giving, as he wrote, "an accurate key to the numerical character of the knots" (Locke 1912: 330). The simplicity of this proof of Locke's decipherment belies the ingenuity necessary to discover it. Locke wrote that the khipu from Huando represented the "most highly developed form of the quipu" and he praised its importance in providing the key to interpreting khipu knots (p. 16).

The pendants on the khipu from Huando display an array of different colours: white, yellow, light blue, dark blue,

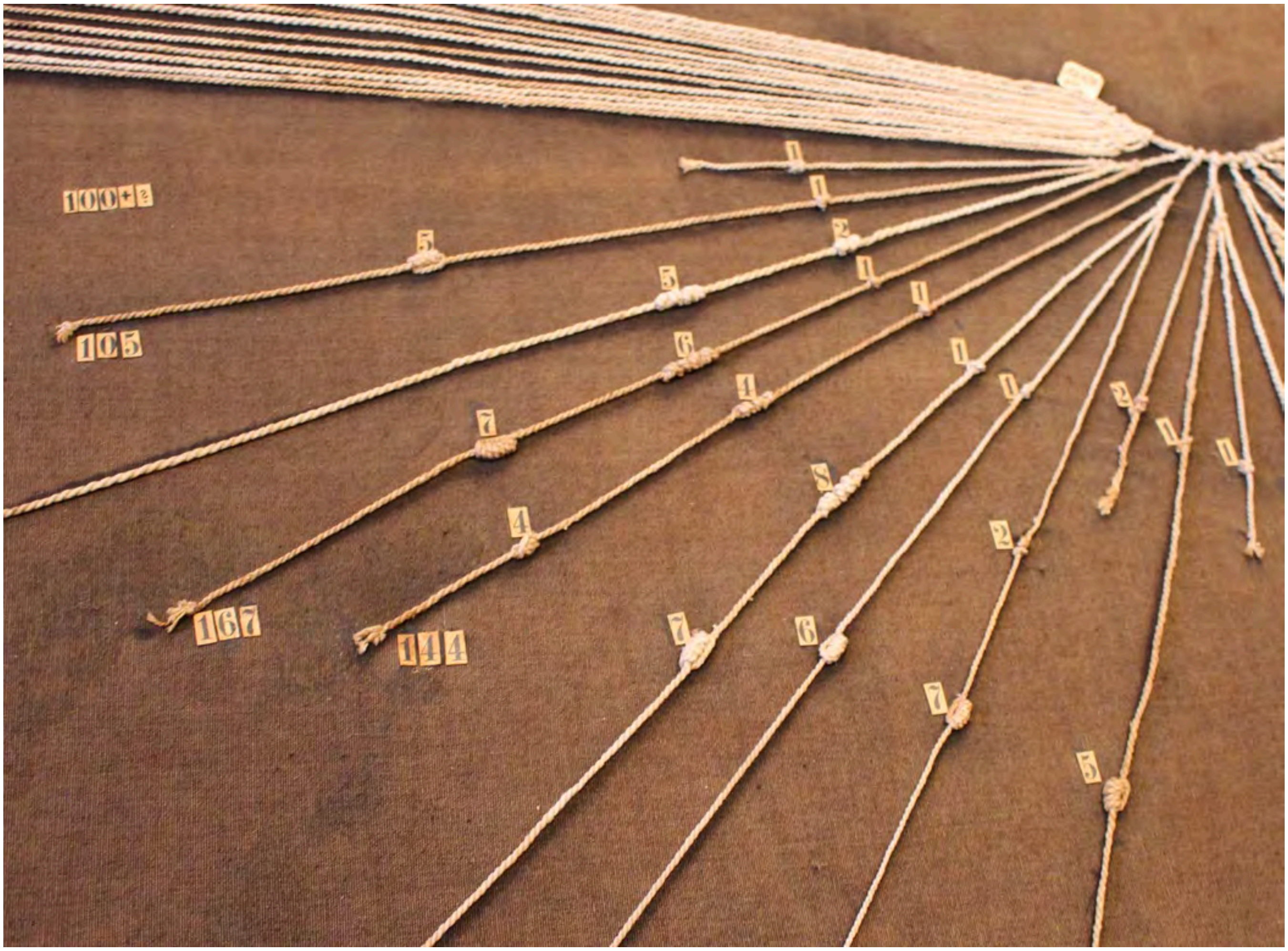

Figure 1. Inka khipu with labels for the numerical knots, MV107764. Vatican Ethnology Museum. Photo by the author.

and varying shades of brown from beige to umber (Locke 1923: 17-18). Most Inka style khipus have at least two colours, and some possess an impressively large range of hues. Guided by Spanish chroniclers who claimed, for example, that white indicated silver and yellow signified gold, Locke concluded that colour sometimes played an important role in indicating what was being enumerated by the knots. "A scheme of roughly suggestive colors was probably in use", Locke wrote (Locke 1923: 32). He speculated that specific colours may have been mandated for certain government khipus, but that for "the great mass of ordinary records", the choice of colour relied upon "the fancy of the maker" (Locke 1923: 32). Locke considered only three attributes of khipus to convey relevant information: knot type, knot position, and – occasionally -- colour. He noted that some pendant cords were thicker than others, but he rejected the possibility that this feature was meaningful for Inka khipus.

Based on his survey of 45 khipus (3 of which were "spurious", and 2 of which were modern), Locke concluded that khipus were used only for recording numbers. They were not utilized for calculations, he stated, nor did the khipus he studied encode a phonetic system of writing. If khipus were intended for any purpose beyond accounting, he argued, it was simply as a memory aide, no more. He viewed the suggestion by various chroniclers that khipus could encode poetry, history, or other forms of narrative to be "bizarre", as Charles Means explained. Locke recognised that some of the exemplars in his small sample of ancient khipus exhibited features that either contradicted or otherwise did not fit his knot typology. For example, Bandelier's contributions to the American Museum of Natural History included sixteen khipus which consist almost entirely of long knots, in which a single pendant may contain seven or more long knots spaced along its length[1]. These khipus violate Locke's principle that there should be a maximum of one long knot per

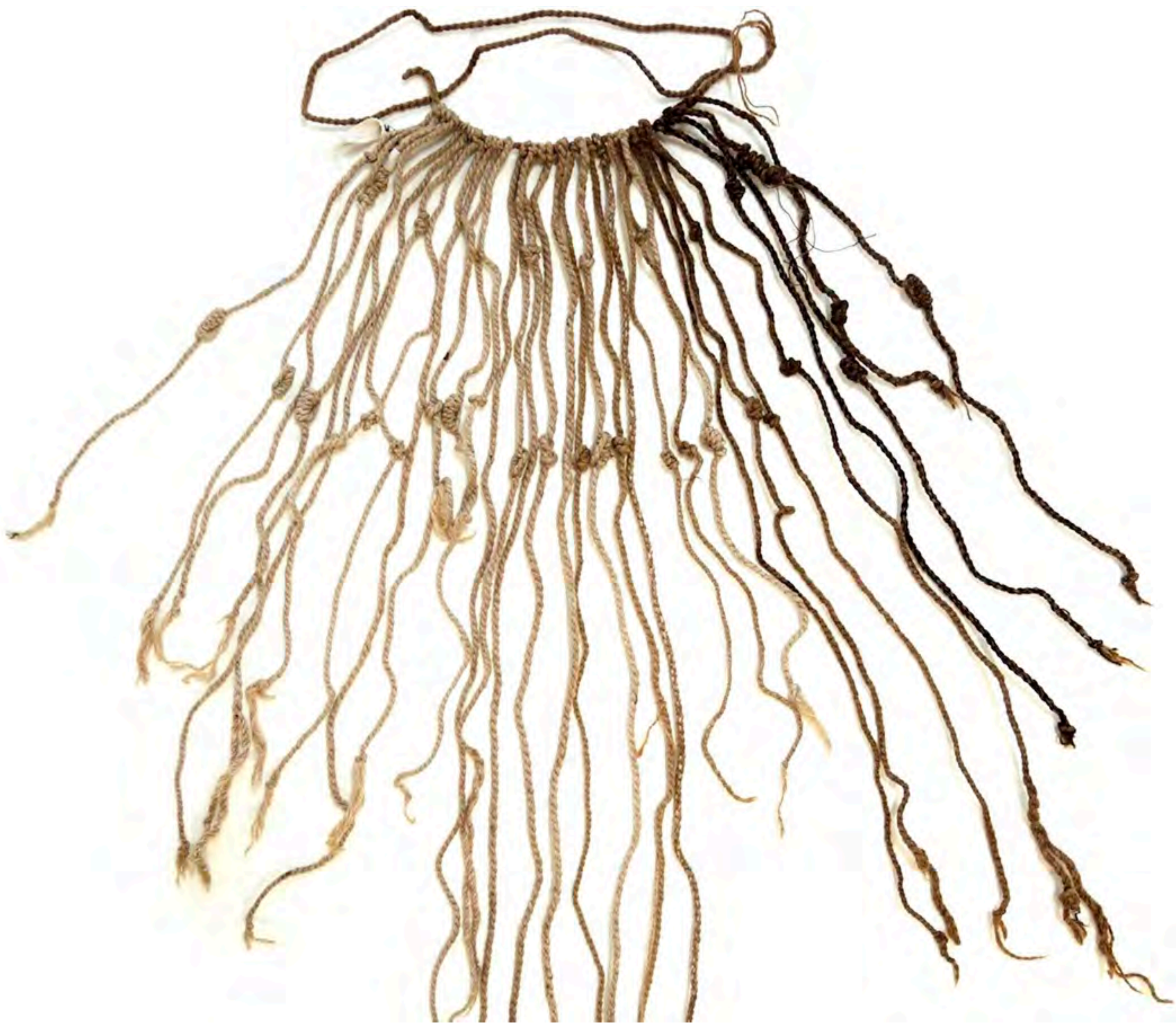

Figure 2. Inka Khipu with horizontal rows of knots. National Museum of Scotland, A.1956.1431. Photo by the author.

pendant and that the long knot, when present, must occur in the "one's" position. However, he explained away this and other anomalies by ascribing them to "inferior" and "degenerate" phases of khipu development, and as therefore not worthy of consideration (Locke 1923: 25). Other "non-Lockean" features on khipus that he studied, such as knots located along the primary cord (e.g., Plate III) or "nether knots" placed below the one's position on a pendant (e.g., on khipu B/8707 from Huando), he simply ignored in his 1923 book. Locke either refused to acknowledge knots that failed to conform to his knot typology, or he considered khipus with anomalous knots to be inferior. Although he was aware of variations in modern khipu knots, he believed that contemporary khipus possessed little relevance to more ancient cords.[2]

1 These sixteen khipus allegedly came from Bandelier's excavations at Cajamarquilla. However, he never published anything about his work at Cajamarquilla, a coastal site covering 167 hectares which was inhabited for over 1000 years. The Cajamarquilla khipus lack archaeological context and the date of their production is unknown.

2 Locke was familiar with variations in modern khipus through the research of Max Uhle. More recently, variation in modern khipus has been noted by numerous scholars, including Cohen 1957; Mackey 1970; Medrano 2021: 93; Mesa y Gisbert 1966; Miranda Rivera 1958; Núñez del Prado (1950) 2005; Prochaska 1983; Soto Flores 1950-1; Salomon 2004.

Locke was a mathematician who conceptualised khipus primarily as a problem in mathematical numeracy, and who was guided by his goal of understanding the cords' role "as a chapter in the extensive history of the abacus", that is, of counting tools. Examples of what is often termed "proto-writing", such as the proto-Cuneiform texts from Uruk, which represent numbers and things and which might possibly offer useful analogies to khipus, would not be identified and published until the 1930s. Given Locke's training and influences, it is not surprising that he would conclude that "the most highly developed form of the quipu... [is] without question numerical in nature" (Locke 1923: 31), and that he would consider any deviations in his samples as "primitive forms" that did not merit further investigation (Locke 1923: 13).

Bandelier, Uhle, and the Relationship between "Inka" and "Modern" Khipus

Locke's mentor, David Smith, brimmed with enthusiasm for an eclectic array of mathematical and scientific books, manuscripts, and artifacts from around the globe. He welcomed the contributions of non-white scholars, such as his co-author Yoshio Mikama, and exuberantly celebrated the achievements of non-European cultures. His open-minded, albeit romanticized views would stand in stark contrast to the anthropologists with whom Locke communicated during the latter's khipu research. In *The Ancient Quipu*, Locke acknowledged the assistance of eight scholars who helped him with his book, all of whom, except for Smith, were anthropologists. Apart from Stewart Culin, a self-taught expert on Korean and Chinese games, these anthropologists were specialists in the Americas who, for the most part, espoused a model of cultural evolution which placed Western civilization at the top of a developmental pyramid above other cultures. Four of these men – Thomas Athol Joyce at the British Museum, Charles Mead and Clark Wissler[3] at the American Museum of Natural History, and G.B. Gordon, a Mayan scholar at the University of Pennsylvania, facilitated Locke's access to the khipus for his research, and it is unclear whether they influenced his understanding of the cords to any appreciable degree. Marshall H. Seville, an archaeologist of Central America who considered Andean material culture to be inferior to that of ancient Mexico (see, for example, Seville 1904), helped Locke with the English translations of passages from the Spanish chroniclers. Bandelier was the sole scholar among those acknowledged who had actual field experience in Andean communities; his book, *The Islands of Titicaca and Koati* (1910), was the only extended ethnographic account of modern Andean culture in English available at the time. Acknowledging the importance of Bandelier's fieldwork, Locke repeatedly cited *The Islands of Titicaca and Koati* throughout *The Ancient Quipu*.

Bandelier practiced anthropology before the cultural relativism of Franz Boas came to dominate the discipline after World War II. The legacy of Boas, who rejected cultural evolution and championed a belief in the equality of cultures, remains the guiding ethos of American anthropology to this day. Bandelier, however, believed passionately in the inferiority of the indigenous peoples whom he investigated. He "disliked the Indians he studied", a later commentator has noted, "and was insensitive toward them, especially during his years in Peru and Bolivia" (Reyman 1996: 4). This same author refers to Bandelier's "racism and sometimes hatred of Indians (especially in South America)" and criticizes Bandelier's adherence to the theory of cultural evolution, which denigrates indigenous peoples as "barbarians" and "savages" (Reyman 1996: 5). As Randell Davis has explained, Bandelier "had been intellectually conditioned to perceive all American Indian groups as exemplifying a lower stage of sociocultural development than Westerners" (Davis 1995: 39-40).

Bandelier's racism and sense of superiority to his subjects pervade his ethnography of the Aymara Indians in the Lake Titicaca area. He referred to living Andean peoples as "primitives" and stated that the native men of Lake Titicaca "are of low stature and have sinister countenances" (Bandelier 1910: 67). He painted their character in the following light:

> "Mistrust is one of the leading traits of Aymara character... the Aymara Indian is as mistrustful of his own people as he is of a stranger" (p. 70).... As the Aymara Indian is naturally of a quarrelsome and rancorous disposition, squabbles in words

3 Clark Wissler is best known for the concept of culture areas. He was also a strong proponent of racial eugenics and advocated eugenic methods against the so-called "lesser races", such as Native Americans. He strongly criticised anthropologists who mistakenly – as he saw it --rated the qualities of "primitives ... much too high" (Ross 1985: 391).

> and deeds are not uncommon... stealing is diligently practiced. They are as dishonest towards each other as towards the owners of the Island" (pp. 87-88).

Bandelier defended the free labour that the native peoples were forced to provide to the landowners, and the unjust conditions under which they farmed the landowners' property and cared for his herds. As the anthropologist explained, "On the whole the proprietors of Titicaca treat their renters with a consideration akin to sacrifice of their own interests.... We had ample opportunity to convince ourselves of how much the Indians abuse the negligence of the owners, or rather their careless good nature; how little they did for the lands of the hacienda, and how the crops raised on them were stolen under the very eyes of the overseer" (p. 79). Bandelier appears to have been especially offended by Aymara ritual practices: "The orgies into which nearly all, if not all, the Indian dances degenerate are not the result of degradation and growing viciousness since the advent of the Spaniards, as is often pretended; they are ancient customs, in which the intemperance displayed takes the character of libations", he noted (Bandelier 1910: 108). He remembered the festivities at the shrine of Our Lady of Copacabana as "a torture on account of the truly infernal uproar" (p. 112). His depiction of the celebration continues: "... the instruments rumbling, thundering, rattling, screeching, howling, and screaming, without any regard of rhythm or harmony; hundreds of ugly voices singing monotonous melodies... the scene is ... so deafening, so utterly devoid of the slightest redeeming feature, that it forms one of the weirdest and, at the same time, most sickening displays imaginable ... the Aymara dances ... all degenerate into an orgy" (p. 114).

Bandelier's contempt for Andean Indians extended to his disdain for their alleged "carelessness" toward any kind of tool or material object:

> "Still the aborigine yet grasps a stone in preference to a hammer, and he ties in preference to nailing. He steals modern tools as diligently as he can, and no nail is safe from him, no end of rope or leather strap, even if they belong to a parcel or to a saddle and their removal endangers the safety of parcel or rider. But after he acquires such civilized implements and auxiliaries, he does not take care of them... This carelessness is exhibited toward everything. The Indian puts on a new shirt and wears it day and night until it is a disgusting rag..." (p. 77).

Unfortunately, Bandelier's racist views, which were cloaked with the guise of "science", coloured Locke's perspective on the khipus and knotted cords used by modern Andean peoples. Bandelier had observed the contemporary Aymara use of khipus but dismissed the cords as overly "simple" (p. 89). Likewise, Locke argued that the modern khipu was a "degeneration of the ancient one", nothing more than "primitive method of accounting" that revealed the "backward state of the present day Indian" (p. 111):

> "The modern 'quipu' still used in some Andean valleys by Indian herders, is but a degeneration of the ancient one, and serves them to keep a record of the sheep in their charge. It is coloured sometimes, but the colour has no meaning and depends upon the fancy of the maker, solely, and the significance of the quipus is found in the knots, the disposition of which indicate units, tens, hundreds, and thousands. Really, this modern quipu is a manifestation of the backward state of the present day Indian, who for lack of proper book learning, is obliged to revert to this primitive method of accounting" (Locke 1923: 111).

Locke's measured tones are mild compared to Bandelier's lurid accounts of alleged Aymara "orgies" and "carelessness" toward all manner of tools and apparel. Locke ascribed the alleged primitiveness of modern khipus to a lack of education, rather than to any innate deficiencies in the Aymara character. Nonetheless, he accepted Bandelier's evolutionary principle that some peoples are more "civilized" than others and dismissed Uhle's belief that contemporary khipus embodied sophisticated aspects of earlier khipu traditions. Of course, since Locke believed that Inka khipus only contained numerical data -- which he already had decoded – it is not clear whether he thought that any additional progress in decipherment was possible or relevant.

The German anthropologist, Max Uhle (1856 – 1944), held contemporary khipus in a much higher estimation than did either Locke or Bandelier (Hyland 2014; Loza 2001; Medrano 2021; Uhle 1895; Uhle 1978). During two separate expeditions to the Lake Titicaca region of Bolivia in 1894 and 1895, Uhle collected ethnographic khipus from Aymara herders. A skilled philologist, Uhle interviewed the men who created the khipus, and kept field notes that carefully recorded their explanations of the cords. In 1895 Uhle published a description of a herding khipu from the Cutusuma hacienda in which he proposed that modern khipus represented a continuity with Inka khipus and could provide insights into more ancient cords (Hyland 2014; Uhle 1895). Uhle based his insights on direct interviews with the khipu creators, rather than relying on the accounts given by the hacienda owners who, in order to justify the exploitation of their Aymara peons, may have had a material interest in denigrating indigenous culture. Uhle's field research was so meticulous that, by comparing his notes to the actual Cutusuma khipu, I was able to identify two additional semiotic elements – ply and knot direction – that had been utilized by the Cutusuma herder to indicate gender and milking status, but that had escaped Uhle's notice (Hyland 2014).

Uhle's interpretation of the relationship between ancient and modern khipus was rejected by Locke, who discussed the Cutusuma khipu in *The Ancient Quipu*, and by most other scholars of his day (see, for example, Guimaraes 1978). It would not be until the latter half of the 20th century that interest in modern khipus would reawaken when scholars such as Carol Mackey, Arturo Ruíz, Carrie Brezine, and Frank Salomon would carry Uhle's vision forward, enlarging our understanding of how Andean peoples continued to create and curate traditional khipus in the 20th and 21st centuries. Locke did not live to see the revival of interest in ethnographic khipus; unfortunately, his source of information on modern Andeans was Bandelier, whose racism was remarkable even for his time. Yet Locke's brilliant work enabled him to decipher the basic features of the khipu numerical system, laying the foundation for all subsequent khipu research.

Non-Lockean Khipu Knots and the Example of "Nether Knots"

Since the publication of *The Ancient Quipu* in 1923, scholars have commented on the existence of various kinds of non-Lockean knots on Inka khipus (e.g. Pereyra Sánchez 2001: 122). Most recently, Kylie Quave detailed the anomalies on nineteen Inka khipus in the collections of the Dallas Museum of Art and the Michael C. Carlos Museum at Emory University (Quave 2009). After finding numerous irregularities in this small sample, Quave urged scholars to study the existence of khipu anomalies which, she argued, may represent "intentionally coded narrative elements that fall somewhere between being mnemonic and being understood regionally" (Quave 2009: 243). She proposed that so-called "anomalies" could be "socially meaningful" (Quave 2009: 248), rather than individual idiosyncrasies, and therefore may be crucial to expanding our ability to interpret Inka khipus. Her article on "deviation and innovation" in khipus highlights the limitations of our current understanding of the ancient cords.

The non-canonical knot variations she found include alterations "in color within or just outside of a knot" (Quave 2009: 245). Figure 1.3 provides an example of this from a khipu in the Museum of Peoples and Cultures in Provo, Utah: a colour change from brown to cream, brown, and black in a long knot. If colour indicates the items enumerated on a pendant, what could be the meaning of a shift in hue within a knot?

Figure 3. Long knot colour change – brown to cream, brown, and black (1978.7.7.1, Museum of People and Cultures, Brigham Young University). Photo by the author.

Other khipu scholars have described additional types of non-Lockean knots on Inka khipus. Anthropologist Erland Nordenskiöld recorded a khipu in the Munich Ethnological Museum (no. 3304, also known as KH0100) containing long knots with 10, 11, 12, or 13 twists; such knots, which have been detected on other khipus as well (such as KH0102), are impossible to read according to the Lockean knot typology (Nordenskiöld 1925: 17). Mathematician Marcia Ascher has published on two khipus from the archaeological site of Pachacamac (KH0114 and KH0115) whose primary cords were tied together in antiquity and whose pendants are knotted entirely with overhand knots (Ascher 2002). She has suggested that these and similar "irregular" khipus may represent labels, identity codes, and other types of numerically coded information, instead of numerical quantities (Ascher 2002: 99-101).

Khipu pendants may also contain unusual knots that do not conform to the tripartite overhand, long, and figure-8 classification. This can be seen, for example, in a khipu, KH0449 (VA16141 b), from an archaeological site on the southern Peruvian coast near Ica. On this khipu, which was tied in antiquity to another khipu with normal Lockean knots, all the pendant knots are bulbous constructions that have been spliced into the pendant cord (Figure 1.4). The "bulb" knots come in different colours, and several may be clustered together on a pendant. Although the bulb knots have been treated as if each had a one-digit decimal value dependent upon its position on the pendant (UR211), their significance is unknown.

Figure 4. "Bulb" knot on KH0449 (VA16141), Berlin Museum of Ethnology. Photo by the author.

Based on her study of 122 khipus believed to date to the Inka era, archaeologist Carol Mackey identified various ways in which khipu knots could diverge from the Lockean knot typology (Mackey 1970: 48-55). Mackey observed five kinds of non-Lockean knots in her survey, beginning with knots located on the primary cord (Mackey 1970: 118). For example, a khipu in her sample from an Inka burial in the coastal Peruvian cemetery of Soniche, has long knots along the primary cord.

Many of the non-Lockean knots, Mackey explained, violate the rules for knot position and type, making it impossible to determine their numerical value. These unreadable knots included: (1) khipus where the pendants have only long and figure 8 knots (see Figure 1.5); (2) khipus where the overhand and long knots are mixed in their positions, with overhand knots in the unit's position, and long knots in the higher decimal zones; and (3) khipus which contain only overhand knots (like the Pachacamac khipus described by Ascher). Mackey speculated that the latter functioned as simple tallies, but she provided no suggestions for how to read the other three kinds of non-Lockean knots.

Finally, Mackey identified a type of non-Lockean knot that I have referred to as a "nether knot" because of its location on the nethermost part of a pendant, below the unit's position. In general, a nether knot is a unit knot (either a long knot or a figure-8 knot) that is located underneath another unit knot; according to Locke's knot typology, "nether knots" should not exist. However, this feature, as Mackey stated, is "rather common" (Mackey 1970, 48), especially when a figure-8 knot is found below a long knot on the same pendant. Figure 1.6 shows an Inka khipu with a "nether knot" – a long knot underneath another long knot on the leftmost pendant in the photo. In the image one can see how the unit positions form an imaginary horizontal line across the five pendants; the nether knot occurs below this line.

"Nether knots" are found in khipus that otherwise conform to the Lockean knot system. Usually nether knots are in the single digits, but sometimes nether knots with greater values occur. For example, an Inca era khipu at St Andrews Univeristy, SH002, has a nether knot with a value of "67". Mackey believed that these extra unit knots resulted when the khipukamayoq realised that they had miscounted and needed to add more to the pendant's value. As she wrote, an explanation for this variant "would be that one or several items were overlooked in the count and, rather than untie the unit knot, another

Figure 5. Close up of a khipu consisting entirely of long and figure-8 knots. This khipu has a probable C14 date of between 1365 and 1384 AD. Photo by the author.

unit knot was added below" (Mackey 1970, 49). Therefore, when Mackey calculated the value encoded by the knots on a khipu pendant, she added the nether knot to the total. Most khipu scholars, such as Hugo Pereyra and Gary Urton, have followed Mackey's lead by adding nether knots to the pendant total. Quave, however, has questioned the practice of adding nether knots to the pendant count. In the notes to her khipu descriptions (available online through Ashok Khosla's Khipu Field Guide), Quave expresses uncertainty about how to deal with nether knots. For example, on khipu KH0600 (QU013) in the Dallas Art Museum, Pendant 92 contains a nether knot; in her notes, Quave asks "is this numerical?". On khipu KH0598 (QU011), Pendant 5 has a nether knot – that is, a figure 8 knot below a long knot with six turns. Quave subtracts the nether knot from the pendant total instead of adding it, while commenting in her notes, "values are only suggested". The Peruvian khipu scholar, Alejo Rojas, has refrained from providing any numerical totals for pendants with nether knots, simply stating that the value of such pendants is unknown.

It is beyond the scope of this article to speculate about the significance of all the different kinds of non-Lockean knots described in the preceding pages. However, there exists direct ethnographic evidence to explain the reason for nether knots on 20th century khipus from the central Peruvian highlands. The testimony of two Andean khipu experts, Mariano Pumajulka and his grandson Mecias Pumajulka, clarifies the meaning of nether knots in the early 20th century. Might their explanations be applicable to Inka khipus? Although khipus undoubtedly changed in the centuries after the Spanish invasion of the Inka Empire, ethnographic analogies can provide us with testable hypotheses for Inka khipus, in this case concerning the significance of nether knots.

Over 20% of the khipus in the Open Khipu Repository (OKR), the largest khipu database in the world, contain

Figure 6. A "nether knot" on Pendant 75 of a khipu C14 dated to between 1475 +/-26 years. (SA002, University of St Andrews). Photo by the author.

one or more pendants or subsidiaries with a nether knot. The magnitude of this issue is therefore significant. If nether knots are incorrectly interpreted, they have the potential to negatively impact attempts to analyse khipu data using statistical methodologies. The frequency of nether knots also calls into question the idea that they represent the rectification of errors. Modern khipu experts, such as Mama Licuna and Taytay Hilario, practice great care and deliberation when knotting their khipus (Hyland, Lee, and Aldave Palacios 2021). It seems questionable to assume that more than one out of every five Inka khipus contains corrected errors.

In November 2015, I interviewed Mecias Pumajulka, an elderly man from the Andean village of Santiago de Anchucaya, who remembered when his community still used khipus to record the contributions of each individual to the community (Hyland 2016; Hyland 2020). When Mecias was young, each kinship group, known as an *ayllu* or "*parcialidad*", in Anchucaya maintained khipus to document how every adult member of the ayllu fulfilled their communal obligations. Mecias had joined his ayllu, *Suni-sika*, as an adult member at the age of fourteen, just as ayllu khipu records were being replaced by notebooks. He recalled that the ayllu level khipus noted an individual's obligation (such as trips to transport goods for communal projects or donations of agave rope for public works) by knots in the upper portion of the khipu pendants, close to the primary cord, while knots on the lower portion of pendant indicated what the person still owed; that is, the remaining part of their contribution that they had failed to carry out. At the yearly accounting ceremony for the ayllu, members were assessed fines for the contributions that they owed. If they paid the fine, the knots were untied, making the cords "clean" ("*limpia*"), but if they did not pay, the khipu remained as it was, with a lower register of numbers to denote their debt. Subtracting the lower knots (what remained to be done) from the higher knots (the total obligation) would provide the numerical amount of what actually had been contributed.

Eighty years earlier, Mecias' grandfather, Mariano Pumajulka, had told anthropologist Julio C. Tello how the community of Anchucaya made khipus to assess each ayllu's contribution to communal works. Tello hailed originally from the highland village of Huarochirí, close to Anchucaya, and welcomed Pumajulka when the latter arrived to visit Tello's home in Miraflores, a suburb of Lima. Tello's transcriptions of Pumajulka's testimony includes a description of "nether knots" on the khipus utilized to record each ayllu's contributions to the village. Pumajulka explained how the village level khipus -- like khipus for the individual ayllus described above -- had two sets of numbers on each pendant. The knots on the top portion of the pendant closest to the primary cord "registered the quantities that correspond to the work that was done and the nether [part] the quantities corresponding to work that was not carried out; ... what was done [was] on the upper half of the pendant in the part closest to the main cord, and what was owed on the other part [of the pendant], further away"[4]. There would be no practical reason to add the upper and lower numbers on each pendant together, since the mandated obligations for the year already were recorded on another khipu, created during the accounting ceremony (the "*watancha*") held the previous year. The upper and nether knots on the pendant represent distinct quantities: what actually was contributed by an ayllu toward a particular obligation, and what still was owed by the ayllu, respectively. This is slightly different from the ayllu khipus, where a pendant's upper knots recorded the person's total obligation, rather than what they had contributed. By Mecias's youth, the different ayllus in the village no longer came together annually to assess each ayllu's contributions and, therefore, the village level khipus were no longer made. Only the ayllu level khipus remained when Mecias was a teenager, and they too were soon abandoned.

In another community in the Huarochirí region, San Andrés de Tupicocha, ancestral khipus play a central role in the investiture of new ayllu leaders, who wear the khipus as a sign of office, as Salomon has described (Salomon 2004). During her 2022 fieldwork in Tupicocha, anthropologist Lucrezia Milillo observed that the khipu worn by the leader of the parcialidad Primer Allauca contains a nether knot similar to those of nearby Anchucaya (personal communication, 28 August 2023). It is believed that the khipus in Tupicocha share the same logic and counting methods with khipus in neighbouring Anchucaya.

In 20th century khipus from the Huarochirí region, nether knots are deliberate. They do not represent the correction of a mistake in counting. While there exists a relationship between the upper and lower knots, adding the two numbers together is rarely meaningful. Subtraction, however, can reveal relevant quantities on khipus with nether knots like those found in ayllu level khipus, where upper knots encode the total obligation while nether knots signify what was left undone. In instances like this, subtraction provides a useful figure, revealing what was contributed in actuality, whether it was how many trips were made to gather building supplies, or how many coils of handmade agave rope were donated for re-roofing the church, etc.

How can we determine whether this ethnographic analogy is applicable to nether knots on Inka khipus? Do the nether knots on Inka khipus represent deliberate and distinct numbers, rather than rectifications of mistakes? If so, were these Inka nether numbers frequently subtracted from the main knots on a pendant, as was the case for the ayllu level khipus in Anchucaya? As Mackey noted, and as a review of khipus in the OKR confirms, the value of the nether knot is less than the value of the upper knots in the vast majority of cases. This observation is congruent with the hypothesis that nether knots were often subtracted from, rather than added to, the main number, but it is not proof. To demonstrate the validity of applying the 20th century meaning of nether knots to more ancient cords, we would need to find a khipu like the one from Huando that confirmed Locke's reading of the knots. That is, we must locate an Inka era khipu with totalizing top cords and nether knots, in which the sum of the pendants matches the totalizing cord only *IF* the nether knots are subtracted from the main knots on the pendants. Given the rarity of khipus with totalizing top cords, it seemed highly unlikely that we could discover such a khipu.

Yet the Inka graveyard on the Huando estate, the same cemetery that yielded the khipu that confirmed Locke's reading of khipu knots, provides a khipu that meets these criteria. Khipu KH0424 (B/8707) has eight groups of pendants along the primary cord, along with totalizing

4 "se registraban las cantidades que corresponden a los trabajos realizados y en la inferior las cantidades correspondientes a los trabajos no realizados ... el haber en la mitad superior del cordon en la parte que se aproxima al cordon matriz; y el debe en la otra parte mas alejada". Archivo Tello, Paquete "Kipu de Anchucaya".

cords. The sum of the first group of six pendants is 596, which equals the number knotted onto its totalizing cord: 596. There are no nether knots on the first group of pendants, but this group establishes the relationship of equivalence between pendant sums and totalizing cords for this khipu. The next four groups of pendants are badly deteriorated, with many broken pendants, so we are unable to read the numbers on the cords. The pendants in the sixth group are complete and their sum – 75 – equals the number knotted onto their totalizing cord: 75. Again, there are no nether knots in this group, but the match between the pendant sum and the totalizing cord confirms the pattern for this khipu.

The totalizing cord for the seventh group is broken, but the totalizing string for the eighth group is complete. In the eighth group of six pendants, there are four pendants with nether knots. Urton, who described this khipu, followed the normal practice of adding the nether knots to the pendants. The resulting sum of the pendants, 109, does *NOT* match the totalizing cord, which is 89. However, if we subtract the nether knots, along with the subsidiary cord on the final pendant in the group, the sum of the pendants is 89.[5]

Pendant group 8, KH0424

P1: 22 – 1 = 21
P2: 15 – 1 = 14
P3: 30
P4: 17 – 6 = 11
P5: 6
P6: 10 – 1 = 9; 9 – 2 (subsidiary) = 7
21+14+30+11+6+7 = **89**

In other words, subtracting the nether knots, as suggested by ethnographic analogy with the modern khipus of the Central Andes, results in a pendant sum – 89 – that is identical to the number – 89 -- on the totalizing cord.

It appears that subtracting the nether knots can lead to the correct values for the seventh group of pendants as well, although it is impossible to determine this for certain because the totalizing cord is broken, and its number is incomplete. The totalizing cord reads one hundred and ten, but the final digit is missing. The sum of the pendants therefore could equal any number between 110 and 119. In Urton's interpretation of the pendants, he adds the nether knots to the upper knots on each pendant, resulting in the sum of 122, which is greater than any potential number on the totalizing cord. However, if we subtract the nether knots, the sum equals 119, which is a possible value for the totalizing cord.

Pendant group 7, KH0424[6]

P1: 27 – 2 = 25
P2: 18 – 2 = 16
P3: 40 – 2 = 38
P4: 25 – 4 = 21
P5: 0
P6: 20 – 1 = 19
25+16+38+21+0+19 = **119**

For KH0424, adding the value of the nether knots to the pendant value – the default practice among many khipu scholars – results in a sum that is higher than the value of the totalizing cord. Subtracting the nether knots, as suggested by the ethnographic analogy with modern Central Andean khipus, leads to a pendant sum that exactly matches the totalizing cord for group 8, and is a possible match for group 7. At the very least, the example of this khipu from Huando should compel khipu scholars to question whether they should automatically add the nether knots to the upper knots on a pendant. It would, perhaps, be better to keep the nether knots as a separate count from the upper knots, under the assumption that each set of knots may indicate a distinct item or quantity. KH0424 also reveals the continuity between ancient khipus and modern ones. Having two registers for distinct numerical quantities on the same pendant is similar to the accounting zones on modern single cord herding khipus (see Hyland and Lee 2021; Mackey 2002).

It generally is accepted that the khipus from Huando date from the time of the Inka Empire, although they have not been radiocarbon dated and little is known about their archaeological provenance other than that they were found in an Inka burial site. Relatively few khipus have either radiocarbon dates and/or an established

5 A description of khipu B/8707 in Locke's *Supplementary Notes* of 1928 indicates that there is a subsidiary containing a long knot with two twists on the sixth pendant in this group. This subsidiary had fallen off by the time Urton examined this khipu decades later. On Pendant 6 in this group, there is a loop with no numerical value in the one's position, above the nether knot. This loop apparently serves as an empty place holder.

6 This reading is based on my own interpretations of the pendant knots, which disagrees in some respects from Urton's.

provenance. As more khipus are radiocarbon dated, we see evidence of non-canonical features of the kinds described by Mackey and Quave, such as nether knots, on Inka era khipus. For example, there are multiple nether knots on the khipu in Figure 1.6 (SA002), whose animal fibre cords were dated to 1475 AD (+/- 26 years), which falls unequivocally in the Inka period.

A sceptical reader may cite the "old wood" problem in the American Southwest, in which lumber was re-used over centuries (see Bowman 1990), to question whether an Inka C14 date actually means that the khipu was made during the Inka Empire. However, unwoven animal fibres are a great deal more fragile than the re-purposed old growth timbre of the American Southwest. If unwoven animal fibres are handled frequently, they either wear out or felt -- that is, the fibres irreversibly interlock and become matted. There is no evidence of felting on the khipu in question, nor that the yarn was handled prior to being fashioned into a khipu. Moreover, the re-use of materials like wood in the American Southwest is driven by scarcity. Both animal fibres and cotton were extremely plentiful in the Inka Empire, so there would be no concern about rarity to motivate the re-use of these breakable substances. Finally, although trees may live for centuries, animals like alpacas and llamas and crops like cotton are relatively short-lived, so a C14 date from their lifespan would be relatively close to the time that their fibre was used to make a khipu. Although khipus may have been made from fibres that had been in storage for several years, there is no reason to suppose that they were created from yarn that was used previously or was centuries old.

The Late Horizon khipu SA002 lacks a totalizing cord that would allow us to confirm that its nether knots should be subtracted from the upper pendant numbers. For example, on Pendant 75, the main knots read "23", with a long knot containing 4 twists tied underneath. Should this pendant be listed as "19" or as "23" and "4"? (Figure 1.7). Perhaps the nether knot for "4" might indicate the amount of grain that must be set aside for the

Figure 7. Pendants 72 – 79 on Khipu SA002, University of St Andrews. On Pendant 75, there is a nether knot with 4 twists below a long knot with 3 twists in the unit position on the pendant. Photo by author.

next year's seed, or it might signify a reduction of an individual's obligations due to illness or infirmity. While we cannot yet say with certainty what the nether knot signifies, it is probably best to describe this pendant as encoding two separate numbers: "23" and "4", while recognizing that "19" is the most likely value. If khipu scholars follow Quave's suggestion to systematically analyse the apparent anomalies that are present in Inka khipus, we may well discover patterns that enable us to better understand such seeming irregularities. Perhaps we may even find clues to deciphering how khipus encoded historical narratives, biographies, and other "bizarre stuff", as Locke termed it.

Locke was a brilliant mathematician whose decipherment of the khipu numerical system grew out of his comparative research on abacuses and modern calculating machines. As khipu studies have matured in the century since the publication of *The Ancient Quipu*, it is time to acknowledge that khipu knots are more sophisticated and varied that he knew. In the absence of a "Rosetta Rope" that could be used to fully decipher pre-Columbian khipus, the example of the nether knots reveals how ethnographic analogies can provide a way forward, enabling us to develop possible models for how non-Lockean features functioned within khipu systems. Andean peoples created khipus for many centuries prior to the Inkas, and for over five hundred years after the fall of Tawantinsuyu. Hidden in the secluded mountain valleys of the Andes may lie answers to many of our questions about khipus.

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