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APPLICATION OF UNCERTAINTY QUANTIFICATION METHODS TO THE PREDICTION OF EFFUSION COOLED COMBUSTOR LINER TEMPERATURE

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ABSTRACT

As far as the preliminary thermal design of gas turbine components is concerned, 1-D codes are still widely used in standard industrial practice. Among the different components, the combustor is one of the most critical ones and its thermal design still greatly affects the reliability and life of the entire engine. During the initial phases of the design process, parameters are often roughly known. For this preliminary phase, a low-order approach is preferred instead of a high-fidelity simulation: the exploration of the whole space is extremely important to better understand the behavior of the system and to focus on the design objectives. Uncertainty quantification (UQ) methods, mainly developed in recent years and applied in many fields, are useful tools for the preliminary design phase and provide support during the whole design process. The objective of this work is to estimate the main sources of uncertainties in the design phase of an aeroengine effusion cooled combustor. The test case is based on a full annular lean-burn combustor, tested during the LEMCOTEC (Low EMissions COre-engine TEChnologies) European project. Among the test points investigated in the experimental campaign, the Approach condition is here analyzed. The inner liner is taken into consideration to investigate the metal temperature. Therm-1D, a 1-D in-house simulation code, is used to model the combustor and the open-source tool DAKOTA is adopted for the uncertainty quantification analysis. The baseline case of the combustor is studied and several uncertainty analyses are investigated. They are divided into 3 main groups: geometrical, tuning modelling parameters and thermal loads. For each group, the most relevant parameters are considered as a source of input uncertainty. In particular, a classical Monte Carlo approach is compared with four innovative polynomial-chaos approaches for each group: Gauss quadrature, total order with LHS sampling, stochastic collocation, and Smolyak. The analyses proved how the last two methods give the best results with a sensible lower amount of simulation (depending on the number of input variables). Lastly, results are compared with experimental data to achieve a better understanding of the most relevant input parameters and the propagation of their uncertainty on the results.

Keywords: Uncertainty Quantification, Gas Turbine, Combustor, Polynomial Chaos, DAKOTA.

NOMENCLATURE

BC	Boundary conditions
C_d	Discharge coefficient
$\overline{E}$	Expected values
FAR	Fuel Air Ratio
HTC	Heat transfer coefficient
LHS	Latin hypercube sampling
MC	Monte Carlo
PC	Polynomial chaos
PCE	Polynomial chaos expansion
PDF	Probability Density Function
QO	Quadrature order
R	Stochastic function
SC	Stochastic collocation
TO	Total order
UQ	Uncertainty quantification
X_T	Mult. factor for gas temperature and FAR
X_V	Mult. factor for gas velocity and momentum
Greeks	
α	Stream-wise inclination angle
β	Beta
μ	Mean value
σ	Standard deviation

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1. INTRODUCTION

The concept of uncertainty is often associated mainly with experimental tests, even if the results of the computational analyses are afflicted by many additional factors of uncertainty. Nowadays it is fundamental to estimate the correct propagation of the uncertainty that influences an analytical result especially from a design point of view. Particularly relevant in this sense is the application to the thermal design of aeroengine combustor liners. As shown in the experimental works of Andreini et al. [1], the design of the liner cooling systems [2] of a combustor is highly critical, since the adiabatic temperature of the wall is influenced by many parameters. Effusion cooling systems represent the state-of-the-art for the combustor liner of gas turbine [1], [3]. This technique uses patterns of holes close together to achieve a more uniform liner coverage, they also exploit slanted perforations, thus increasing the length of the holes and leading to a significant internal heat pick-up. This feature allows to reduce the thermal load on the liner with less cooling air, a fundamental requirement in modern lean combustors developed to abate polluting emissions. The critical issues and costs related to experimental measurements on real components from one hand and the complexity and modelling limitations of CFD investigations of the involved multi-physics problem, on the other hand, have led to the development of a 1-D code that allows a preliminary thermal design with short run times and a limited computational effort. To increase the robustness of the 1-D code developed by the University of Florence [4], [5], an Uncertainty Quantification analysis (UQ) is conducted using non-intrusive methods.

The approach used is to vary the input data of the numerical code and to evaluate the variation of the system response; then the statistics on the results are calculated. The procedure described is known as the Monte Carlo method [6], which will be described. A probabilistic approach based on the application of spectral methods allows to evaluate the propagation of uncertainty in numerical codes without increasing too much the computational cost. Based on the probabilistic application of spectral methods, it is possible to evaluate the propagation of uncertainty in numerical codes without increasing the computational cost. Stochastic expansion methods allow building an efficient solution that connects the solution provided by the numeric code to the input variables with a predefined statistical distribution. These methods are based on the theory developed by Wiener [7]. The first application was in mathematics on hypergeometric polynomials designed by Askey [8]. For the quantification of uncertainty, the most interesting methods are the Polynomial Chaos Expansions (PCEs), developed by Kardaniakis-Xiu [9] based on the Askey scheme, and Stochastic Collocation (SC).

This work proposes a comparison between different spectral methodologies and the various polynomial orders; then compared with the results obtained by applying the Monte Carlo method. Several works of uncertainty quantification applied to gas turbines are available in the literature. For film cooling systems the results shown by D'Ammaro et al. [10] showed almost identical results between PC and the MC method, but with a reduction of 10 times the computational effort for the PC approach. Similar analyses were conducted by W. Shi [11] to consider the effects of geometric variations, related to the manufacturing process, on the effectiveness of film cooling provided by fan-shaped hole. The comparison between PC and SC has shown by Montomoli et al. [12] for thermal loads on a nozzle of a gas turbine indicated a difference in results of less than one percent. However, a comparison of the different techniques of UQ was also made by Durocher et al. [13] analyzing the formation of NO_x under different flame conditions, the results have highlighted a certain discrepancy between the results obtained.

2. UQ THEORY

Non-intrusive uncertainty quantification methods allow to derive how uncertainties propagate within a numerical code, so it is possible to know how much an input variable impacts the final solution. Through this approach, it is possible to obtain probabilistic information using, however, a discrete approach. With uncertainty quantification techniques an uncertainty variable, in input of the code, is modeled with a probabilistic distribution [14]. This variable is sampled within the set distribution and the number of samples depends on the type of method used. The computational cost varies according to the uncertainty quantification technique used, the most expensive is the Monte Carlo method, which requires a high number of simulations to reach the statistical convergence. The expected value $\bar{E}(x)$ is obtained by averaging the sample over the population.

$$\bar{E}(x) \approx \bar{R}(x) = \frac{1}{N}\sum_{i=1}^{N} R_i \quad (1)$$

If the sample size N tends to infinity the error committed assuming the calculated average equal to the average of the population tends to zero. Optimized sampling techniques can accelerate the convergence of the method, e.g. with the Latin Hypercube sampling the error committed is reduced, compared to random sampling, by the square root of the sample size [15].

$$o(err)_{LHS} = \frac{1}{\sqrt{N}} \quad (2)$$

The most interesting approaches are based on spectral methods, with these techniques it is possible to minimize the computational costs. They are based on the polynomial approximation and create a stochastic functional between uncertain inputs and code outputs, as shown in the formula below.

$$R(x,\xi) = \sum_{i=1}^{\infty} \alpha_i(x)\psi_i(\xi) \quad (3)$$

With the polynomial chaos expansion (PCE) method [16] the interpolation polynomial bases ψ_i are calculated, while with a stochastic expansion (SC) the polynomial coefficients α_i are computed. When a PCE method is used the approximating polynomial can be truncated to a desired order (total order expansion) or considered in its entirety (quadrature order or also called tensor-product expansion), this involves a different number of simulations required. The evaluation number depends on the n variables considered, the polynomial order p used for the approximation and by the sampling grid. For total order expansion with LHS, the number of simulations to carry out is the following.

$$N_s = \frac{(n+p)!}{n!p!} \tag{4}$$

As pointed out in [17] this number must be at least doubled to increase the robustness of the analysis.

Otherwise, for quadrature order with sampling based on Gauss grid, the number of simulations is shown below

$$N_s = \prod_{i=1}^{n}(p+1) \tag{5}$$

Smolyak grid [18] can be used to reduce the simulation number required, this is a reduction of the Gauss grid and is much more convenient when the problem has a high number of uncertain variables. The number of evaluations of this grid depends on a number of the variables and by the level of approximation imposed.

3. TEST CASE

The present work investigates a single annular combustor designed in the EU-funded research program LEMCOTEC, additional information about the test case can be found in previous works [19], [20]. The prototype can be considered an improvement of the combustor designed, manufactured and tested in a previous research program (NEWAC) and depicted in FIGURE 1. The fluid dynamics behaviour is characterized by air passing through a dump diffuser and diverted within the cowl and to the inner/outer annuli, where it cools the liner and is partly bled outside. Once inside the cowl, it flows through the swirler and the dome cooling system. An impingement-cooled heat shield protects the dome, which provides also slot cooling in the first part of the liner. Such liners are cooled also by staggered effusion holes.

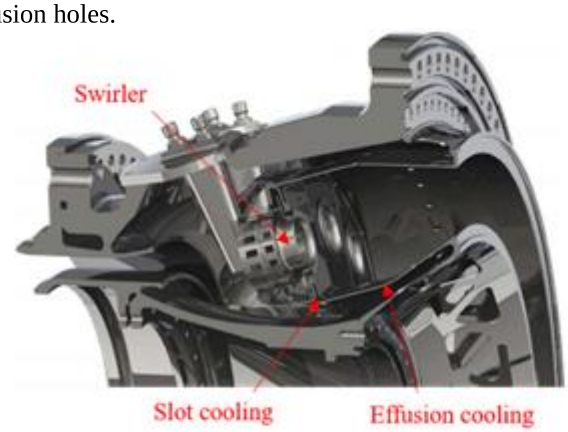

FIGURE 1: NEWAC COMBUSTOR [19]

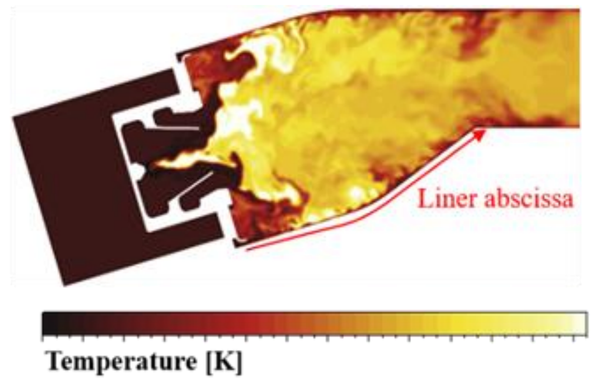

FIGURE 2: EXPECTED TEMPERATURE FIELD AT APPROACH CONDITION FOR THE LEMCOTEC COMBUSTOR [19]

The goal of this work is to investigate different UQ methodologies to determine the performance of the different approaches as well as to determine some guidelines for future activities. Concerning the analysis, 2 inputs are chosen with truncated Gaussian distribution: streamwise angle of the effusion cooling holes α and the C_d: mean value and standard deviation are respectively 30°, 2.5° and 0.7, 0.05. Truncation is chosen at $\pm 2\sigma$. The inner liner only is considered in this work (see FIGURE 1).

4. METHODOLOGIES

The current investigation is based on the use of two main tools: an in-house 1-D thermal design code for aeroengine combustors and the DAKOTA toolkit [21].

4.1.1 Therm-1D

One-dimensional codes are simulation tools that rely on simplified approaches to perform project evaluations and/or optimizations. Low-order equations and correlations are used here to represent the physical phenomena involved in the problem, providing rapid results. For this reason, approaches such as these are ideal in the early stages of engineering design. The code used in this work is called Therm-1D and is used for the preliminary design of combustor cooling systems. Once a configuration has been defined, it is used both to define a preliminary cooling arrangement and to evaluate performance under different operating conditions. The procedure can predict wall temperature and liner heat loads by combining a well-proven, internally developed, industrial flow network solver with a standard 1D thermal transfer model based on the Lefebvre methodology [21]. For a detailed description, the interested reader is referred to [4] and [5]. FIGURE 3 and FIGURE 4 shows schematic representations of the operations carried out by the different codes of the procedure, which are called upon to solve the calculation of the fluid-wall conjugate.

The first input is represented by main geometrical features and boundary conditions at inlet and outlet ports of the flow network, consisting of mass-flow rate, total pressure, temperature, and static pressure respectively. Starting from that information, the prediction of the air-split is performed by the aero-solver, which makes use of specific models to calculate pressure drops trough the different sections. The internal heat transfer is evaluated having the availability of a wide range of thermal correlations. The 1D equation for conduction is resolved to determine the equilibrium temperature across the liner thickness when all the thermal loads are defined. The procedure involves two iterative cycles. In the internal cycle, the wall heat transfer is resolved until it converges with the calculation of the 1D conduction. The radiation and convection for the "gas side" and the radiation for the "cooling side" are iteratively evaluated here to obtain the metal temperature of the liner. This first result is provided as input to the overall cycle in which the convection on the cold side, the heat-sink effect and the parameters of the main flow network are evaluated. The calculation continues until the convergence or arresting criteria are achieved, providing an output in terms of liners temperature and main characteristics of a cooling system such as adiabatic effectiveness distribution and coolant characteristics.

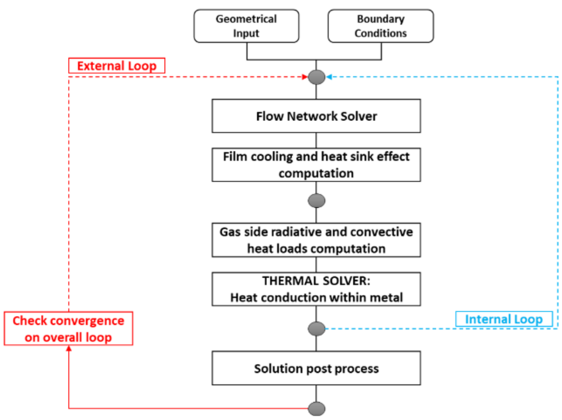

FIGURE 3: THERM-1D PROCEDURE

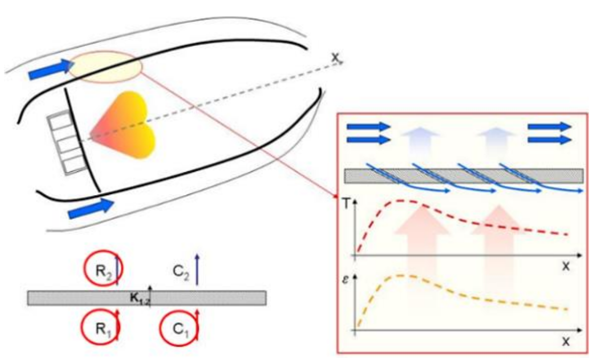

FIGURE 4: THERM-1D THERMAL EVALUATION

In this study, a network representative of a cross-section passing through the axis of the injector is modeled. Geometric information is provided as sections for air passages and liner thickness, as well as the type and arrangement of cooling characteristics. The boundary conditions are established based on the characteristics of the mission point in which the real hardware is tested, the mesh parameters are thus defined for the calculation of the finite elements to ensure proper discretization of the solid walls.

4.1.2 DAKOTA

DAKOTA toolkit [22] provides a flexible and extensible interface between simulation codes and iterative analysis methods. DAKOTA contains algorithms for optimization with gradient and non-gradient-based methods; uncertainty quantification with sampling, reliability, and stochastic expansion methods; parameter estimation with nonlinear least squares methods; and sensitivity/variance analysis with design of experiments and parameter study methods. These capabilities may be used as components within advanced strategies such as surrogate-based optimization, mixed-integer non-linear programming, or optimization under uncertainty.

Such coupling is often referred to as “black-box,” as DAKOTA does not know the internal details of the computational model, obviating any need for its source code. Such loose coupling is the simplest and most common interfacing approach DAKOTA users employ. DAKOTA and the simulation code exchange data by reading and writing short data files. During operation, DAKOTA automatically executes the user’s simulation code by creating a separate process external to DAKOTA.

The main advantage of DAKOTA is that it processes files, so it doesn't have to interact directly with the code that solves the problem in question.

In the work routine set up for this study, the required simulations are carried out in serial mode and only at the end of this, the uncertainty quantification analysis is carried out. The whole procedure has been automated through a user-defined interface script to make Therm1D and DAKOTA code communicate each other. Going into more detail, DAKOTA samples the uncertain variables according to the probability distributions imposed, then the values obtained are written in the appropriate inputs of the Therm1D code. Once the thermal problem is solved, the solution is passed back to DAKOTA that stores it. This procedure is repeated until the number of simulations is equal to the required number, defined by the uncertainty quantification method used for the analysis. When all the required outputs are obtained, DAKOTA performs the proper UQ and sensitivity analysis

4.2 UQ ANALYSIS

In this activity several uncertainty quantification methods are investigated for different types of variables, the results obtained by spectral methods are validated with Monte Carlo analysis. Polynomial chaos expansion, both quadrature order and total order truncation are studied, as well as the stochastic collocation method.

Four macro-analyses were carried out for each sensitivity analysis, conducted using the Sobol indexes [23] to estimate which parameter has the greatest impact on the output quantity. All the distribution characteristics for the input variables chosen for the four UQ analysis are summarised in TABLE 1.

In the first analysis, two geometrical variables were considered: the inclination angle of the effusion cooling holes and their discharge coefficient; both have been modeled using a normal distribution.

For the probability distribution of the uncertainty of the inclination angles of the holes, the average value coincides with the nominal one, equal to 30°. In accordance with Bunker [24] a variation of 5° has been considered, which implies a standard deviation of 2.5° for the sampling probability distribution.

The mean and standard deviation for the distribution is 30° and 2.5°, while the other one is 0.7 and 0.05.

This analysis was used to assess the impact of the polynomial order on the response surface.

In the second analysis, the heat transfer coefficients of gas side, coolant side and of heat sink effect were considered uncertain. The three variables were modeled with three multiplicative factors distributed according to a uniform distribution since in the code they are set by the user. The probability distribution of the gas side HTC enhancement factor of the gas side has been limited to allow at most a variation of 30% concerning the nominal value, while for the other two factors a variation of 20% was granted.

With the third analysis, the thermal and kinematic quantities of the boundary conditions in the combustion chamber are considered. Two multiplicative factors are used, the first one acts on gas temperature and fuel air-ratio (X_T); while the second one on the velocity and momentum of the gas (X_V). Uncertainties are modeled with normal distributions to take into account the variations of the sizes concerning the nominal working point, both distributions are characterized by having an average unit value, but for thermal quantities a standard deviation of 0.05 was imposed, while for kinematic quantities a standard deviation of 0.25 was considered. All uncertainty variables modeled with normal distributions have been truncated at $\pm\ 2\sigma$ to avoid sampling points too far from the nominal conditions.

A fourth analysis was carried out, in which all the uncertain variables were considered at the same time to validate the Smolyak grid and globally evaluate which of them has the greatest impact on the liner wall temperature.

	Distribution	μ	σ	LIM_{SUP}	LIM_{INF}
Angle (α)	Normal	30°	2.5°	35°	25°
C_d	Normal	0.7	0.05	0.8	0.6
HTC_{COLD} Factor	Uniform	/	/	1.2	0.8
Heat Sink Factor	Uniform	/	/	1.2	0.8
HTC_{HOT} Factor	Uniform	/	/	1.3	0.7
X_T	Normal	1	0.05	1.1	0.9
X_V	Normal	30°	0.25	0.5	1.5

TABLE 1: PROBABILITY DISTRIBUTIONS OF INPUT MAGNITUDES

	Geom	HTC	Boundary C.	Total
Monte Carlo	10-100-1000	10-100-1000	10-100-1000	10-100-1000
Tensor Quad	4	8	4	/
Total Order	12	20	12	/
Stochastic C.	4	/	/	/
Smolyak	/	/	/	15

TABLE 2: NUMBER OF EVALUATIONS REQUIRED

The analyses carried out and the relative number of simulations required, in the best possible configuration, are summarised in TABLE 2.

5. RESULTS

As far as the results are concerned, 4 types of analyses have been carried out as already mentioned and results are presented in each dedicated sub-chapter here. However, for the sake of clarity, the main results are also reported here in order to better understand the UQ methodology adopted. Furthermore, the HTC and the boundary conditions analysis are explained in detail. Lastly, an overall analysis accounting for all the 7 parameters taken into consideration has been performed, including a sensitivity to the MC analysis.

5.1 GEOMETRICAL ANALYSIS

The objective was to explore and analyze different UQ methodologies to subsequently carry out the analyses with the optimum method. As mentioned in the chapter "methodology" the results show the comparison between the MC analysis and different polynomial chaos technique. Regarding the PC approach, only the optimum method is here reported: the graphs will show the MC in comparison with the Quadrature order (1st order) and the total order (2nd order) approximation only. Additional orders were previously studied [25] but the results proved how these methods are the most computational cost-effective method with an optimum grade of accuracy for the results. The stochastic collocation has not been included since the results are the same as the QO method. FIGURE 5 reports the mean, maximum and minimum values for the liner wall temperature. It is clear that both PC approaches perform approximately the same results as the one obtained by a MC analysis with 1000 LHS samples. Moreover, it is important to notice that the QO method requires only 4 evaluations to perform the analysis. It is important to remind that the liner abscissa has been discretized in 712 points by Therm-1D; therefore, 712 different polynomials chaos analyses have been created, one for each value of non-dimensional abscissa. Furthermore, it is reminded that the results of the polynomial chaos approaches provided here are the ones obtained by 1000 LHS samples on the surrogate model (PC) created.

FIGURE 5 shows important information about the possible output range of the values and, therefore, about the standard deviation associated with each liner zone. However, it does not say anything about the probability of each event. FIGURE 6 provides the output probability for the geometrical analysis for the MC analysis. The ones representing the polynomial chaos approach are roughly the same and are not reported for the sake of brevity. It is important to notice that the graph reports the probability in terms of PDF. This graph can give additional information with respect to the previous one. To determine the output of most uncertain zones, one has to focus not only on the range but also on the probability associated with each zone. For liner non-dimensional abscissa < 100, the range is moderate and the PDF has a highly red zone around the middle (the median), therefore this can be considered a low-uncertain zone. On the other hand, the non-dimensional abscissa between 150 and 200 is the most uncertain zone: not only the range is the maximum, but also the probability associated is highly uniform (almost blue). Lastly, for example, the zone at 650 has approximately the same range as the zone at 250, however, the latter features more uncertainties since the PDF is more uniform and spread across all the zone.

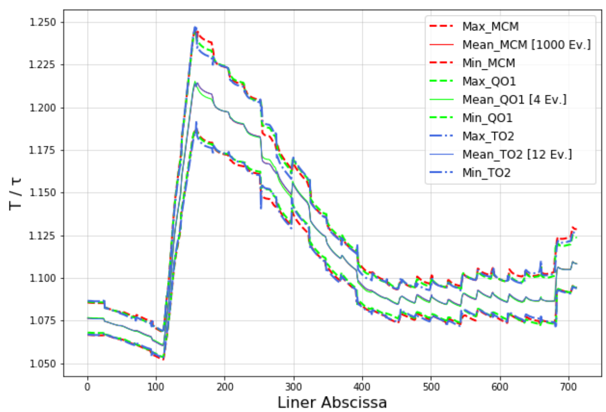

FIGURE 5: GEOMETRICAL ANALYSIS; MEAN, MAXIMUM AND MINIMUM VALUES

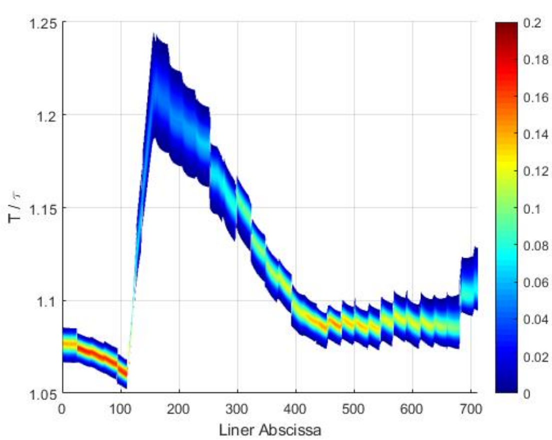

FIGURE 6: GEOMETRICAL ANALYSIS; OUTPUT PROBABILITY

Another important result is the sensitivity analysis reported through Sobol's indices. FIGURE 7 represents the indices through a cumulative histogram. The peaks and valleys shape is because 712 independent polynomials chaos analyses (and therefore Sobol's indices) have been created. The discharge coefficient of the holes plays a major role regarding the liner wall temperature, accounting for almost 75% over the whole liner.

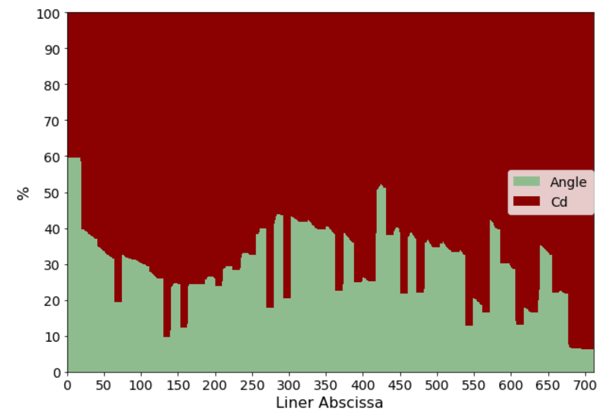

FIGURE 7: GEOMETRICAL ANALYSIS; SENSITIVITY ANALYSIS

5.2 HTC ANALYSIS

This analysis aims to investigate the influence of the heat transfer tuning factors on the results. Three different factors have been considered, the one acting on the cold side, the one acting on the hot side and the one for the internal holes. Their variation has been obtained with multiplicative factors. Again, the study showed a perfect agreement among the MC analysis and the stochastic expansion methods. Following the guiding principles adopted in the previous sub-chapter, the graphs are here presented in the same way. FIGURE 8 focuses on the maximum and minim values and, therefore, on the output range. It is clear how the range is now bigger than the previous analysis. However, the trend is approximately the same and the same considerations can be deduced.

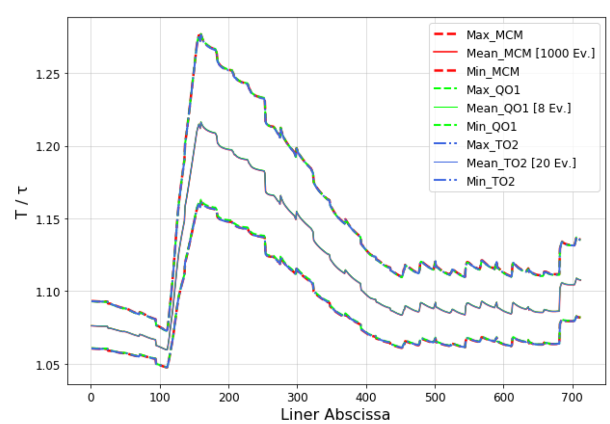

FIGURE 8: HTC ANALYSIS; MEAN, MAXIMUM AND MINIMUM VALUES

On the other hand, FIGURE 9 provides the output probability distribution for the analysis for the MC method. The most uncertain zone remains the one at the non-dimensional abscissa 150-200 and it is even bigger due to the greater input uncertainty considered. However, a different trend could be seen towards the end part of the liner. In fact, while for the geometrical analysis the final part of the liner featured an increment of both the range and the uncertainty, here the trend appears to be almost constant. The maximum variability reaches 40% in the zone immediately after the slot cooling.

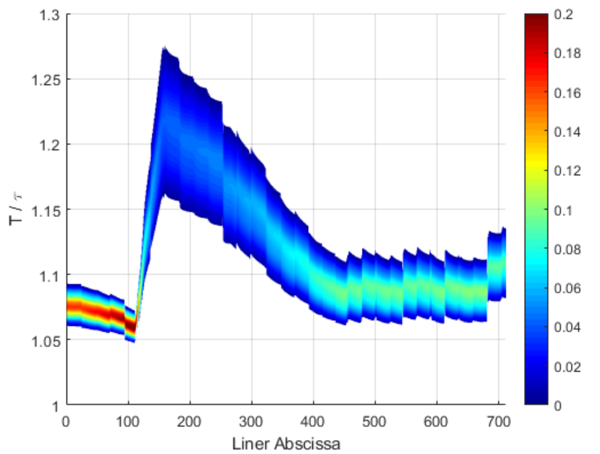

FIGURE 9: HTC ANALYSIS; OUTPUT PROBABILITY

As for the geometrical investigation, also for this study, a sensitivity analysis has been performed and the results are provided in FIGURE 10. The hot side has almost 10 times the influence of the cold side, it's dominant for the liner output temperature, followed by the internal heat pick-up and lastly by the cold side. In particular, the latter has a very limited influence on the final results, as confirmed by previous studies on effusion cooling systems [26], [27]. Therefore, it appears evident once again that film cooling and heat sink effects are fundamental to ensure an adequate life of the component under high hot side thermal loads.

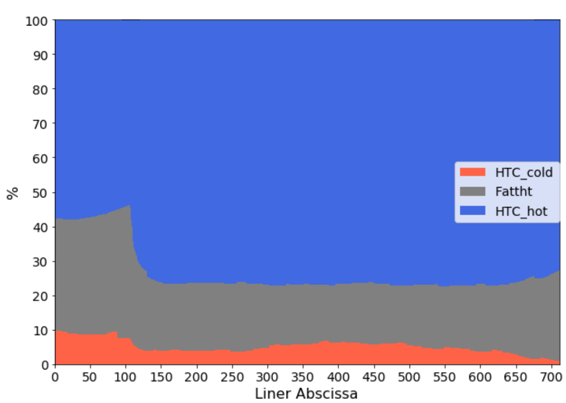

FIGURE 10: HTC ANALYSIS; SENSITIVITY ANALYSIS

5.3 BC ANALYSIS

This analysis has the objective to determine the effects of the boundary conditions imposed in the input files on the results. Again, multiplicative factors have been imposed to perform the input variation with the assigned distribution. FIGURE 11 provides the output range and, therefore, a measure of the standard deviation associated with each non-dimensional abscissa. The trend is similar to one already seen for the previous analyses and, in particular, to the geometrical one. The range is the highest among the three analyses considered and it is due to the input uncertainties considered. The different methods perform approximately the same results.

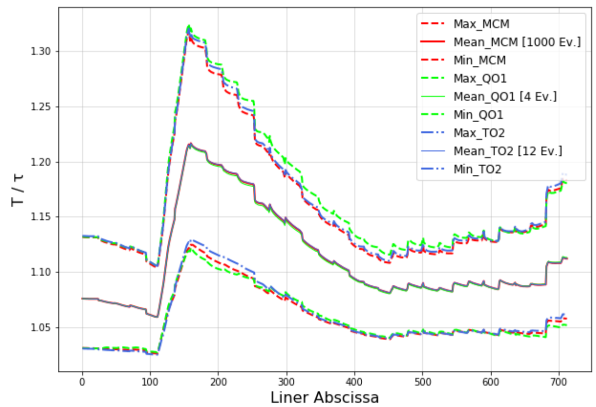

FIGURE 11: BC ANALYSIS; MEAN, MAXIMUM AND MINIMUM VALUES

FIGURE 12 provides the output probability distribution for this particular analysis for the MC analysis. Stochastic expansion methods feature approximately the same result. Even if the general trend is the same as the previous ones, a slightly more red-zone can be seen toward the middle-end part of the liner, meaning lower uncertainty in this zone. A variation of 20% on the temperature conditions of the hot gas can cause a maximum variability up to 54% on the liner wall temperature.

The sensitivity analysis through Sobol's indices (FIGURE 13) shows a particular feature. In the first part of the liner, in the presence of the slot cooling, the factor acting on thermal parameters plays a major role for the output temperature. On the contrary, in the final part of the liner, the factor acting on kinetic variables is dominant for the liner wall temperature. In the middle part of the liner the situation is balanced.

It has been decided also to include the experimental points to better understand the capability of the solver. FIGURE 14 includes some experimental data, whose scattering highlights the variability of the liner temperature on the centerline of different sectors of the combustor. As it is possible to notice, the range of the calculated PDF is somewhat representative of the experimental uncertainty. It is visible the potentiality of the UQ analysis and the results can now properly be assumed reasonable. However, from the picture, it is evident that the methodology used is not capable of justifying the gap with the experimental data measured in the test, which is reasonably to be ascribed to the adiabatic wall temperature distributions rather than the hot side HTC. Rather, it can quantify the uncertainty associated with each zone.

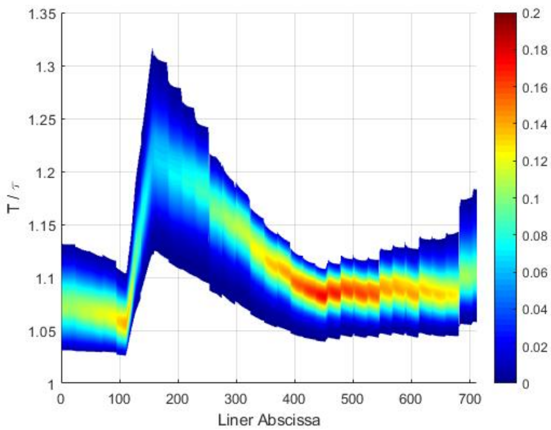

FIGURE 12: BC ANALYSIS: OUTPUT PROBABILITY

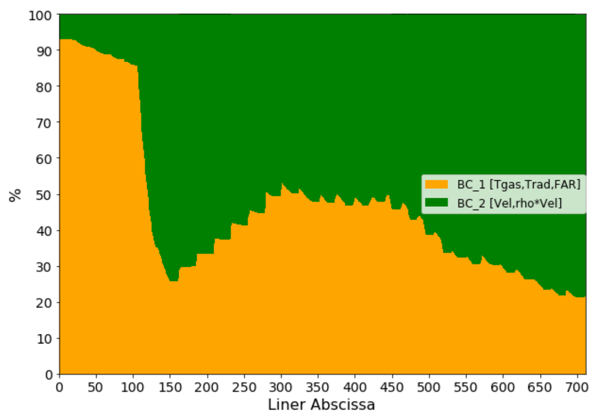

FIGURE 13: BC ANALYSIS; SENSITIVITY ANALYSIS

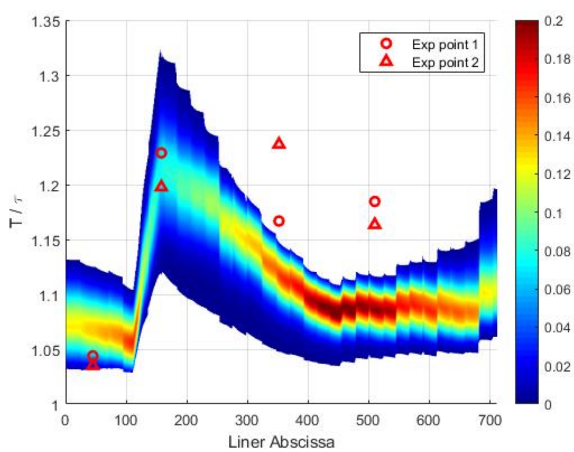

FIGURE 14: OUTPUT PROBABILITY INCLUDING EXPERIMENTAL DATA

Comparison with experimental data reported in FIGURE 14 suggests that the underestimation observed in the region between liner abscissa 300 and 500, is probably due o an overestimation of the adiabatic film effectiveness produced by the effusion cooling system. This is also motivated by the reduced effect on the predicted metal temperature due to the uncertainty of the investigated parameters. FIGURE 15 shows the distribution of the adiabatic effectiveness along liner abscissa, including the minimum and maximum limits obtained by the UQ analysis. It is evident that the predicted adiabatic film effectiveness in the central region is not significantly influenced by the investigated uncertainties. An additional source of uncertainty, not included in the present investigation, could be ascribed to the criteria adopted to model film superposition. The classical criteria proposed by Sellers is here used [28], which assumes a perfect overlapping of consecutive film layers produced by effusion jets. As observed in previous investigations carried out by the same research group, the presence of a highly three-dimensional flow field produced by the swirling jet issued by the burner may greatly affect the flow field in the near-wall region, with local flow structures impinging on the film layer and therefore significantly limit the superposition process [29]. Specific sensitivity to the uncertainty related to the superposition criteria may point out this problem but this type of detailed analysis was out of the scope of the present work.

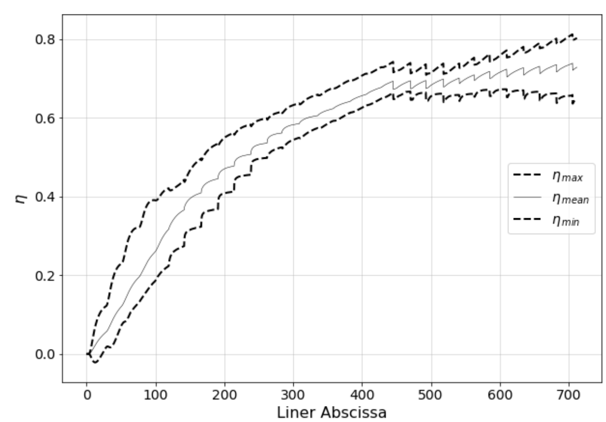

FIGURE 15: ADIABATIC EFFECTIVENESS TREND

5.4 OVERALL ANALYSIS

The last analysis aims to investigate the effects of all the seven parameters, comparing the overall influence with respect to the single analysis. The first consideration is based on FIGURE 16. This picture represents the standard deviation for the three analyses previously considered. The problem arises trying to consider the overall effect on the liner temperature. Obviously, to correctly assess the overall influence of these parameters on the liner temperature the single analyses are not sufficient. Therefore, an overall analysis has been performed.

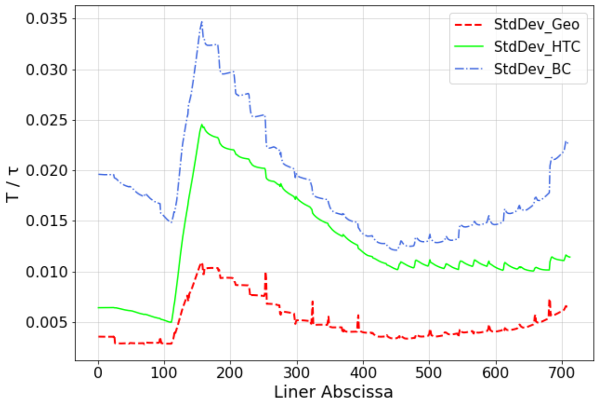

FIGURE 16: STANDARD DEVIATION FOR THE THREE ANALYSES

The first investigation was a sensitivity analysis to the MC samples size. LHS sampling has been used and 5 different sizes have been selected: 50, 100, 250, 500, 1000 (FIGURE 17). The results show how the 1000 samples case is necessary to correctly assess the uncertainty. Furthermore, based on additional analyses, this sample size can be considered a good compromise between the exact solution and the computational cost. As expected, the trend is slightly different from the ones of the previous analyses and the uncertainty is now greater. The importance of this analysis is to assess the exact uncertainty combining all the seven parameters together.

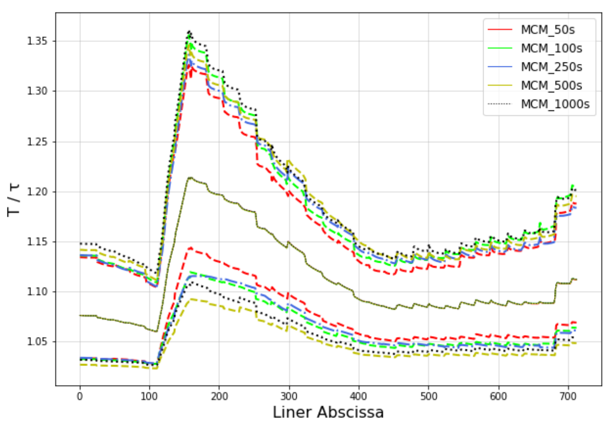

FIGURE 17: MC ANALYSIS

In order to further investigate the Uncertainty Quantification methodologies, the Smolyak grid has been tested and the results are presented in FIGURE 18. It is clearly visible that the results are very similar to the ones obtained with the MC analysis, proving how this methodology can be efficient even with 15 evaluations for 7 variables. Three different levels have been tested and the results are comparable.

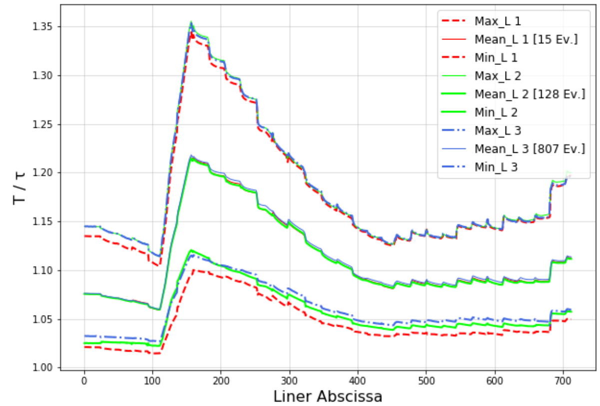

FIGURE 18: SMOLYAK GRID ANALYSIS

To fully understand the capability of the UQ techniques and, in particular, of the stochastic expansion processes, the polynomial chaos approach has been also applied with the total order (2nd order) and the quadrature order (1st order) approximation. Results are presented in FIGURE 19 in conjunction with the Smolyak grid (level = 1) and the reference case MC analysis with 1000 samples size.

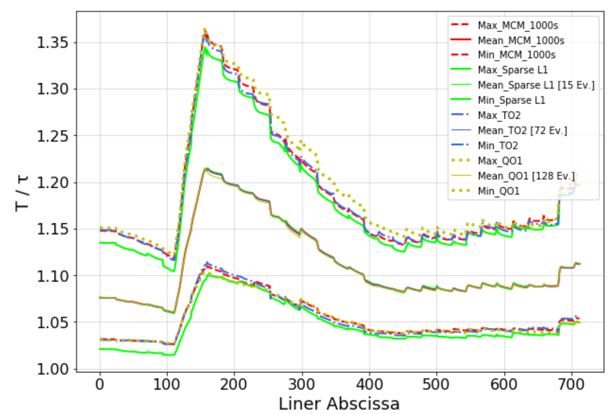

FIGURE 19: OVERALL ANALYSIS COMPARISON

Similar to what already presented in the previous analyses, FIGURE 20 shows the probabilistic view for the overall analysis. The zone with major uncertainty is the zone immediately after the slot cooling, in which not only the range is at his maximum value, around 20%, but also the probability distribution is broadly spread, resulting in a high uncertain zone in which all the events (the liner temperature) are most likely to happen.

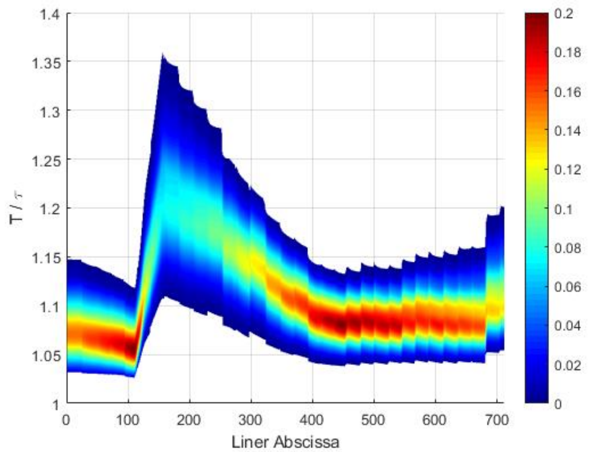

FIGURE 20: OVERALL ANALYSIS PROBABILISTIC GRAPH

Finally, a sensitivity analysis has been performed and the results are presented in FIGURE 21. Sobol's indices are presented for the primary parameters. The geometric parameters have a very low influence on the final temperature of the liner. The thermal boundary conditions have a dominant effect in the first part of the liner, whereas in the middle part the situation is more balanced: here, the two parameters affecting the boundary conditions and the hot side HTC have approximately the same Sobol's indices. In the final part of the liner the boundary condition involving the velocity and the density gains relevance. This information is of primary importance looking to a potential optimization phase: it is worth spending resources to optimize the most influencing parameters, if possible.

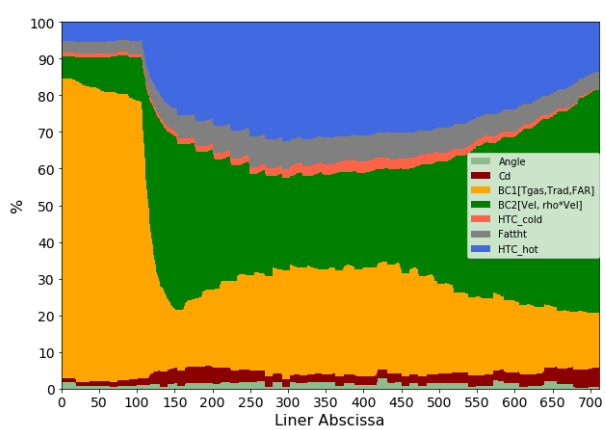

FIGURE 21: SENSITIVITY ANALYSIS

6. CONCLUSION

The software DAKOTA has been used to perform uncertainty quantification analysis regarding the preliminary design of a combustor. The test case analyzed is the LEMCOTEC combustor, designed and tested as part of an EU project. The results achieved are presented in terms of liner wall temperature. The main objective was to investigate different analyses with different methods. The classical approach is the Monte Carlo analyses. However, recent methodologies of stochastic expansion methods have been investigated and the results proved their efficiency. Four different simulations were carried out, evaluating for each one the liner wall temperature, the probability associated to each zone and the sensitivity analysis. In the first three analyses, the influence of the effusion cooling holes geometry, the influence of the heat transfer tuning factors and the magnitudes related to the boundary conditions have been respectively evaluated. In the last simulation, all the variables have been considered simultaneously. Only 4 evaluations are required with the polynomial chaos expansion using the Gauss grid for 2 uncertainty variables, while 8 evaluations for 3 variables, against the 1000 requests to reach a good convergence for the Monte Carlo method. The problem related to the number of parameters variation, which involves the exponential increase of the evaluation number, is partially solved using the Smolyak grid. For the complete analysis with seven uncertain variables, the method requires only 15 simulations. In quantitative terms, the maximum difference between the results obtained with spectral methods and those obtained with the Monte Carlo method is in the order of 1%, proving the efficiency of the stochastic expansion methods. Regarding the sensitivity analysis for the geometrical variables, it was found that the variation in terms of the discharge coefficient is more impacting than the angle. At the same time, when compared to other variables, the geometric ones have a limited impact on the liner wall temperature. This greater impact is also evident from the probabilistic graphs, as there is greater variability in wall liner temperature associated with a greater standard deviation. The range is approximately 25% for geometric analysis and 54% for boundary conditions one. The maximum uncertain zone is the one close to temperature peak: here, the maximum excursion is recorded and the probability associated to this zone has a wider PDF.

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