PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 68, No. 6
June 2007
pp. 925–930

A calculation of the physical mass of sigma meson

J R MORONES-IBARRA and AYAX SANTOS-GUEVARA
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma, de Nuevo León,
Ciudad Universitaria, San Nicolás de los Garza, Nuevo León, 66450, México
E-mail: ayax_santos@yahoo.com

MS received 24 January 2007; revised 26 March 2007; accepted 11 April 2007

Abstract. We calculate the physical mass and the width of the sigma meson by considering that it couples in vacuum to two virtual pions. The mass is calculated by using the spectral function, and we find that it is about 600 MeV. In addition, we obtained 220 MeV as the value for the width of its spectral function. The value obtained for the mass is in good agreement with that reported in the Particle Data Book for the σ-meson, which is also named $f_0(600)$. This result also shows that σ-meson can be considered as a two-pion resonance.

Keywords. Sigma meson; scalar mesons; light mesons.

PACS Nos 14.40; 14.40.Cs; 13.75.Lb

1. Introduction

Even though the existence of the scalar–isoscalar σ-meson is subject to controversy [1], the study of its properties and structure in vacuum and in nuclear matter has recently attracted growing interest because of the role it plays in some theoretical models and the increasing experimental evidence of its existence [2]. It has been assumed that the σ-meson participates as an intermediate particle in several processes in vacuum and in hot and dense nuclear matter [3]. Particularly, the σ-meson is of wide interest because in theoretical models of the nuclear force, the sigma model is responsible for the attractive part of the nuclear potential [4], where it is attributed to the exchange of a single σ-meson.

In addition, the σ-meson plays the role of the chiral partner of the pion in the sigma model, which is a toy model for the interaction between nucleons and pions, with $SU(2)_{\rm L} \otimes SU(2)_{\rm R}$ symmetry [5–8]. This model was originally developed by Gell-Mann and Levy [9]. In the Gell–Mann and Levy model, the σ-meson plays the same role as the Higgs particle in the Weinberg–Salam theory [7,9], in the sense that the nucleon gets mass when the $SU(2)_{\rm L} \otimes SU(2)_{\rm R}$ symmetry is spontaneously broken. Moreover, in chiral perturbation theory the σ-meson enters as an essential part to adjust the theory to the experimental data [10], and in the interactions between mesons, the σ appears as a resonance, showing up as a pole in the T-matrix

[11,12]. In [6,13,14] they propose some experimental possibilities for investigating the behaviour of the σ-meson in hot and in dense nuclear matter.

On the other hand, the existence and the composition of the σ-meson, as a $\overline{q}q$ meson or as a $\pi\pi$ resonance is still under discussion [13,15,16]. We conclude here that it can be considered as a two-pion resonance.

There are several ways to define and determine theoretically the mass and the width of unstable particles. In [17], the mass of a particle is defined as the pole in its complete propagator; this definition is used also in [18–20]. On the other hand, authors in [21,22] make use of the spectral function to define the mass of a particle. In [23,24] the S-matrix formalism is used to determine the mass and the width of a meson.

In this work we define the physical mass and the width in terms of the spectral function. We study the spectral function of σ-meson in vacuum when this meson couples through strong interaction to two virtual pions. Defining the physical mass of a particle as the magnitude $|k|$ of its four-momentum for which the particle spectral function $S(k)$ gets its maximum, we find a closed expression for the regularized self-energy function of the σ-meson and so we obtain an exact analytical function for its spectral function. The σ-meson self-energy is calculated in the one-loop level and the propagator is computed by summing over ring diagrams, in the so-called random phase approximation (RPA) [25], which is characterized by the calculation of the self-energy to one-loop order. To carry out the summation we use the Dyson equation. The real part of the self-energy is ultraviolet divergent and it is regularized by using a simple subtracted dispersion relation, which preserves the symmetries of the theory.

2. Formalism

The spectral function of a particle is defined as -2π times the imaginary part of its propagator. The technique of defining the physical mass of a particle in terms of its spectral function is used extensively in [21,26,27] and it is well-established. This method will be used here for calculating the physical σ-meson mass.

In order to evaluate the physical mass of the σ-meson, we need, firstly, to calculate its dressed propagator $\Delta(k)$ in vacuum, where k is the four-momentum of the propagating meson. The expression for the dressed σ-meson propagator is obtained from the Dyson equation [4,28]

$$i\Delta(k) = i\Delta_0(k) + i\Delta_0(k)\left[-i\Sigma(k)\right] i\Delta(k), \tag{1}$$

where $i\Delta_0(k) = i/(k^2 - (m_\sigma^0)^2 + i\epsilon)$ is the free σ-meson propagator, with m_σ^0 and $\Sigma(k)$ being the bare mass and the self-energy of σ, respectively. The self-energy $\Sigma(k)$ contains all the information about the interactions of the meson with the quantum vacuum. So in order to determine the self-energy $\Sigma(k)$ we must specify the dynamical content of our model.

Our starting point is the interaction Lagrangian density $L_{\sigma\pi\pi}$ which describes the $\sigma - \pi$ dynamics, given by

$$L_{\sigma\pi\pi} = \frac{1}{2} g_{\sigma\pi\pi} m_\pi \vec{\pi}\cdot\vec{\pi}\sigma, \tag{2}$$

Figure 1. Sigma meson self-energy diagram. The dashed lines represent the σ-meson and the dotted lines the pion field.

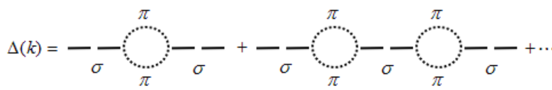

Figure 2. Sigma meson full propagator in the chain approximation. The right side is the diagrammatical representation of the chain approximation to the dressed propagator.

where $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ represents the Cartesian components of the pseudoscalar π-meson field, σ is the scalar σ-meson field, $g_{\sigma\pi\pi}$ is the coupling constant, and m_π is the mass of the π-meson [29].

The influence of the interaction of σ-mesons with virtual pions is introduced through the modification of the free propagator in the one-loop approximation. This is shown graphically in figure 1. This Feynman diagram contributes to the σ-meson self-energy. The dashed lines represent the σ-meson and the dotted lines represent the pion field. We will calculate the full propagator in the chain approximation, which consists of infinite summation of the one-loop self-energy diagrams [19]. The diagrammatical representation of the modified propagator is showed in figure 2, and the analytical expression is given by $i\Delta(k)$ in eq. (1).

The solution for $\Delta(k)$ in the Dyson equation (1) is given by

$$\Delta(k) = \frac{1}{[\Delta_0(k)]^{-1} - \Sigma(k)} = \frac{1}{k^2 - (m_\sigma^0)^2 - \Sigma(k)}. \tag{3}$$

On the other hand, the analytical expression for the self-energy $\Sigma(k)$ is given by [5,29]

$$-i\Sigma(k) = \frac{3}{2} g_{\sigma\pi\pi}^2 m_\pi^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{q^2 - m_\pi^2 + i\epsilon} \frac{1}{(q-k)^2 - m_\pi^2 + i\epsilon}, \tag{4}$$

where the coefficient 3/2 for the pion loop comes from the three isospin states and the permutation symmetry factor [29].

We will work in the σ-meson rest frame, for which $k^\mu = (k^0, \vec{0})$.

Carrying out the integration of $\Sigma(k)$ with respect to q_0 by using the Cauchy residue theorem, and integrating in the q_0 complex plane, we obtain

$$\Sigma(k) = -\frac{3}{8\pi^2} g_{\sigma\pi\pi}^2 m_\pi^2 \int_{-\infty}^{\infty} \frac{\vec{q}^{\,2}\,\mathrm{d}\,|\vec{q}|}{\sqrt{\vec{q}^{\,2} + m_\pi^2}\,[4(\vec{q}^{\,2} + m_\pi^2) - k_0^2 - i\epsilon]}. \tag{5}$$

The real and imaginary parts of eq. (5) can be separated by using the well-known formula

$$\frac{1}{x \pm i\varepsilon} = P\frac{1}{x} \mp i\pi\delta(x). \tag{6}$$

The integration of the imaginary part is readily obtained, given the result

$$\mathrm{Im}\,\Sigma(k) = \frac{-3g^2_{\sigma\pi\pi}m^2_\pi}{32\pi}\left(1 - \frac{4m^2_\pi}{k^2}\right)^{1/2} \tag{7}$$

for $k^2 \geqslant 4m^2_\pi$ and zero for $k^2 < 4m^2_\pi$. We can see the characteristic threshold value $k^2 \geqslant 4m^2_\pi$ for the production of real $\pi - \pi$ pairs from the σ-field.

On the other hand, the real part of $\Sigma(k)$ is ultraviolet divergent, and therefore it needs to be regularized. The regularization of Re $\Sigma(k)$ will be done by using a simple subtraction dispersion relation [30], which is given by the expression

$$\Sigma(t) = \frac{t}{\pi}\int_0^\infty \frac{\mathrm{Im}\,\Sigma_{\sigma\pi}(t')}{t'(t'-t) - i\epsilon}\mathrm{d}t', \tag{8}$$

where the imaginary part in the integrand is taken from eq. (7). On the other hand the real part of the integral in eq. (8) is simply $\Sigma(t) - \Sigma(0)$, being convergent [30].

Now let us write the identity

$$\mathrm{Re}\,\Sigma(k) = \mathrm{Re}\,\Sigma(k) - \mathrm{Re}\,\Sigma(0) + \mathrm{Re}\,\Sigma(0), \tag{9}$$

where $\mathrm{Re}\,\Sigma(0)$ is an infinite quantity that cancel the infinite terms of $\mathrm{Re}\,\Sigma(k)$. We define now the finite quantity $\mathrm{Re}\,\Sigma^R(k) = \mathrm{Re}\,\Sigma(k) - \mathrm{Re}\,\Sigma(0)$ as the regularized real part of the σ-meson self-energy, and the renormalized mass m_σ through $m^2_\sigma = (m^0_\sigma)^2 + \mathrm{Re}\,\Sigma_{\sigma\pi}(0)$. With this definition we take $m_\sigma = 600$ MeV as the experimental value of the mass for sigma.

From eq. (8), we obtain for the real part

$$\mathrm{Re}\,\Sigma^R(k) = \frac{-3g^2_{\sigma\pi\pi}m^2_\pi k^2}{32\pi^2}P\int_{4m^2_\pi}^\infty \frac{\left(1 - \frac{4m^2_\pi}{x'}\right)^{1/2}}{x'(x' - k^2_0)}\mathrm{d}x'. \tag{10}$$

The integration can be carried out directly giving the result

$$\mathrm{Re}\,\Sigma^R(k) = \frac{-3g^2_{\sigma\pi\pi}m^2_\pi}{16\pi^2}\left(1 + cI_0\right), \tag{11}$$

where

$$c \equiv 1 - \frac{4m^2_\pi}{k^2}$$

and

$$I_0 = \frac{1}{2\sqrt{c}}\ln\left|\frac{\sqrt{c}-1}{\sqrt{c}+1}\right| \quad \text{with } c > 0.$$

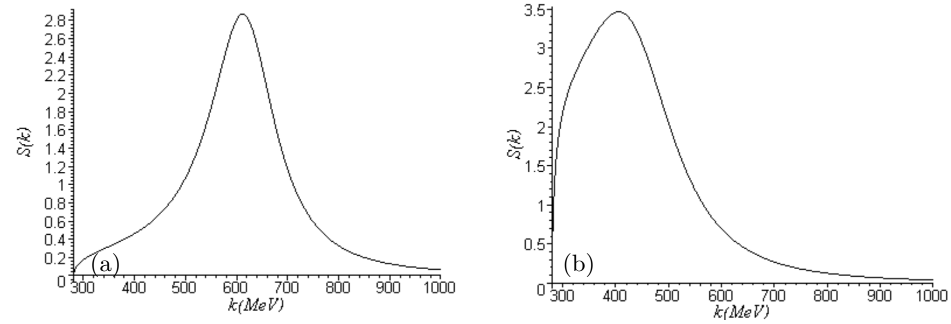

Figure 3. (**a**) Sigma meson spectral function for the parameter $m_\sigma = 600$ MeV. (**b**) Sigma meson spectral function for the parameter $m_\sigma = 441$ MeV.

The expression for the renormalized self-energy $\Sigma^R(k) = \mathrm{Re}\,\Sigma^R(k) + i\mathrm{Im}\,\Sigma(k)$, constructed from eqs (7) and (11), is the main result of this work.

The propagator given by eq. (3), takes the form

$$\Delta(k) = \frac{1}{k_0^2 - m_\sigma^2 - \mathrm{Re}\,\Sigma^R(k) - i\mathrm{Im}\,\Sigma(k)}. \tag{12}$$

From the definition of the spectral function $S(k)$ given above, we have

$$S(k) = -\frac{2\pi\mathrm{Im}\,\Sigma(k)}{\left[k_0^2 - m_\sigma^2 - \mathrm{Re}\,\Sigma^R(k)\right]^2 + \left[\mathrm{Im}\,\Sigma(k)\right]^2}. \tag{13}$$

Substituting $\mathrm{Im}\,\Sigma(k)$ and $\mathrm{Re}\,\Sigma^R(k)$ from eqs (7) and (11) into eq. (13), we obtain a closed expression for the spectral function. The parameters in eq. (13) are the reported m_σ value for the σ-meson mass, which we take as 600 MeV, and the bare $\sigma\pi\pi$ coupling constant $g^2_{\sigma\pi\pi} = 12.8$ [29]. The plots of $S(k)$ given by eq. (13) are shown in figures 3a and 3b.

As we can see, $S(k)$ gets its maximum value at $k = 600$ MeV, according to the reported value for the mass of σ [31]. On the other hand, we obtained the value of 220 MeV for the width, taking at one half of the maximum value of the spectral function.

In a recent work [32], a mass of about 441 MeV and a width of 540 MeV for the σ-meson are reported. Both values are far from our values. However, if $m_\sigma =$ 441 MeV, we obtain the graph showed in figure 3b, with a peak at 400 MeV and a width of 220 MeV. The values obtained for the mass are far from the two-pion threshold due to the fact that the peak of the spectral function depends on the choice of the parameter m_σ. As we can see in [16] a very wide and disperse set of values for the mass and width of sigma has been reported, depending on different theoretical models.

3. Conclusion

We have succeeded in obtaining, in our calculations of the σ-meson physical mass, a value which is in the range of its experimental value with that reported in the

Particle Data Book for $f_0(600)$. This calculation gives a relatively small value for the width compared with that in [31]. The use of a simple subtraction in the dispersion relation has allowed us to renormalize the σ-meson self-energy, given a closed analytical expression. The fact that we could obtain the physical mass of the σ-meson by considering that it couples in vacuum to two virtual pions, is a strong indication that the σ-meson can be considered as a two-pion resonance.

References

[1] A A Natale and C G Roldão, nucl-th/0204004 (2 Apr. 2002)

[2] Stephan Narison, *Proceedings of the Possible Existence of the Sigma Meson and its Implications to Hadron Physics*, Kyoto, Japan, 2000, pp. 123–132

[3] M K Volkov, A E Radzhabov and N L Russakovich, *Phys. Atom. Nucl.* **66**, 997 (2003)

[4] B D Serot and J D Walecka, *Adv. Nucl. Phys.* **16** 1 (1986)

[5] K Saito, K Tsushima, A Thomas and A Williams, *Phys. Lett.* **B433**, (1998)

[6] T Kunihiro, *Significance of the sigma meson in hadron physics (QCD) and possible experiments to observe it*, hep-ph/9905262 (1999)

[7] C S Lim, *Implications of sigma models in the standard model and beyond*, hep-ph/0010109

[8] E van Beveren, F Kleefeld, G Rupp and M Scandron, *Mod. Phys. Lett.* **A17**, 1673 (2002)

[9] M Gell-Mann and M Levy, *Nuovo Cimento* **16**, 705 (1960)

[10] H Q Zheng, Z Y Zhou, G Y Qin and Z G Xiao, hep-ph/0309242 (21 Sep. 2003)

[11] A Dobado and J R Peláez, *Phys. Rev.* **D47**, 4883 (1993)

[12] J A Oller, E Oset and J R Peláez, *Phys. Rev.* **D57**, 074001 (1999)

[13] L Roca and E Oset, *Nucl. Phys.* **A723**, 129 (2003)

[14] T Komada, M Ishida and S Ishida, *Phys. Lett.* **B508**, 31 (2001)

[15] M Ishida, *Prog. Theor. Phys. (Suppl.)* **149**, 190 (2003)

[16] N A Tornqvist, *Proc. of Possible Existence of sigma meson and its implication to hadron physics* (Sigma Meson 2000), YITP Kyoto, June 2000, Kyoto 102, No. 5 (2001); KEK-proceedings 2000-4

[17] H Shiomi and T Hatsuda, *Phys. Lett.* **B334**, 281 (1994)

[18] Y Kim, R Rapp, G E Brown and M Rho, *RHIC Physics and Beyond 17* (American Institute of Physics, 1999) CP482

[19] H C Jean, J Piekarewicz and A G Williams, *Phys. Rev.* **C34**, 1981 (1994)

[20] A K Dutt-Mazumder, B D R A Kundu and T De, *Phys. Rev.* **C53**, 790 (1996)

[21] M Asakawa, C M Ko, P Lévai and X J Qiu, *Phys. Rev.* **C46**, R1159 (1992)

[22] M Hermann, B Friman and W Nörenberg, *Nucl. Phys.* **A560**, 411 (1993)

[23] A Bernicha, G López Castro and J Pestieau, *Phys. Rev.* **D50**, 4454 (1994)

[24] M Feuillat, J L Lucio and J Pestieau, *Phys. Lett.* **B501**, 37 (2001)

[25] D Lurié, *Particles and fields* (Interscience Publishers, London, England, 1968) p. 266

[26] G Chanfray and P Schuck, *Phys. Rev.* **C57**, 8 (1998)

[27] S H Lee, *Phys. Rev.* **C57**, 927 (1998)

[28] S A Chin, *Ann. Phys. (N.Y.)* **108**, 301 (1977)

[29] G Wolf, B Friman and M Soyeur, *Nucl. Phys.* **A640**, 129 (1998)

[30] E Lifshitz and L Pitaevskii, *Relativistic quantum theory* (Pergamon, New York, 1974) vol. IV, part 2, pp. 415–416

[31] W-M Yao *et al*, *J. Phys.* **G33**, 1 (2006); http://pdg.lbl.gov

[32] H Leutwyler, *Int. J. Mod. Phys.* **A22**, 257 (2007)