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REPORT

WILEY

Use them or lose them: Are manipulatives needed to assess numeracy and geometry performance in preschool?

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Abstract
In two studies, we investigated whether using three-dimensional (3D) manipulatives during assessment aided performance on a variety of preschool mathematics tasks compared to pictorial representations. On measures of children's understanding of counting and cardinality ($n = 103$), there was no difference in performance between manipulatives and pictures, with Bayes factors suggesting moderate evidence in favor of the null hypothesis. On a measure of children's shape identification ($n = 93$), there was no difference in performance between objects and pictures, with Bayes factors suggesting moderate evidence in favor of the null hypothesis. These results suggest flexibility in the materials that can be used during assessment. Pictures, or 2D renderings of 3D objects, which can be easily printed and reproduced, may be sufficient for assessing counting and shape knowledge without the need for more cumbersome concrete manipulatives.

KEYWORDS
assessment, geometry, manipulatives, mathematics, numeracy, preschool

1 | INTRODUCTION

Teachers frequently use manipulatives to promote mathematical understanding (Bryan et al., 2007; Uribe-Flórez & Wilkins, 2010). Concrete materials help ground learning in a more familiar representation, instead of requiring children to rely on symbolic notation (Sarama & Clements, 2009, 2016). Research on embodied cognition suggests actions may be particularly helpful because they constrain attention and support the development of more complex understanding (Barsalou, 2003; Martin & Schwartz, 2005; Weisberg & Newcombe, 2017). For example, when manipulating blocks as part of a guided mathematics lesson, manipulatives can focus the learner's attention to the mathematical relations between the sets (Moyer-Packenham & Westenskow, 2013, 2016). However, manipulatives can be distracting, and require extra time and resources for teachers to introduce the materials (Burns, 1996; McNeil & Jarvin, 2007; Petersen, 2013). During assessment, children typically do not have extended interaction with manipulatives, which is important for supporting the representations of the material (Barsalou, 2003; Martin & Schwartz, 2005; Weisberg & Newcombe, 2017). Thus, in some contexts the costs of manipulatives (both in terms of time and resources) may outweigh the benefits. For example, if manipulatives become more beneficial after extended exposure and practice in an educational setting, then using a novel set of manipulatives during assessment may not aid in measuring their knowledge. To test whether manipulatives are needed to assess children's early mathematical understanding, we investigated whether using physical objects (vs. pictures) influenced *performance* on measures of preschool children's mathematical skills (counting, cardinality, and shape identification).

1.1 | Manipulatives in the classroom

It is important to consider the use of manipulatives during mathematics activities in two broad categories: assessment and instruction. The benefit of using physical objects in mathematics *instruction* has long been discussed within early education (Montessori, 1917; Piaget, 1970). Meta-analytic work suggests that manipulatives are broadly effective during instruction (Carbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013); however, manipulatives also have drawbacks that can limit their effectiveness (Carbonneau et al., 2013). Teacher guides emphasize that teachers carefully plan how to introduce manipulatives to prevent children from using them in non-mathematical ways (Burns, 1996; Laski et al., 2015). To effectively teach with manipulatives, educators should use them over a long period of time, carefully choose materials that will not be distracting, and be explicit during instruction on how the manipulatives relate to the mathematical concept being demonstrated (Laski et al., 2015).

The belief of the benefits of manipulatives are also reflected in the Common Core Standards, which can provide guidance on what children's understanding should look like during assessment (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The standards for Kindergarten mathematical understanding refer to children's ability to use objects to show an understanding of counting and cardinality (CCSS.MATH.CONTENT.K.CC.B.4.A; CCSS.MATH.CONTENT.K.CC.B.4.B; CCSS.MATH.CONTENT.K.CC.B.5) or to compare numbers (CCSS.MATH.CONTENT.K.CC.C.6). However, other areas of mathematical understanding such as operations and algebraic thinking (CCSS.MATH.CONTENT.K.OA.A.2; CCSS.MATH.CONTENT.K.OA.A.3; CCSS.MATH.CONTENT.K.OA.A.4) or numbers and operations in base-ten (CCSS.MATH.CONTENT.K.NBT.A.1), the Standards suggest children may show their understanding using pictorial representations instead of objects. For geometry understanding, such as shape identification, the standards make no mention of whether children should use pictorial representations or physical objects (CCSS.MATH.CONTENT.K.G.A.2). Comparing across these areas, it would suggest that using objects is particularly important for counting and cardinality but less so for other areas of mathematical understanding.

1.2 | Manipulatives during assessment

Early mathematical assessments have incorporated objects into the assessment process (Clements et al., 2008; Ginsburg & Baroody, 2003; Griffin, 2000; Jordan et al., 2010; Weiland et al., 2012), with assessors providing manipulatives for some questions (e.g., questions about cardinality or arithmetic) but not others (e.g., story problems or verbal counting prompts). For these assessments children are provided with manipulatives during early trials to help them represent the sets being asked about instead of needing to form an internal representation (Griffin, 2000).

Given the importance of assessment in guiding instruction (Lee & Ginsburg, 2009; Raudenbush et al., 2020), it is crucial that decisions regarding when to use manipulatives during assessment are based on research findings. However, any benefits of using manipulatives are likely related to both the concept being targeted and the specific task being used. In studies comparing children's performance when counting objects versus counting actions or sounds, two-to three-year-old children performed better when they used the physical objects, likely because the objects provided a persistent representation of the set (Schaeffer et al., 1974; Wynn, 1990). For children's geometry understanding, children sometimes perform better with three-dimensional (3D) manipulatives (Stevenson & McBee, 1958) though in some cases children show similar performance when using pictures as they do with 3D objects (Sowell, 1989). Thus, it is not always clear whether 3D objects during assessment are needed to accurately measure children's knowledge, or whether pictorial representations are sufficient. This is important given that assessments should be as simple to assemble and administer as possible so that teachers can easily use them to measure student learning and plan course content accordingly (Purpura & Lonigan, 2015; Weiland et al., 2012).

2 | CURRENT STUDY

Although meta-analytic work has shown an advantage to using manipulatives during instruction (Carbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013), it is unclear whether these advantages translate to assessment, where the goal is often to quickly and accurately measure children's understanding. To investigate the role of manipulatives in performance during mathematics assessment, we performed analyses on two different datasets that included measures of counting and cardinality or shape identification. These concepts are of particular interest given that the Common Core Standards make explicit reference to using physical objects for counting and cardinality and no specific reference to the format of assessment for shape identification. The data from each of these studies come from two different datasets investigating children's early academic skills. As part of the larger data collection, each study included a measure that used both pictorial representations and physical stimuli, allowing for a within-child comparison of the effects of manipulatives on children's performance. In the first study, children ($n = 103$) completed a one-to-one correspondence measure as well as a measure of their understanding of the cardinality principle. In the second study, children ($n = 93$) completed a shape identification task in which they were shown a shape and had to provide the correct name. In both studies, the 2D and 3D materials were presented in the same way (e.g., in the counting task, the materials for a set of eight items were presented in one line for both 2D and 3D trials), with only the materials themselves differing across presentation format.

For each of the tasks, we investigated whether the presentation format (pictorial representation or physical object) influenced performance. Previous work with preschool and early elementary aged children has suggested some benefit of manipulatives, especially compared to stimuli that did not remain visible (Schaeffer et al., 1974; Wynn, 1990) while other studies have reported null effects for the influence of physical objects on performance (Petersen, 2013; Sowell, 1989). These competing findings suggest support for the null hypothesis. To test for the likelihood that there is no effect, we conducted a series of Bayesian analyses comparing children's performance on each task variation.

3 | STUDY 1: COUNTING AND CARDINALITY

3.1 | Participants

The first set of analyses included data from a larger study investigating early numeracy skills. One hundred three children ($M_{age} = 4.33$, $SD = 0.63$; 54.4% female, 45.6% male; 47.6% White, 38.8% Black, and 13.6% other race or ethnicity) were assessed individually by an experimenter in a quiet area of their school for counting and cardinality tasks.

3.2 | Measures

The measures of interest for the current questions were selected from a broad set of early mathematics measures. These tasks included items broadly measuring children's counting skills including verbal counting (i.e., how high can you count), subitizing, estimation, counting on from a given number or counting back from a given number, counting subsets, and questions about cardinality. Because we were interested in the role of manipulatives in children's performance, we focused on tasks that had both 2D and 3D stimuli, which were the counting and cardinality tasks. Items were completed in a fixed order, with the counting and cardinality items coming after children were asked to identify whether the experimenter made counting errors while counting sets of 5 or 10 items (Clements & Sarama, 2007; Purpura & Lonigan, 2013). Because of the fixed order of the administration, all children would have been similarly influenced by previous items in the measure, allowing for a direct investigation of within-child differences on the counting and cardinality items. Children were not given feedback on their responses.

3.2.1 | Counting

This task assessed the children's understanding of one-to-one correspondence. Children were shown a set of linearly arranged items and asked to count the set. For each trial type, children were asked about sets of 3, 4, 8, 16 and 20 items. Each child completed trials for each set size across three different presentations: black dots, a picture of a toy car, or a physical set of blocks (see example images of the materials in the Supplementary Materials). All three presentation types were presented in a similar way with quantities presented in one line (for 3, 4, and 8) or two (for 16 and 20). Children completed trials in a fixed order, first completing the trials with dots, then pictures, and then blocks. For the blocks, children were presented with all the blocks for a trial at once in the same arrangement that the images were presented in (e.g., a set of eight presented in one line). Children were allowed to touch or move the blocks as they wished but the experimenter did not specifically direct them to manipulate the blocks. Children were given the same prompt as they were for the dots and pictures. Children received credit for a given trial if they were able to correctly tag each item once while following the correct counting sequence. A total score ranging from 0 to 5 was computed for each presentation type (each format showed similar levels of reliability; $\alpha = 0.71$ for dots; $\alpha = 0.70$ for pictures of toy cars; $\alpha = 0.73$ for blocks).

3.2.2 | Cardinality

After each trial of the one-to-one correspondence measure, children were asked to identify the total number of items in the set. The set remained visible while children were asked to respond to the cardinality question. Note that a correct response required the child to provide a cardinal label for the set that matched the last word of the count, regardless of if the count was correct. For example, if a child counted a set of three as 1, 2, 3, 4 and said 'four' in response to the cardinality question, the score for counting would be incorrect, but the cardinality item would be counted as correct because their response matched the last word of the count sequence—indicating that they understood the last word of the count is an appropriate response to questions about cardinality (Fuson & Hall, 1988). Children completed the same set sizes as the one-to-one correspondence measure (3, 4, 8, 16, and 20) and did so for the three different presentation types (i.e., dots, a picture of a toy car, and 3D blocks). All three presentation types were presented in a similar way with quantities presented in one line (for 3, 4, and 8) or two (for 16 and 20). The items remained visible while the how many? question was asked so as not to tax the children's memory skills. A total score ranging from 0 to 5 was computed for each presentation type (each format showed similar levels of reliability; $\alpha = 0.72$ for dots; $\alpha = 0.82$ for pictures of toy cars; $\alpha = 0.82$ for blocks).

3.3 | Analytic procedure

Because our question of interest for each task involved the possibility that the null is true (i.e., that there is no difference between presentation type), we conducted Bayesian *t*-tests to determine the likelihood of this hypothesis.[1] Under the null hypothesis we expect an effect size of 0. Thus, we define H_0: $\delta = 0$. The alternative hypothesis is two-sided, H_1: $\delta \neq 0$, and we assumed that δ was distributed as a Cauchy distribution with scale $r = 0.707$ (following Rouder et al., 2012).

4 | RESULTS

Table 1 provides the descriptive statistics and zero-order correlations for the counting and cardinality tasks for each of the presentation types. Each of the presentation formats was positively and significantly related to their performance on the other task format types. Figure 1 shows the number of correct responses for each of the presentation types for each task.

> **RQ1.** Does presentation format influence performance on a counting (one-to-one correspondence) measure?

For the one-to-one counting measure, we found a Bayes factor of $BF_{01} = 8.53$ when comparing performance on dots versus objects and a Bayes factor of $BF_{01} = 3.79$ when comparing performance on pictures versus objects, which means that the observed data are several times more likely under H_0 than H_1 (see Supplementary Materials analyses comparing presentation format for each set size). When comparing the 2D

TABLE 1 Descriptive statistics and correlations for the counting and cardinality tasks.

	M (*SD*)	1	2	3	4	5	6
Counting							
1. Dot	2.71 (1.45)	—					
2. Picture	2.80 (1.39)	0.75	—				
3. Object	2.67 (1.46)	0.75	0.78	—			
Cardinality							
4. Dot	2.89 (1.79)	0.65	0.64	0.62	—		
5. Picture	2.92 (1.77)	0.60	0.75	0.72	0.82	—	
6. Object	2.91 (1.82)	0.63	0.71	0.75	0.82	0.88	—

Note: All correlations are significant at $p < 0.001$. Each task was scored out of a possible five.

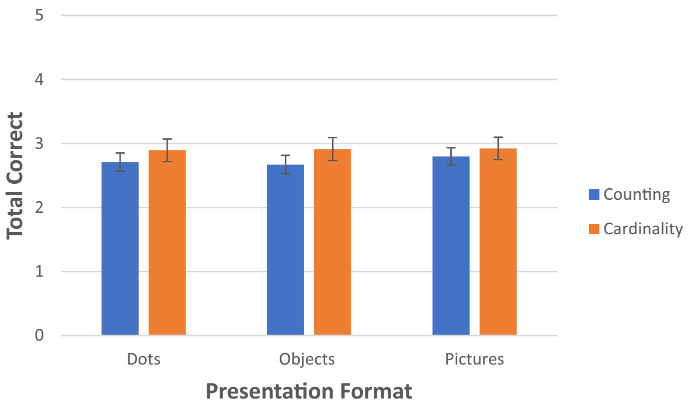

FIGURE 1 Total number of correct responses (out of five) for each presentation type. Error bars reflect ±1 SE.

formats (dots vs. pictures) to each other, we found a Bayes factor of $BF_{01} = 6.37$. Bayes factors between 3 and 10 can be interpreted as moderate evidence in favour of the null hypothesis (Jeffreys, 1961), suggesting that children's performance on the one-to-one correspondence measures was not influenced by whether or not items were 3D physical manipulatives.

RQ2. Does presentation format influence performance on a cardinality measure?

For the cardinality measure, we found a Bayes factor of $BF_{01} = 9.01$ when comparing dot arrays to the objects and a Bayes factor of $BF_{01} = 9.11$ when comparing the pictures to the objects, which means the data are several times more likely under H_0 than H_1 (see Supplementary Materials analyses comparing presentation format for each set size). When comparing the 2D formats (dots vs. pictures) to each other we found a Bayes factor of $BF_{01} = 8.83$. Because children may be able to identify the cardinality of smaller set sizes (up to 4 items) without needing to count, we also investigated whether the pattern of performance for each format differed for small set sizes (3 and 4) compared with larger set sizes (8, 16, and 20). For the small set sizes, we found a Bayes factor of $BF_{01} = 8.63$ when comparing dot arrays to the objects and a Bayes factor of $BF_{01} = 4.49$ when comparing the pictures to the objects, which means the data are several times more likely under H_0 than H_1. When comparing the 2D formats (dots vs. pictures) to each other we found a Bayes factor of $BF_{01} = 4.46$. For larger set sizes, we found a Bayes factor of $BF_{01} = 9.16$ when comparing dot arrays to the objects and a Bayes factor of $BF_{01} = 8.18$ when comparing the pictures to the objects, which means the data are several times more likely under H_0 than H_1. When comparing the 2D formats (dots vs. pictures) to each other we found a Bayes factor of $BF_{01} = 8.39$. Bayes factors between 3 and 10 can be taken as moderate evidence in favour of the null hypothesis (Jeffreys, 1961), suggesting that children's performance on the cardinality measures was not influenced by whether or not items were 3D manipulatives.

5 | STUDY 1: DISCUSSION

When examining children's counting and cardinality skills, we found that whether or not children were using manipulatives did not influence their performance during assessment. Children's early counting and cardinality skills are important predictors of their later mathematics achievement (Geary et al., 2018; Nguyen et al., 2016), and students learn more when teachers can match instruction to their developmental level (Raudenbush et al., 2020). Because manipulatives offered no performance difference when it came to assessing children's understanding of counting and cardinality researchers and educators may be able to focus on a simpler administration of assessments using 2D printouts of materials. However, counting and cardinality are only one aspect of children's early mathematical development. For other areas, such as geometry, using 3D representations of materials may help support children's performance on assessments (Stevenson & McBee, 1958).

6 | STUDY 2: SHAPE IDENTIFICATION

To further investigate the role of manipulatives in children's early mathematical performance, we used data from a second dataset that included a measure of children's shape identification skill. This dataset included a broader battery of tasks measuring children's early academic skills. The shape identification task was chosen for the analysis because it included both pictorial and physical representations of 3D shapes, allowing for a test of whether the use of physical manipulatives influenced performance.

6.1 | Participants

For the analyses investigating children's shape identification skills, 93 children ($M_{age} = 4.35$, $SD = 0.61$; 53.8% female, 46.2% male; 58.1% White, 18.3% Latino, 14.0% Black, and 9.6% other race or not reported) completed the shape identification tasks. The measures were completed while working one-on-one with an experimenter in a quiet area of their school as part of a larger battery of tasks measuring children's mathematical understanding that included a broad numeracy measure, measures of children's executive functioning skills, and their spontaneous focusing on numerosity. There was not a fixed administration for each task, with experimenters administering each task as they were able to over the course of the session. For the current analyses we focused on the measures that had both 2D and 3D representations for the same questions, which were items focusing on shape identification.

6.2 | Measures

6.2.1 | Shape identification

To measure shape identification, children were asked to identify specific 3D shapes (e.g., '*Point to the pyramid.*') from sets of 3D shapes. The six specific shapes children were asked to identify included a cylinder, sphere, cone, cube, pyramid, and rectangular prism. First, children were asked to identify each of the six shapes from arrays of coloured pictures, which were depicted via pages of an assessment binder (see Supplementary Materials for example images of the materials). Children needed to clearly point to the correct shape to receive credit for the trial. Sets of four shapes (pictures of 3D representations) were presented as response options for each trial and the specific shapes presented varied from trial to trial. Second, children were asked to identify each of the six shapes from a set of foam shapes. Four foam shapes were displayed in front of the child as response options for each trial and the specific shapes presented varied trial to trial but were consistent across similar trials for the 2D representation and 3D physical shapes. Children were free to interact with the materials if they wished to do so. A total score ranging from 0 to 6 was computed for the pictures and the 3D manipulatives. Each format showed similar levels of reliability ($\alpha = 0.70$ for pictures; $\alpha = 0.70$ for 3D manipulatives).

7 | RESULTS

Table 2 provides the descriptive statistics and zero-order correlations for the presentation types. Each of the presentations was positively and significantly related to the other presentation type ($r = 0.46$, $p < 0.001$). Figure 2 shows performance broken down by stimulus format.

RQ3. Does presentation format influence performance on a shape identification measure?

A Bayesian *t*-test using the same assumptions highlighted in the Analytic Plan section for Study 1 was conducted to test for differences across presentation format.[2] We found a Bayes factor of $BF_{01} = 3.75$ when comparing performance on the pictures of the shapes to the physical shapes (see Supplementary Materials analyses comparing presentation format for each shape). This means the observed data are 3.75 times more likely under H_0 than H_1. This provides moderate evidence in favour of the type of material used during this assessment having no effect (Jeffreys, 1961).

8 | GENERAL DISCUSSION

The present analyses suggest that pictures and other 2D renderings may be sufficient for assessing counting and shape knowledge without the need for more cumbersome and distracting concrete manipulatives. On the measures of counting and cardinality understanding, children performed similarly when the stimuli consisted of dots, pictures, or physical objects. On the measure of shape identification, children performed similarly across the 3D physical objects and 2D pictures of those 3D objects. Given the lack of difference in performance across formats, 2D materials can be developed and used as a lower-cost alternative to needing to supply manipulatives during assessment.

TABLE 2 Descriptive statistics and correlation for shape identification task.

	M (*SD*)	1	2
1. Pictures	2.47 (1.32)	—	
2. Objects	2.66 (1.24)	0.46	—

Note: Correlation is significant at $p < 0.001$.

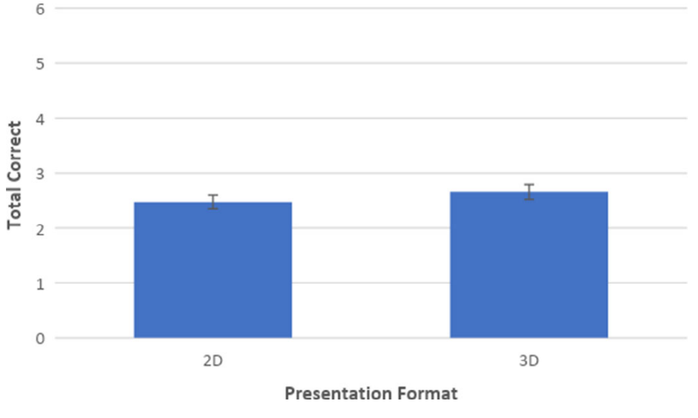

FIGURE 2 Total number of shapes (out of six) that children correctly identified. Error bars reflect ±1 SE.

Pictures also provide a more straightforward and quicker route for administration, which is crucial for brief assessments (Purpura et al., 2015; Weiland et al., 2012). These results should provide researchers and educators with flexibility in deciding the materials that may be used during assessment of counting, cardinality, or shape identification.

The null findings suggest that researchers and educators may use simple, 2D representations when administering assessments for preschoolers' understanding of counting and shape identification. Previous suggestions for using manipulatives have cited the time or monetary cost for establishing manipulatives in the classroom, suggesting teachers set aside additional time prior to introducing the materials to ensure that children are using them in a mathematical way (Burns, 1996; Laski et al., 2015). In many assessments (Clements et al., 2008; Ginsburg & Baroody, 2003; Griffin, 2000; Jordan et al., 2010), and the current study, children are provided with objects but not required to manipulate or have an extended interaction with them. Research in embodied cognition suggests that extended manipulation or action with objects is needed to support attention and the ability to represent the materials (Barsalou, 2008; Weisberg & Newcombe, 2017). One reason why there were no differences in assessment when manipulatives were used may be because the benefits of manipulatives are realized over the course of time after extended interaction.

Although the current study suggests that it is not necessary to use physical manipulatives when *assessing* children's counting and shape identification skills, this should not be generalized to children's *learning*. That is, previous work has suggested that there can be benefits for children's development of mathematical skills when educators use manipulatives to teach concepts (Carbonneau et al., 2013). The use of manipulatives is also an integral part of early educational methods, such as Montessori instruction (Laski et al., 2015).

8.1 | Limitations and future directions

Several important limitations that should be considered when generalizing the current findings. The findings highlight children's performance across three different early mathematics measures: counting, cardinality, and shape identification. Although these tasks tap into important aspects of early mathematical development where manipulatives are commonly used, future work will need to determine whether there is any added benefit of manipulatives for other mathematical assessments (e.g., arithmetic, measurement, or patterning). For more advanced concepts such as arithmetic, children may benefit from seeing the concrete representations of the operations being performed (e.g., mathematical equivalence instruction benefits from starting concrete and fading to abstract; Fyfe et al., 2014). Work with virtual manipulatives also has suggested that when they are designed to focus children's attention to specific details this can help them better understand the concept being tested (Moyer-Packenham & Suh, 2012). For example, children learning about place-values may benefit from being able to manipulate and group sets of items into base-ten configurations in ways that simply seeing 2D representations may not. Future work can explore whether such concepts may particularly benefit from the addition of manipulatives during the assessment process. It is also important to note that the current sample is limited to the preschool age range. As children gain experience with manipulatives, particularly during instruction, any benefits of using manipulatives during assessment may become clearer. One argument for the benefits of manipulatives is that they provide a concrete context for children to make a connection to abstract mathematical concepts (Sarama & Clements, 2009). When initially making this connection, children's understanding may be more context dependent, showing a greater reliance on physical manipulatives.

Future work can expand the tasks and type of manipulatives used to determine when and where to use objects during assessment. Previous work has suggested that perceptual richness may provide a benefit when using physical manipulatives to assess children's counting skills (Petersen & McNeil, 2013). Virtual manipulatives (interactive, visual representations) can also help support children's understanding (Moyer-Packenham & Westenskow, 2013). Such virtual representations can support children's geometry skills during instruction (Sarama & Clements, 2016). Future work can investigate the effects of different types of manipulatives alongside 2D stimuli to determine the effects of physical materials on both instruction and assessment. This may be particularly important as educators incorporate technology into assessment within the classroom. Last, it is important to consider the generalizability of these results to additional populations across the socioeconomic spectrum as well as outside of the United States. For example, Chinese mathematics teachers may place less of an emphasis on the use of manipulatives than their US counterparts (Cai & Wang, 2010). If manipulatives are less emphasized in children's early learning environment than the use of objects during assessment may add in a level of unfamiliarity and distraction that could influence performance.

8.2 | Conclusions

Parent and teacher surveys suggest that caregivers view manipulatives as important for children's early mathematical understanding (Cannon & Ginsburg, 2008; Uribe-Flórez & Wilkins, 2010). Current recommendations (National Association for the Education of Young Children, 2010) as well as the Common Core State Standards also suggest using manipulatives within the classroom to teach mathematics. However, in the current study we found that manipulatives may not be necessary when assessing children's understanding of one-to-one correspondence, cardinality, or shape identification. This suggests that researchers and educators may be able to identify children quickly and accurately understanding without needing to rely on cumbersome and costly manipulatives, at least for children's numeracy (i.e., counting and cardinality) and geometry (shape recognition) knowledge. Researchers may be able to develop straightforward, scripted sets of assessments that they can share with teachers and caregivers can then easily access and use to gauge children's early mathematical understanding. Ultimately, the current study suggests researchers and educators have flexibility in the materials that can be used during assessment—either physical objects or pictures can be used to assess children's counting, cardinality, and shape identification.

AUTHOR CONTRIBUTIONS

Connor O'Rear: Conceptualization; writing – original draft; writing – review and editing. **Erica L. Zippert:** Conceptualization; writing – original draft; writing – review and editing. **Patrick Ehrman:** Writing – original draft; writing – review and editing. **Lauren Westerberg:** Project administration; writing – original draft; writing – review and editing. **Christopher Lonigan:** Conceptualization; writing – review and editing. **David J. Purpura:** Conceptualization; project administration; writing – original draft; writing – review and editing.

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PEER REVIEW

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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ENDNOTES

[1] Taking a frequentist approach and conducting a repeated-measures ANOVA with presentation type for the one-to-one correspondence task as the repeated measure and controlling for age shows a non-significant difference between

presentations ($p = 0.311$). A similar pattern is shown using a repeated-measures ANOVA with presentation type for the cardinality task ($p = 0.511$).

[2] Taking a frequentist approach and conducting a repeated-measures ANOVA with presentation format (pictures vs. objects) entered as the within-subjects factor, controlling for age, shows a non-significant effect of format on performance ($p = 0.504$).

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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