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THREE ESSAYS ON PRODUCT DESIGN: INCORPORATING CAPACITY, REMANUFACTURING, AND ENVIRONMENTAL FRIENDLINESS

BY

WENJUN GU

DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Business Administration
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2012

Urbana, Illinois

Doctoral Committee:

Professor Dilip Chhajed, Co-Chair
Associate Professor Nicholas Petruzzi, Co-Chair
Assistant Professor Yunchuan Liu
Doctor Baris Yalabik, University of Bath

Abstract

A product line design problem studies the optimal product line and corresponding quality and price for each product. This problem is critical for the success of businesses in both manufacturing and service sectors, and has been an important research focus for decades in both areas of marketing and operations management. To contribute in this line of discussion, this dissertation comprises of three essays in which we investigate how product line design problem interacts with such operations issues as capacity, remanufacturing, and reverse logistics, and examine how the results of these interactions affect the environment. Throughout this dissertation, we focus on the design of product quality, a single dimensional vertical differentiation which represents all more-is-better attributes of a product.

In the first essay, we investigate how "back-end" capacity constraint interacts with "front-end" pricing decisions. Specifically, we assume capacity is consumed in both fixed (depends on the length of a product line) and variable (depends on product quality) ways. A product design model for segmented market is used to derive and analyze the optimal product line strategy. We find that lower capacity introduces operations cannibalization which reduces the length of the optimal product line. We also show that if longer product line consumes more fixed capacity (e.g., for changeover or setup), then the resulting economy of scale could make offering a standard product optimal, which completely contradicts the classic product line design results.

In the second essay, we study remanufacturing and its environmental consequences within the context of product design. In particular, production costs and consumer valuations are considered as functions of quality and are differentiated based on whether the product is non-remanufacturable, remanufacturable, or remanufactured. Given this, the firm maximizes its profit by determining whether or not to remanufacture and, if so, how much to remanufacture. Correspondingly, we examine the environmental consequences of these optimal remanufacturing decisions using a quality-dependent measure which focuses on resource extraction and waste disposal. We show that true “green” consumers should not only value remanufacturable products but also value remanufactured products. We also find that consumers’ higher willingness-to-pay and the firm’s low production cost can potentially lead to worse environmental consequences in addition to higher profit.

In the final essay, we extend the model in the second essay by incorporating an exogenous collection rate and by considering social and environmental welfare in the decision making process. Examining the collection rate, we confirm that an increase in the collection rate generally benefits both the firm and the environment. We also find that there exists a threshold of collection rate above which collecting more units yields no effect on either profitability or environmental friendliness. Examining social and environmental welfare, we find that considering environmental welfare benefits the environment due to both lower quality and a smaller sales volume. In contrast, considering consumer welfare hurts the environment due to a much larger sales volume. This once again underscores that it is consumption that hurts the environment.

To my parents, for their love and support.

To a better environment.

Acknowledgments

My thanks go to Dilip and Nick for being my mentors. Without their trust, I would have doubted myself. Without their guidance and wisdom, I would have been lost and achieved nothing. My thanks also go to Frank and Baris, for serving on my dissertation committee. Special thanks to Sanjeev for his invaluable suggestions both professionally and personally. Many thanks to Enno and Gopesh for their encouragement throughout my Ph.D. study.

My friends in Champaign Urbana also deserve thanks. Thanks to my officemates, Abhijeet, Young Choul, Lu, Prasanna, Sai, Tianqin, Xuefeng, Yifan, and Ying, for making my time in the great Surveying Building more interesting. Big thanks to my teammates in Tiger Soccer Club, Di, Feng, Jeff, Nate, Pulin, Shen, Xiao, and Ye, for giving me a perfect place to blow off some steam. Many thanks to Huibing and Yu for being my roommates and friends. My thanks also go to Bin, for helping me like a sister.

I would like to thank my mother and father. Without their unconditional love, support, or understanding, it would be impossible for me to get this far. And thanks to Lili, for always believing in me and being there for me.

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Chapter 1

Introduction

1.1 Problem Overview

Product design is critical to a manufacturer's success. A well designed product will facilitate a firm implementing its strategy, while an ill designed product might force a firm to bankruptcy. For example, Apple Inc. launched its first *iPod* on October 23, 2001. Now about ten years later, Apple Inc. has developed a full product line that includes Classic *iPod*, the touchscreen *iPod Touch*, the video-capable *iPod Nano*, and the compact *iPod Shuffle*. Inspired by its success with *iPod*, Apple further developed two new product lines, namely *iPhone* in January 2007 and *iPad* in April 2010. The successful introduction of these product lines has not only saved the company but also significantly increased its stock price. As of January 03, 2011, Apple Inc. was worth more than $300 billion, the second most valuable company in the world (Praetorius 2011). For this reason, among many others, product line design problems have been one of the focal interests in marketing, operations management, and engineering design.

Recent years, especially the last decade, have witnessed the booming of reverse supply chain management. Product return, the "cost center" of a firm's operation, has been gradually turned into a new "revenue center" (Guide and Van Wassenhove 2009). On the one hand, top management has changed their view on the reverse supply chain. More systematic theories and tools are applied to managing product returns. Instead of sitting in the warehouse, returned products are collected, sorted, remanufactured, and then sold to consumers through marketing endeavors. On the other hand, consumers and the society as a whole are becoming increasingly aware of the environmental consequences of product production and consumption. More consumers are willing to buy "greener" products, even at a price premium (Cremer and Thisse 1999, Sengupta 2011). As a result, the remanufacturing industry increased to $63 billion business by 2005.

Indeed, this surging "green wave" has presented enormous opportunities for the 21st century. Whether firms can seize these opportunities is critical to their success. Also important is how firms take on the inherent challenges from legal, technological and marketing perspectives. On the legal side, a number of legislations have been introduced in Europe, North America, and Japan in the last two decades. These laws and regulations mandate manufacturers to reduce waste and take responsibilities for the entire life-cycle of their products (Webster and Mitra 2007). On the technological side, to take advantage of the cost savings, manufacturers must design and produce new remanufacturable products which would require more costly materials and more efficient technology (Lee and Bony 2007). Finally, on the marketing side, remanufactured products may not be well received by consumers because these products have been used (Guide and Li 2010). Also, selling remanufactured products could cause significant cannibalization to the new products, hence impeding the profitability (Atasu et al. 2008). Therefore, to ride this green wave, firms need to systematically incorporate remanufacturing and the reverse supply chain in their decision-making processes.

Many researches have shown that remanufacturing generally leads to higher profitability. However, does this necessarily translate into environmental friendliness, one of the most important benefits of remanufacturing? This question is difficult to answer. To make things worse, remanufacturing could potentially have negative environmental implications. For example, in ship remanufacturing, unprocessed scrap metals are oftentimes abandoned on the beach (CNN.com, 2008). In fact, firms proliferated across many industries are guilty of greenwashing in the sense that they ride the "green wave" of environmental consumerism without necessarily considering whether or not their actions actually benefit the environment (Orange 2010). Indeed, the implicit presumption that remanufacturing benefits the firm as well as the environment warrants more rigorous examination.

1.2 The Objective and the Plan

This dissertation consists of three essays, each of which models a product design problem within a specific operations management context. The first essay (Chapter 2) addresses the question of how a capacity constraint affects a monopoly firm's optimal product line decisions. The second essay (Chapter 3) studies the interaction between remanufacturing and product design. In this setting, it also examines the environmental consequences of remanufacturing by comparing the environmental damage associated with optimal remanufacturing solutions to the damage associated with optimal non-remanufacturing solutions. Building on the second essay, the third essay (Chapter 4) extends the model of Chapter 3 by investigating the impact of collection rate on the profitability and environmental friendliness of remanufacturing.

In Chapter 2, we study the interaction between product line design and limited capacity in a monopolistic setting. Classic product line design literature (i.e. Moorthy 1984, Kim and Chhajed 2000) has discussed this problem extensively by studying cannibalization and its implications. Here we consider an operations constraint, limited capacity, which introduces to the product line design problem a new trade-off between quality and quantity. On the one hand, offering high quality can extract more surplus with each product but lead to small quantity. On the other hand, offering low quality can boost the sales volume but significantly reduce the profit margin of each product. Hence, we consider this new trade-off in the product line design problem, and ask how does limited capacity affect the optimal product line decisions? Specifically, how does limited capacity interact with market cannibalization and affect the optimal quality and price decisions?

To address these questions, we consider a monopoly firm serving a market with two consumer segments that differ in size and valuation of quality. The firm faces resource constraints in the form of limited capacity. Its production technology dictates that capacity is consumed during changeovers or setups as well as during actual production. Specifically, setup capacity (capacity consumption per setup) depends on the length (number of product types) of the product line, with longer product lines consuming more capacity for setups. In contrast, variable capacity (capacity consumption per product) increases linearly in product quality to reflect that it consumes more capacity to manufacture a product with higher quality than it is to manufacture one with lower quality. In our paper, capacity consumption depends on both quality and quantity (as in Yayla-Küllü et al. 2011), whereas in existing literature capacity consumption depends only on quantity (as in Dobson and

Yano 2002, Tang and Yin 2010).

In Chapter 3, we study remanufacturing and its environmental consequences in the context of product design. A firm has the option to design a non-remanufacturable product or a remanufacturable one and to specify a corresponding quality. These design choices affect both the production costs and the consumer valuations associated with the product. On the cost side, remanufacturable products cost more to produce originally, but less to remanufacture, than non-remanufacturable products cost to produce. Analogously, on the consumer side, remanufacturable products are valued more, but remanufactured products are valued less, than non-remanufacturable products are valued. Given this, we investigate the environmental consequences of remanufacturing by first defining a measure of environmental damage that, ultimately, is a function of what is produced and how much is produced, and then applying that measure to assess the environmental damage associated with the firm's optimal strategy relative to the environmental damage associated with the firm's otherwise optimal strategy if a non-remanufacturable product were designed and produced. In doing so, we bridge the gap between profit maximization and environmental friendliness to ascertain the extent to which the two are complementary and to identify key factors of compatibility when they are not complementary.

To operationalize the firm's design for remanufacturing problem with a parsimonious model that captures remanufacturing fundamentals, we follow the lead of Ferrer and Swaminathan (2006) and Atasu et al. (2008) by formulating a two-stage analytic framework. We consider a firm serving a continuous market. At the beginning of stage 1, the firm first determines whether to design a remanufacturable product or a non-remanufacturable one and, correspondingly, establishes the quality of the chosen product. Then the firm sets the selling price for the product and sells an amount accordingly, as dictated by the specified consumer market's heterogeneity. Finally, to conclude stage 1, consumers who purchase the product extract its consumption value and then either discard the remains (which is the case if the product was designed to be non-remanufacturable) or return the remains (which is the case if the product was designed to be remanufacturable). The returned amount, if applicable, thus establishes a supply constraint on the number of units that can be remanufactured for resale. Given that, at the beginning of stage 2, the firm's decision is to set its optimal product portfolio, that is, to determine how many units of new versus remanufactured products to produce and what associated prices to set accordingly for each product type in stage 2.

In Chapter 4, we extend the model of Chapter 3 to explore the effect of collection rates on a monopolist's optimal design for remanufacturability. In particular, we investigate the conditions under which the firm makes its products remanufacturable when only a certain fraction of used products is returned at the end of each period. In that context, we ask, how does the optimal product mix between new and remanufactured products and optimal product quality change according to the collection rate? And, how do these decisions affect the environment? Moreover, we compare and contrast the manufacturer's optimal design for remanufacturability to the design that would maximize social and environmental welfare.

In contrast to our model in Chapter 3, we operationalize the design for remanufacturability problem in Chapter 4 with an infinite-horizon model. There are two key distinctions: 1) no initial period, which means that cannibalization between new and remanufactured products explicitly exists in each period; and 2) no ending period, which means that each new product can potentially be remanufactured. At the beginning of the planning horizon, the decision maker determines whether to design a remanufacturable product or a non-remanufacturable one, and establishes the corresponding quality. Given this product design, at the beginning of each period, the decision maker sets its optimal product portfolio, that is, determines how many units of new versus remanufactured products to produce and what associated prices to set accordingly in each period. Then, consumers make purchasing decisions and determine whether to discard or to return the remains after consumption. Finally, to conclude each period, the firm's reverse logistics collects a certain fraction of used products, which establishes a supply constraint on the number of units that can be remanufactured for resale.

Chapter 2

Product Line Design With Capacity Constraint

2.1 Introduction

The product line design problem involves the study of the optimal product variety, and corresponding quality and price for each product in the product line. This problem is critical for success of business in both manufacturing and service organizations. Thus, it has been an important research focus for more than four decades by scholars in both marketing and operations management. Marketing literature establishes that higher product variety can result in higher market share and revenue because it allows a firm to better meet the special needs of consumers. However, the potential gain from greater product variety must be balanced against the lower unit cost with fewer variants (Lancaster 1990). The operations literature has studied this tradeoff between revenue and cost, and has offered some prescriptions under such settings as competition (Alptekinoğlu and Corbett 2008, Villas-Boas 2009, Tang and Yin 2010), lead time and congestion (Dobson and Yano 2002, Chayet et al. 2011, Alptekinoğlu and Corbett 2010), and higher overhead and administrative costs (Netessine and Taylor 2007). In this chapter, however, we study capacity as yet another important factor in determining the optimal product variety.

Indeed, firms are always operating under operational constraints such as time, labor, equipment, space, and inventory (Goldratt and Cox 1992). These resource constraints can lead to competition among multiple products as manifested in many industries. For example, in the manufacturing sector, the constraint could be the total machine time which is limited by the total working hours and the number of assembly lines. Products which require different amount of processing time and serve different consumer segments share the same assembly line, and hence compete against each other for machine time (Dobson and Yano 2002, Chayet et al. 2011). Consequently, a firm has to optimally allocate its limited machine time among competing products to maximize its profits. Other examples include customer service centers which serve both premium and general customers, and aircraft accommodating seats for different classes (Yayla-Küllü et al. 2011).

When making product line design decisions, a firm faces challenges posed by capacity constraint on both the revenue and cost sides of the problem. On one hand, offering higher variety enables the firm to extract more surplus from consumers by better matching products with consumer preferences. But limited capacity may prevent the firm from reaching out to every consumer or from offering the best quality. On the other hand, offering lower variety reduces unit production cost due to economies of scale in the production process. But lack of differentiation prevents the firm from extracting more surplus from consumers with higher willingness to pay. Some research has addressed the trade-off between high revenue and low cost (e.g., Chayet et al. 2011, Tang and Yin 2010, Yayla-Küllü et al. 2011).

In this chapter, we aim to study the product line design problem of a monopoly firm who serves a segmented market under limited capacity. Specifically, we investigate the effect of capacity on the firm's optimal product line strategy, and identify conditions under which this optimal strategy deviates from the corresponding classic prescription under unconstrained capacity. We also examine the manifestation of capacity in quality and price, as well as in consumer welfare and profitability.

To address these questions, we consider a segmented market with two consumer segments that differ in size and valuation of quality. Serving this market is a monopoly firm who has limited capacity and whose production technology dictates that capacity is consumed during changeovers or setups as well as during actual production. On one hand, setup capacity (capacity consumption per setup) depends on the length of the product line, with longer product line consuming more capacity for setups. On the other hand, variable capacity (capacity consumption per product) increases linearly in product quality, representing the notion that it consumes more capacity to manufacture a product with higher quality than to manufacture one with lower quality. Therefore, in this chapter, capacity consumption depends on both quality and quantity (as in Yayla-Küllü et al. 2011), whereas in many existing literature capacity consumption depends only on quantity (as in Dobson and Yano 2002, Tang and Yin 2010).

Our model considers product quality as an endogenous design decision. This enables us to explicitly formulate capacity consumption with regards to quality. This modeling construct allows richer dynamics between quality and capacity, and is potentially more realistic for production scenarios where products with different quality demands different amount of capacity. Consequently, our modeling framework allows us to derive more insights into the trade-off between operations and marketing prescriptions. The main results of this chapter are summarized as follows:

- Capacity introduces cannibalization from the operations side as opposed to cannibalization from the market side which manifests because of consumer differentiation. These two types of cannibalization exhibit similar yet different effects on the firm's optimal product line decisions. First, an increase in cannibalization reduces the "length" of a product line (product variety) regardless of the sources of cannibalization. However, lower capacity increases cannibalization because of reductions in capacity for the high-quality products, while larger consumer differentiation increases cannibalization because of the firm's inability to implement price discrimination. Secondly, operations cannibalization impacts the qualities of both products while market cannibalization only affects the quality of the low-end product when offering two products is optimal. Moreover, capacity restriction does not affect the "width" of a product line (quality differentiation between two products). Therefore, the quality differentiation between products should remain the same even if the firm experiences changes in capacity due to any reason, whether it is downsizing, acquisition, or outsourcing.

- Offering a standard product for the whole market could be optimal when capacity is limited, although such a strategy is never optimal when capacity is abundant. When the two consumer segments are close in their valuation and capacity is moderate, offering a standard product for both segments is better than customizing only for the high-end segment due to a larger sale volume of the standard product, and it is also better than offering two products due to savings in setup capacity. Hence, capacity constraint offers another explanation of why offering a standard product is still practiced in many industries. Note that the differentiation in valuation between the two segments is the fundamental driver in determining whether or not offering a standard product is optimal. If the two segments are sufficiently apart, for example, then this strategy is never optimal. Instead, the firm offers two products when capacity is high and offers a niche product for the high segment only when capacity is low.

- In addition, limited capacity also exhibits strong influence over the firm's operational decisions, such as capacity utilization. For example, the firm may choose to idle some capacity when there is a large setup capacity associated with offering two products. This occurs when capacity is moderate, in which case the firm has enough capacity to offer the targeted segment with its efficient quality. But, large setup capacity makes it unattractive to offer two products because the remaining capacity after setup is not sufficient to offer high enough qualities to attract consumers.

- Intuitively, limited capacity generally leads to lower quality and price for consumers as compared to the unconstrained capacity case. However, this is not necessarily true. In fact, limited capacity may lead to higher quality, price and consumer welfare for certain segment(s). For example, as opposed to offering two differentiated products, when offering a standard product for the whole market is optimal, the low segment may receive higher quality and hence higher price. This consequently translates to higher consumer welfare for the high segment.

The rest of the chapter is organized as follows. In §2.2, we review relevant literatures. Then in §2.3, we lay out the model assumptions for both consumers and the firm, and we develop and solve models for each product line design strategy. In §2.4, we discuss the impact of capacity on the firm's optimal product line decisions such as capacity utilization, quality, price, consumer welfare as well as the firm's profit. And we conclude in §2.5.

2.2 Literature Review

Our research is related to two streams of literature. One stream is the literature on product line design. It started with Mussa and Rosen (1978)'s seminal research on vertical product differentiation in quality, which was further developed by Moorthy (1984) to tailor for segmented market. These marketing research emphasize "front-end" issues, that is to extract maximum surplus from heterogeneous consumers. They demonstrate the effects of market cannibalization in a product line with multiple products. Given cannibalization, this model is further developed to study a variety of issues, such as consumers' patience about time-to-market (Moorthy and Png 1992), distribution channel structure (Villas-Boas 1998), competition and horizontal consumer taste (Desai 2001), multiple quality-type attributes (Kim and Chhajed 2002), consumer evaluation cost (Villas-Boas 2009), and consumer variety seeking behavior (Sajeesh and Raju 2010). These works generally consider no explicit operational cost but assume a convex relationship between quality and corresponding variable cost. The results suggest that cannibalization leads to quality distortion to the lower-valuation segment. As cannibalization increases, the low segment should be dropped altogether. Moreover, a single standard product for all would never be optimal in this setting.

Also building on the stylized product line design model of Mussa and Rosen (1978) are many research papers on the operations and marketing interface. These works emphasize on the "back-end" issues, that lead to lower operational cost. For example, Desai et al. (2001), Kim and Chhajed (2002), and Heese and Swaminathan (2006) study the implications of product commonality and resulting cost saving effect on product line decisions. Krishnan and Zhu (2006) investigate impact of development cost in addition to variable cost. Shao (2007) studies system flexibility in a competition setting and learns that first-mover with flexibility may suffer from low profitability when the level of flexility is public information to its competitor. Netessine and Taylor (2007) examine the production technology with the classic economic order quantity model representing the make-to-stock cost. Chayet et al. (2011) study the effect of congestion in the form of queuing models.

The second stream of literature of interest to us is on resource constraint. This literature generally assumes exogenous product design and demand, and focuses on satisfying consumer demand with limited resources. For example, Cohen et al. (1988), Zhang (1997) and Mirchandani and Mishra (2002) propose that limited resources should be prioritized according to products or consumer classes. In particular, the product with higher profitability or consumer class with higher willingness to pay should be assigned with higher priority. Dobson and Yano (2002), Tang and Yin (2010), and Yayla-Küllü et al. (2011) study the effect of capacity with exogenous product design but endogenous pricing scheme which determines the demand of each type of product. Dobson and Yano (2002) assume that each product serves independent market which allows the authors to focus on the effect of capacity sharing in the absence of cannibalization. In contrast, Tang and Yin (2010) and Yayla-Küllü et al. (2011) study capacity with cannibalization such that multiple products serve the same uniformly distributed market. Tang and Yin (2010) consider product differentiation only in consumer valuation, while Yayla-Küllü et al. (2011) consider products differentiation in both quality and variable capacity consumption. Nevertheless, their results show that with limited capacity, a firm chooses its product line strategy by balancing marginal profitability against marginal capacity consumption.

In this chapter, we continue this line of discussion on the impact of capacity in product line design. The questions we ask are closely related to those in Tang and Yin (2010) and Yayla-Küllü et al. (2011). However, our model differs from these two papers in that we consider endogenous quality and model capacity consumption that depends on quality. Firstly, we assume segmented market which allow us to examine the rich dynamics between quality, quantity and capacity. Secondly, capacity is consumed in both fixed and variable fashion. In particular, the fixed setup capacity depends on product types, while the variable capacity requirement per product depends on the quality of the product. Here, our interpretation of capacity as a quality-dependant resource constraint is not only more realistic than the quantity interpretation in Tang and Yin (2010), but also enables us to explicitly capture the trade-off between quality and quantity. Moreover, a longer product line requires more capacity to setup than a short product line does, which differentiate this chapter from Yayla-Küllü et al. (2011). This assumption captures the notion of economy of scale in the sense that lower product variety means smaller setup capacity requirement and hence more effective capacity for production.

2.3 The Model

In this section, we first introduce and discuss our assumptions for the consumers and the firm. Then, we formulate and solve the decision models for each product strategy. Finally, we compare these strategies to develop the optimal strategy.

2.3.1 Modeling Assumptions

Consumers. We consider a market with two vertically differentiated consumer segments which differ in size and valuation of quality. Within each segment, consumers are homogenous. We use subscripts h and l to denote the high- and low-valuation consumer segments, respectively. The high-valuation segment is characterized with size n_h and valuation of quality v_h, whereas, the low-valuation segment is characterized with size n_l and valuation of quality v_l. Consumers purchase a product to maximize their non-negative surplus defined by the difference between consumer's utility and product price. For a product of quality q and price p, a customer with valuation v will derive a surplus of $vq - p$. If only one product is offered to the market, then consumers will purchase this product if the resulting surplus is non-negative (i.e., $vq - p \geq 0$). If two products are offered to the market, then consumers will choose the product which provides them with higher non-negative surplus. If purchasing either product yields the same surplus, then consumers will choose the product that is designed for them (i.e., high segment chooses high-quality product, while low segment chooses low-quality product).

We allow partial coverage of a consumer segment if the total number of products offered is smaller than the size of the targeted segment. In this case, some consumers in this segment purchase the product while others do not purchase because of limited supply of products. Which specific consumer in a segment purchases the product is not important because of consumer homogeneity within each segment[1].

Firm. We consider a monopoly firm seeking to maximize its profit by offering the market with one or two products. Each product is differentiated by a single dimension "quality", denoted by q, which captures all more-is-better features (e.g., Moorthy 1984, Kim and Chhajed 2000). Generally, a

[1]For the same reason, we do not consider strategic consumers in this chapter.

production process involves multiple steps and each step requires different inputs. The intricate relationships between various inputs and quality lead to increasing marginal cost in quality. Consistent with the extant product line design literature (e.g., Moorthy 1984, Kim and Chhajed 2000, Netessine and Taylor 2007), we use cq^2 to model the variable cost per unit to manufacture a product with quality q, where $c > 0$ is the cost coefficient.

The firm has a limited capacity, denoted by K. Its production technology requires both fixed and variable consumption of capacity. First, a fixed capacity is required for each product type for set-up activities such as calibrating machines and preparing materials. By definition, this setup capacity depends only on the number of product types. Accordingly, we denote this setup capacity with b_m, where $m = 1, 2$ is the number of product types. Without loss of generality, we normalize b_m such that $b_1 = 0$ and $b_2 = b > 0$. Here, b is the extra setup capacity for offering two products, hence represents the cost of product variety. Second, a quality dependent variable capacity is required for each product. And we use aq to model the variable capacity per unit required for manufacturing a product with quality q, where coefficient a denotes the variable capacity required to offer unit quality. In the context of this chapter, $a > 0$ captures the notion that high quality implies finer workmanship or more features, thus warranting more capacity (Krishnan and Zhu 2006).

In our modeling context, we follow the convention of Alptekinoğlu and Corbett (2008) by assuming that variable capacity requirement depends linearly on quality. This assumption can also be justified from the bottleneck perspective in operations management. Generally, a production process involves multiple steps and each step requires different inputs. But the capacity of a process is solely determined by the bottleneck step which is characterized by the lowest output rate. Hence, it is reasonable to assume that the variable capacity required at this step increases at a constant rate in quality represented by a linear function. Take the bottling industry for example. In this industry, quality can be represented by the volume of a bottle, whereas the capacity can be represented by the total machine time. Given that the machine has a constant filling rate, the time required to fill up a bottle is determined by the volume of the bottle, a relationship that can be represented by a linear function.

We summarize all the notation in Table 2.1. Consistent with the prod-

Table 2.1: Summary of Notation

Symbol	Definition
v_i, n_i	Valuation of quality and size for segment $i = h, l$
a,	Marginal capacity for each unit of quality
b	Setup capacity for a product line
R	Market cannibalization factor, $R = n_h/n_l(v_h/v_l - 1)$
x_i	Quantity of the $i = h, l$-quality products (decision variables)
q_i	Quality levels for product $i = h, l$(decision variables)
p_i	Price levels for product $i = h, l$ (decision variables)

uct design literature (Moorthy and Png 1992, Krishnan and Zhu 2006), we denote $q_h^e = v_h/2c$ and $q_l^e = v_l/2c$ as the "efficient" quality for the high and low segment[2], respectively. Similarly, we denote $q_h^u = v_h/2c$ and $q_l^u = v_l/2c(1 - n_h(v_h - v_l)/(n_l v_l))$ as the unconstrained quality when two products are offered Moorthy and Png (1992), respectively. Note that $q_h^e = q_h^u$ while $q_l^e > q_l^u$ indicating that cannibalization leads to a lower quality for the low segment. Consequently, to serve the market with unconstrained quality requires a minimum capacity of $b + \frac{a(n_h+n_l)v_l}{2c}$, in which b represents the set-up capacity, and $\frac{a(n_h+n_l)v_l}{2c}$ represents the capacity required to manufacture n_h products of quality q_h^u and n_l products of quality q_l^u. We define this capacity threshold as the unconstrained capacity $\bar{K}$. When $K \geq \bar{K}$, our problem is unconstrained and is equivalent to that in Moorthy (1984). Hence in this chapter, we focus on the case where $K < \bar{K}$.

Facing capacity constraint, the firm may choose to offer different number of products catering to different segments. Specifically, the firm can customize one product for the high segment only, which we refer to as the Niche strategy (N); or offer a standard product targeted at both segments, which we refer to as the Standard strategy (S); or customize one product for each segment, which we refer to as the Product-line strategy (L). We use superscripts N, S, L to represent the Niche, Standard and Product-Line strategy, respectively. In the remainder of this section, we develop and solve the profit-maximizing model for each strategy.

[2]Here, efficient quality for a segment is defined as the optimal quality if a single product is offered to the segment.

2.3.2 Single Product Strategies

A single product strategy represents the case when the firm offers one product to the market. If the Niche strategy is implemented, then the firm designs a single product so that it appeals only to consumers in the high segment. Correspondingly, the firm faces the following problem

$$\max_{x,p,q} \Pi^N = x(p - cq^2) \tag{2.1}$$

$$s.t. \quad v_h q - p \geq 0 \tag{2.2}$$

$$x - n_h \leq 0 \tag{2.3}$$

$$axq - K \leq 0 \tag{2.4}$$

And all decision variables $x_h, q_h, p_h \geq 0$. Constraint (2.2) is the participation (or incentive) constraint which ensures that customers in the high segment derive non-negative surplus from purchasing the niche product[3]. Inequality (2.3) and (2.4) are the quantity and capacity constraints for the Niche strategy. Accordingly, the incentive constraint (2.2) is binding because nothing prevents the firm from setting a price which extracts all the surplus from consumers in the high segment. That is, $p(q) = v_h q$.

Similarly, if the Standard strategy is implemented, then the firm designs a single product and targets it at both segments. Correspondingly, the firm faces the following problem

$$\max_{x,p,q} \Pi^S = x(p - cq^2)$$

$$s.t. \quad v_l q - p \geq 0 \tag{2.5}$$

$$x - (n_h + n_l) \leq 0 \tag{2.6}$$

$$axq - K \leq 0$$

And again all decision variables $x, q, p \geq 0$. The Standard strategy problem resembles the Niche strategy problem except for the participation constraint and quantity constraint. Specifically, (2.5) is the incentive constraint which ensures that customers in the low segment derive non-negative surplus from

[3]Consumers in the high segment will derive from the niche product zero surplus which is always higher than the surplus derived by consumers in the low segment from the same product. Hence this niche product does not appeal to the low segment.

purchasing the standard product[4]. And (2.6) is the quantity constraint to ensure that the total quantity of standard product should be no more than the total size of the two segments.

LEMMA 2.1. *For a single product strategy problem defined in §2.3.2,*

(i) quantity constraint (2.3) and (2.6) are always binding;

(ii) capacity constraint (2.4) is binding when capacity is low, i.e., $K \le K^N$ for the Niche strategy or $K \le K^S$ for the Standard strategy.

where $K^N = an_h v_h/(2c)$ and $K^S = a(n_h + n_l)v_l/(2c)$.

Lemma 2.1 establishes that the quantity and capacity constraint will be tight for a single product strategy. Accordingly, we can write the profit as a function of q, i.e., $\Pi^N(q) = n_h(v_h q - cq^2)$ for the Niche strategy, which is concave in q. Optimizing this profit over q, we obtain the following proposition.

PROPOSITION 2.1. *Let $x^{i*}, q^{i*}, p^{i*}, \Pi^{i*}$ denote the conditionally optimal quantity, quality and price decisions as well as the associated profits for strategy $i = N, S$. If a single product strategy is implemented, then*

	x^{i*}	q^{i*}	p^{i*}	Π^{i*}
Niche	n_h	$\frac{v_h}{2c}(1-(1-\frac{K}{K^N})^+)$	$v_h q^{N*}$	$\frac{n_h v_h^2}{4c}(1-((1-\frac{K}{K^N})^+)^2)$
Standard	$n_h + n_l$	$\frac{v_l}{2c}(1-(1-\frac{K}{K^S})^+)$	$v_l q^{S*}$	$\frac{(n_h+n_l)v_l^2}{4c}(1-((1-\frac{K}{K^S})^+)^2)$

Proposition 2.1 summarizes the optimal quantities, qualities, prices and the corresponding profits for both the Niche and Standard strategies. Note again that K^N and K^S represent the corresponding thresholds capacity below which the firm following a single product strategy will exhaust all capacity while above which the firm will offer the corresponding efficient quality level and idle any extra capacity more than these thresholds.

Examining first the optimal quantities, and from Lemma 2.1 (i), we find that the firm should always completely cover the targeted segment(s). As capacity increases, the firm can obtain non-negative increase in profit by either increasing quantity or increasing quality. The marginal return on capacity is higher if this unit of capacity is utilized to increase quantity than

[4]Consumers in the high segment always buy as they derive more surplus from the standard product than consumers in the low segment do.

if it is utilized to improve quality as long as the intended segment(s) is not fully covered. As a case in point, consider the Niche strategy. For given quantity $x < n_h$,

$$\frac{\partial \Pi^N(x(K), q)}{\partial K} = \frac{v_h - cq}{a} > \frac{v_h - 2cq}{a} \geq \frac{v_h}{a} - \frac{2cK}{a^2 x} = \frac{\partial \Pi^N(x, q(K))}{\partial K} \geq 0$$

holds where the second inequality follows because $K \geq axq$. In other words, the firm is better served by increasing quantity first before improving quality when capacity is not enough to cover the intended segment(s) with the corresponding efficient quality. Examining the optimal profits, we find that there exists a maximum profit which is obtained when capacity is sufficiently high (specifically, $K \geq K^i$ for the strategy $i = N, S$). When capacity is lower than the corresponding capacity threshold, the firm obtains a lower profit, which reduces quadratically in the difference between available capacity K and the corresponding threshold. The bigger is this difference, the lower is the firm's profit.

2.3.3 Product Line Strategy

In the Product-Line strategy, the firm customizes one product for each segment. Correspondingly, the firm faces the following problem,

$$\max_{x_i, p_i, q_i} \Pi^L = \sum_{i=h,l} \left[x_i (p_i - c q_i^2) \right] \tag{2.7}$$

$$s.t. \quad v_i q_i - p_i \geq 0, \quad i = h, l \tag{2.8}$$

$$v_h q_h - p_h \geq v_h q_l - p_l \tag{2.9}$$

$$v_l q_l - p_l \geq v_l q_h - p_h \tag{2.10}$$

$$x_h - n_h \leq 0, \tag{2.11}$$

$$x_h + x_l \leq n_h + n_l \tag{2.12}$$

$$b + a \sum_{i=h,l} x_i q_i - K \leq 0. \tag{2.13}$$

Again (2.8) is the incentive constraint for segment $i = h, l$, (2.11) and (2.12) are the corresponding quantity constraints, while (2.13) is the the capacity constraint, and (2.9) and (2.10) are self-selection constraints. These self-selection constraints ensures that consumers in the high segment always derive positive utility from either product. In our modeling context, this means that consumers in the high segment seek for the high-quality product first and for the low-quality product later if the high-quality product is not available. Thus, these conditions do not rule out the possibility that a fraction of the high segment buys the high-quality product while another fraction buys the low-quality product.

The participation constraint (2.8) will be binding for the low segment as nothing prevents the firm from extracting all the rent from consumers in the low segment, or $p_l = v_l q_l$. Also, the self-selection constraint (2.9) is binding, indicating that the firm can extract surplus from the high segment up to a point where the high segment is indifferent between either product, or $p_h = v_h q_h - (v_h - v_l)q_l$. Replacing the prices for high- and low-quality products in (2.7), the profit function can thus be written as

$$\Pi^L(x_h, x_l, q_h, q_l) = x_h(q_h v_h - cq_h^2 - q_l(v_h - v_l)) + x_l(v_l q_l - cq_l^2) \qquad (2.14)$$

subject to constraints (2.11), (2.12), and (2.13). Similar to Lemma 2.1 for the Niche strategy, we have the following lemma for the Product-line strategy which follows directly from the corresponding Lagrange model.

LEMMA 2.2. *For the Product-line problem defined in §2.3.3, i) quantity constraints (2.11), (2.12) are always binding; ii) capacity constraints (2.13) is always binding.*

The intuition behind Lemma 2.2 is as follows. First, both segments should be completely covered under the Product-Line strategy. Offering every customer in the high segment with high-quality product gives the firm higher marginal profit. While, offering every customer in the low segment with low-quality products gives the firm lower marginal cost. Second, the firm should exhaust available capacity under the Product-Line strategy. If the limited capacity is not exhausted, then utilizing one more unit of capacity can increase product quality, which leads to higher price and higher profit margin. Consequently, we can solve (2.14) and arrive at the following result.

PROPOSITION 2.2. *If the Product-Line strategy is implemented, a firm facing* *limited capacity ($K < \bar{K}$) designs its product qualities, such that*

$$q_h^L = \frac{(K-b)}{a(n_h+n_l)} + \frac{(v_h-v_l)}{2c}$$
$$q_l^L = \frac{(K-b)}{a(n_h+n_l)} - \frac{n_h(v_h-v_l)}{2cn_l}, \tag{2.15}$$

and the corresponding profit $\Pi^L(K) = \frac{n_h v_h^2}{4c} + \frac{n_l v_l^2}{4c}(1-R) - \frac{c(\bar{K}-K)^2}{a^2(n_h+n_l)}$. $K^L \leq K < \bar{K}$ *and* $R \leq 1$*, where* $K^L = \bar{K} - \frac{a(n_h+n_l)v_l}{2c}(1-R)$.

Proposition 2.2 characterizes the optimal product line quality with regards to K. These results can be interpreted as follows. First, effective capacity after setup is evenly allocated resulting in a base quality of $\frac{K-b}{a(n_h+n_l)}$ for each product. Then to differentiate the two products, $\frac{an_h(v_h-v_l)}{2c}$ amount of capacity intended for producing low-end products is reallocated to producing the high-end product. As a result, quality of the low-end product is reduced by $\frac{n_h(v_h-v_l)}{2cn_l}$, while that of the high-end product is increased by $\frac{(v_h-v_l)}{2c}$. Second, with unlimited capacity, the two products should have qualities of q_h^u and q_l^u, respectively. Because of limited capacity, quality of each product is reduced by

$$q_h^u - q_h^* = q_l^u - q_l^* = \frac{\bar{K}-K}{a(n_h+n_l)},$$

so that the firm can offer enough volume to cover each segment.

One interesting result is that the "width" of the product line (defined by the differentiation in quality between the products and represented by $\Delta q = q_h - q_l$) remains the same whether or not capacity is limited, i.e., $\Delta q^* = \Delta q^u = \frac{(n_h+n_l)(v_h-v_l)}{2cn_l}$. This illustrates the effects of market cannibalization. That is, in order to implement price discrimination through consumer self-selection, the firm must differentiate the qualities of its two products by increasing the quality of the high-end product and reducing the quality of the low-end product.

In contrast, the second perspective demonstrates the effects of limited capacity on the quality differentiation. Due to limited capacity, the firm cannot offer the same product qualities as the unconstrained case. Instead, it reduces the qualities for both products by the same level. And this level of reduction is proportional to the difference between available capacity and the unconstrained capacity. The lower available capacity the firm has, the bigger is the quality reduction. When available capacity approaches the unconstrained level, the reduction in quality becomes zero. When available capacity approaches K^L, the reduction in quality is the largest, at a value of $\frac{v_l}{2c} - \frac{n_h(v_h - v_l)}{2cn_l}$, in which case the quality of the low-end product approaches zero. Thus, the available capacity should be more than K^L for offering two products to be feasible.

2.3.4 Optimal Product Line Strategy

Let K^{ns} denote the capacity threshold where the firm is indifferent between the Niche strategy and the Standard strategy, and is obtained by setting $\Pi^N(K) = \Pi^S(K)$. Similarly, let K^{sl} and K^{nl} denote the corresponding capacity threshold where the firm is indifferent between the Standard and the Product-Line strategies, and between the Niche and the Product-Line strategies, respectively. By comparing the conditionally optimal profits for all three strategies, we have the following proposition about the optimal product line strategy.

PROPOSITION 2.3. *The optimal product line strategy for a firm facing limited capacity is as follows:*

(i) If $\max[K^{sl}, K^{nl}] \leq K$ *and* $v_h < v_l(n_h+n_l)/n_h$*, then the firm implements the Product-Line strategy;*

(ii) If $K^{ns} \leq K < K^{sl}$*, then the firm implements the Standard strategy;*

(iii) If $0 < K < \min[K^{ns}, K^{nl}]$ *and* $v_h < v_l(n_h + n_l)/n_h$*, or if* $v_h \geq v_l(n_h + n_l)/n_h$*, then the firm implements the Niche strategy.*

Proposition 2.3 characterizes the optimal strategy for given capacity and consumer valuation. To better understand the results in Proposition 2.3, we graphically represent the optimal strategy space in Figure 2.1. We set the parameters at $v_l = 1$, $n_h/n_l = .6$, $b = .1$, and $a = c = 1$ for all figures in this chapter unless otherwise stated. Notice that it is the ratios of v_h/v_l and n_h/n_l as well as a/c that determine the structure of the optimal strategy.

When capacity is not constrained, we know from (Moorthy 1984, Moorthy and Png 1992) that a firm offers a product line if market cannibalization is weak (i.e., $v_h < v_l(n_h + n_l)/n_h$ as in region (iv)) and offers only a high-quality product for the high segment if market cannibalization is strong (i.e., $v_h \geq v_l(n_h + n_l)/n_h$ as in region (v)), which corresponds to the area of Figure

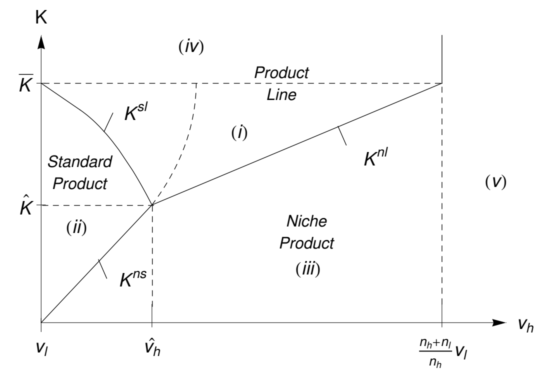


Figure 2.1: Optimal Strategy in Capacity and Consumer Valuation

2.1 in which $K \geq \bar{K}$. When capacity is constrained, this classic result can be challenged depending on the level of capacity and consumer valuation. On the one hand, if capacity is close to the unconstrained level (as in region (i)), then offering two products is optimal as it helps the firm to segment the market and to benefit from a large demand. On the other hand, if capacity is sufficiently lower than the unconstrained level, then segmenting strategy is no longer optimal due to corresponding low qualities. Instead, the firm finds it more profitable to offer a single product so as to avoid the extra setup capacity associated with offering two products. Specifically, the firm offers a standard product for both segments (as in region (ii)) if the two consumer segments are close to each other in valuation; and it offers a niche product for the high segment (as in region (iii)) if the two consumer segments are far apart.

Figure 2.1 suggests that there are certain threshold values of capacity and valuation for a strategy to be viable. The following proposition formalizes these existential results.

Proposition 2.4. *There exist $\widehat{v_h}$, $\widehat{K}$, and $\widehat{b}$ such that*

i) Offering a standard product to both segments is never optimal when consumer segments are sufficiently apart in valuation (i.e., $v_h \geq \widehat{v_h}$), regardless of capacity level;

ii) Offering two products is never optimal when capacity is low (i.e., $K < \widehat{K}$) regardless of consumer valuations.

where $\widehat{b} = K^S \frac{(\sqrt{n_l(n_l+n_h)}-n_l)}{2n_h+n_l}$, $\widehat{K} = K^{ns}(\widehat{v_h})$, *and*

$$\widehat{v_h} = \begin{cases} v_l + \frac{2bv_l}{K^S}(-1+\sqrt{1+\frac{n_l}{4n_h}(1+\frac{2K^S}{b})}), & b \leq \widehat{b} \\ v_l + \frac{bv_l}{K^S}\sqrt{\frac{n_l}{(n_h+n_l)}}(-1+\sqrt{\frac{n_l}{n_h}(-1+\frac{2K^S}{b}\sqrt{\frac{(n_h+n_l)}{n_l}})}), & b > \widehat{b} \end{cases}$$

The first part of Proposition 2.4 demonstrates that offering a standard product is never optimal if consumer segments are sufficiently apart from each in valuation (i.e., $v_h > \widehat{v_h}$ where $\widehat{v_h} = arg[K^{sl}(v_h) = K^{ns}(v_h)]$). We know that, on the upside, offering a standard product can be optimal because it benefits from a larger sales volume (compared to offering a niche product) as well as no setup capacity (compared to offering two products). On the downside, offering a standard product does not implement price discrimination, the loss of which hinges on the level of cannibalization. When $v_h < \widehat{v_h}$, weak cannibalization reduces the need to differentiate quality which renders offering a standard product optimal. When $v_h \geq \widehat{v_h}$, however, relative strong cannibalization dictates quality differentiation between products. In particular, if the capacity is relatively low, then the firm achieves this quality differentiation by customizing only for the high segment so as to avoid setup capacity requirement. If the capacity is relatively high, then the firm customizes for both segments. Notice that $\widehat{v_h} < \sqrt{\frac{n_h+n_l}{n_h}}v_l$, where the latter represents the threshold valuation for the high segment above which offering a standard product is always dominated by offering a niche product.

The second part of Proposition 2.4 demonstrates that lower capacity reduces product variety. More interestingly, we find that offering two products is never optimal when capacity is below a certain threshold $\widehat{K}$. We know from Proposition 2.2 that the firm reduces the qualities of both products as capacity decreases from the unconstrained level. This reduces the benefit of market segmentation due to lower profit margins. When $K \geq \widehat{K}$, the firm still benefits from implementing segmentation because the resulting qualities and profit margins are sufficiently high (i.e., region (*i*)). When $K < \widehat{K}$, however, this benefit becomes trivial. The firm is better off offering a single product to avoid the extra setup capacity, specifically, offering a standard product if consumer valuations are close (i.e., region (*ii*)) and offering a niche product if consumer valuations are apart (i.e., region (*iii*)).

Note that when the setup capacity $b = 0$, we have $\widehat{v_h} = v_l$ and $\widehat{K} = 0$. The implications of b is discussed in the next section.

2.3.5 Operations Cannibalization and Cost of Variety

Our discussion about the optimal strategy indicates that limited capacity introduces *operations* cannibalization because offering low-quality products cannibalizes the capacity that could be used to offer high-quality products. This is different from *market* cannibalization introduced by asymmetric information about consumer valuation. Specifically, operations cannibalization introduced by limited capacity intensifies as capacity decreases. And the effects of stronger operations cannibalization on the optimal strategy can be illustrated in two ways. First, operations cannibalization decreases the qualities for both segments when offering either a standard product or two products is optimal. This is different from market cannibalization which only distorts the quality of the low-quality product.

Second, strong operations cannibalization reduces the length of a product line to obtain higher profit margin from the high segment, as is similar to the effect of market cannibalization. However, a shorter product line associated with strong operations cannibalization can also be attributed to the setup capacity b, which recall from §2.3.1, represents the extra setup capacity required for offering two products relative to offering a single product. In particular, larger b favors shorter product line and smaller b favors longer product line. Obviously, for single product strategies, a decrease in b yields no effect on the firm's decisions. For the Product-Line strategy, however, a decrease in b means that less capacity is required for setup hence more capacity is available for actual production. In other words, decreases in b translate to higher qualities for both products which, in turn, translate into more benefit from customizing for both segments. Specifically, we have the following result on the setup capacity b.

Proposition 2.5. *Large setup capacity decreases product variety. Offering a standard product is optimal only when the setup capacity is sufficiently large, i.e.* $b > \beta_1$, *where*

$$\beta_1 = \begin{cases} K^S(1-2R)\left(\sqrt{1+\frac{n_l R^2}{n_h(1-2R)^2}}-1\right) & v_h \leq \frac{2(n_h+n_l)v_l}{(2n_h+n_l)} \\ K^S\frac{\sqrt{n_l}}{\sqrt{(n_h+n_l)}}\left(1-R-\sqrt{1-R\frac{(v_h+v_l)}{v_l}}\right) & \frac{2(n_h+n_l)v_l}{(2n_h+n_l)} < v_h \leq \sqrt{\frac{(n_h+n_l)}{n_h}}v_l \end{cases}$$

Recall that the extra setup capacity b represents the cost of product variety. In this context, β_1 denotes the minimum level of setup capacity above which the cost of variety is sufficiently high (i.e., $b > \beta_1$) such that offering a standard product could be optimal. Note that β_1 increases in v_h and $\lim_{v_h \to v_l} \beta_1 = 0$. This indicates that for given $b > 0$, there exists a corresponding $\widehat{v_h}$ such that if $v_h < \widehat{v_h}$, then $b > \beta_1$ is true. In other words, as long as offering two products requires extra setup capacity, the corresponding cost of variety could render offering a standard product optimal for small consumer differentiation. Graphically, this result means that region (ii) in Figure 2.1 always exists for positive values of b. In contrast, if $b = 0$, then there is no cost of variety. Hence offering a standard product is never optimal regardless of capacity and consumer valuation. However, the effects of operations cannibalization remain the same as the firm offers a niche product when capacity is low (i.e., $K < K^{nl}$) and offer two products when capacity is sufficiently high (i.e., $K \geq K^{nl}$).

2.4 Discussion of Results

In this section, we investigate how the dynamics between limited capacity and cannibalization affect capacity utilization and correspondingly, product variety, quantity, as well as qualities. In particular, we illustrate how these decisions change in capacity for a given set of market and production parameters, and compare these decisions to those with unconstrained capacity (Moorthy 1984).

2.4.1 Capacity Utilization

Recall from §2.3 that when capacity is limited, an increase in capacity allows a firm to achieve higher profit by increasing either product quality or quantity. Thus, as a natural extension, we explore in this section whether a firm would necessarily exhaust all available capacity when following the optimal product line strategy. This is answered by the following proposition.

Proposition 2.6. *A firm facing limited capacity may not necessarily exhaust all capacity. For given v_h, the firm may idle some of its capacity as follows:*

(i) *If* $\frac{2(n_h+n_l)v_l}{(2n_h+n_l)} < v_h$ *and* $b > \beta_2$*, idle* $K - K^N$ *units of capacity for* $K^N < K < \min[K^{ns}, K^{nl}]$

(ii) *If* $v_h < \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$ *and* $b > \beta_3$*, idle* $K - K^S$ *units of capacity for* $K^S < K < K^{sl}$*;*

where $\beta_2 = \frac{an_l v_l(\sqrt{(n_h+n_l)/n_l}-1)}{2c}(1-R)$*, and* $\beta_3 = K^S R\sqrt{\frac{n_l}{n_h}}$*.*

Proposition 2.6 presents some interesting circumstances in which a firm with limited capacity chooses not to use all available capacity. This capacity idling behavior only occurs when a single product strategy is optimal[5]. In these cases, increases in capacity do not affect product quality because the quadratic cost function dictates that it is never optimal to offer a consumer segment with a quality higher than the corresponding efficient level. In the meantime, neither do increases in capacity affect the firm's optimal strategy because the resulting qualities will be too low to justify higher product variety (e.g., switching from the Niche or Standard strategy to the Product-Line strategy) or higher quantity (e.g., switching from the Niche to the Standard strategy). Hence, the firm is better-off to idle any capacity that is more than the capacity thresholds K^N (for the Niche strategy) or K^S (for the Standard strategy).

From Proposition 2.6, we find that both consumer valuations and setup capacity play critical roles in the firm's capacity idling behavior. In particular, capacity idling occurs when consumer segments are neither too close (i.e., Proposition 2.6 (*i*)) nor too far apart (i.e., Proposition 2.6 (*ii*)). If consumer segments are close to each other in valuation (i.e., if $v_h < \frac{2(n_h+n_l)v_l}{(2n_h+n_l)}$, violating the condition of Proposition 2.6 (*i*)), then the marginal return of additional capacity increases faster when producing for both segments than it does when customizing only for the high segment. As a result, the firm switches its optimal strategy from offering a niche product to offering a standard product before the capacity is sufficient to offer the high segment with its efficient quality. On the other hand, if consumer segment are far apart (i.e., if $v_h > \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$, violating the condition of Proposition 2.6 (*ii*)), then the marginal return of additional capacity is always higher when customizing for only the high segment than it is when producing a standard product for both segments.

[5] Recall from Lemma 2.2 that the Product-Line strategy always exhausts all capacity for $K \leq \bar{K}$.

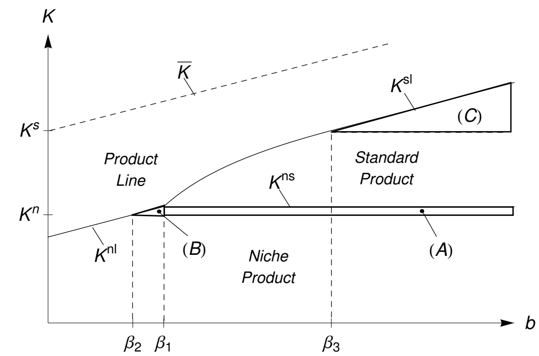


Figure 2.2: Impact of Setup Capacity b with $v_h = 1.5$

Proposition 2.6 also establishes setup capacity b as another driver for the firm's capacity idling decision. To better illustrate the impact of b, we provide a variant representation of the optimal strategy in Figure 2.2 in which the horizontal axis now represents the setup capacity b instead of the valuation for the higher segment as in Figure 2.1. Figure 2.2 demonstrates that capacity idling occurs only when the setup capacity is sufficiently large (regions (A),(B),(C)). Recall that setup capacity represents the additional capacity required to set up for two products as compared to the capacity required to set up for a single product. By definition, b only affects the Product-Line strategy but not the single product strategies. Hence, one would expect that the larger b is, the more reluctant the firm would be to switch to offering two products, hence the more likely capacity idling would occur.

Now comparing the size of idled capacity to setup capacity b, we have the following result.

PROPOSITION 2.7. *A firm facing limited capacity never idles more than b units of capacity.*

Proposition 2.7 demonstrates an important characteristic of capacity idling, i.e., the firm never idles more than the setup capacity. Recall that capacity idling occurs only when the firm has sufficient capacity to offer efficient quality to the intended segment. Given the conditions specified in Proposition 2.6, if the firm acquires b more units of capacity, then it should offer two products rather than a single product (e.g., regions (B) and (C)), or offer a standard product rather than a niche product (e.g., region (A)). Note that if the firm offers a single product and exhausts its capacity, then acquiring b more units of capacity does not guarantee switch to a new strategy.

2.4.2 Product Variety and Quantity

When capacity is not constrained, we know from (i.e., Moorthy 1984) that consumer valuation is the only driver for a firm's choice of product variety and quantity. Specifically, larger differentiation in valuation between the two consumer segments (e.g., high cannibalization) leads to lower product variety and quantity, in which case the firm is better-off serving only the high segment. When capacity is limited, however, we find that lower capacity is another driver for lower product variety and quantity. In particular, the firm reduces its product variety and quantity when capacity is limited. As we have discussed in §2.3, higher variety requires extra setup capacity which translates to lower effective capacity for the production and lower quality for each product. In contrast, lower variety can avoid this setup capacity and hence leave more effective capacity for production. In a similar vein, larger quantity means capacity is shared among more products hence each product consumes less capacity which again translates to lower quality. But smaller quantity means that each product can consume relatively more capacity and consequently, have higher quality and profit margin.

Moreover, we find that reductions in product variety and quantity do not necessarily happen simultaneously. For example, when consumer segments are close to each other in valuation (i.e., in the case when $v_h < \widehat{v_h}$ as illustrated in Figure 2.1), lower capacity makes it profitable for the firm to reduce its product variety so as to avoid setup capacity, but lower capacity does not affect the total quantity because the firm is better-off serving both segments. Only when capacity is significantly low does the firm reduce product quantity so as to have higher quality for one segment.

2.4.3 Product Quality and Price

In this section we investigate the optimal quality and price for each consumer segment. Specifically, we ask two questions: (*i*) How do they change in capacity? and (*ii*) how do they compare with the corresponding unconstrained levels. Here, we answer the questions for product quality. First, we know from §2.3 that product qualities are non-decreasing in capacity for a given strategy. However, qualities may decrease at the capacity thresholds where the firm's optimal strategy switches. If the firm switches from offering a niche

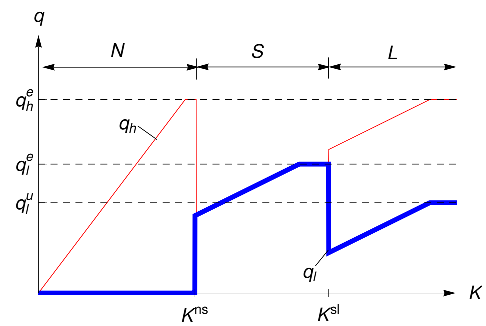


Figure 2.3: The Impact of Capacity on Product Qualities

product to offering a standard product, the high segment receives lower quality while the low segment receives higher quality (from 0 to positive quality) because the firm reallocates some capacity from serving the high segment to serving both segments. In contrast, if the firm switches from offering a standard product to offering two products, then the high segment receives higher quality while the low segment receives lower quality because, conversely, the firm reallocates some capacity from serving the low segment to increasing the quality for the high segment.

Comparing the optimal product quality for each segment under limited capacity to the corresponding unconstrained qualities q_h^u and q_l^u, we find that both the high and low segment generally receive lower-than-unconstrained quality when capacity is limited. However, the low segment may receive higher quality when capacity is limited than it does when capacity is unconstrained. This occurs when offering a standard product is optimal and capacity is moderate, as summarized in the following proposition.

PROPOSITION 2.8. *Under limited capacity, consumer segments generally receive lower qualities than they do under unconstrained capacity. However, the low segment receives higher quality under limited capacity, if* $K^S(1-R) < K \leq K^{sl}$.

Note that the range of capacity in Proposition 2.8 exists when consumer segments are relatively close in valuation and setup capacity is relatively large. In this case, close consumer valuations make it profitable for the firm to serve the low segment, while large setup capacity prevents the firm from offering two products. More interestingly, the low segment may even receive its corresponding efficient quality q_l^e. Figure 2.3 graphically illustrates these results about product quality by depicting the quality received by each

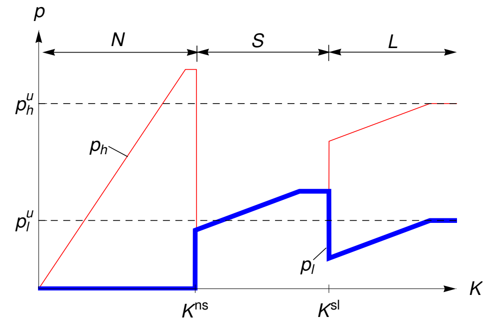


Figure 2.4: The Impact of Capacity on Product Prices

segment as a function of K. In this figure, N, S, and L, respectively, refer to the zones where Niche, Standard, and Product Line strategies are optimal. Note that $v_h = 1.5$ and $b = 0.4$ which corresponds to the case $b > \beta_1$.

Next, we investigate the price for each consumer segment and find similar results as for qualities. Specifically, the prices for both segments generally increase in capacity but may increase or decrease when the firm's optimal strategy switches. These changes are due to the corresponding changes in quality. If the firm switches from offering a niche product to offering either a standard product or two products, then the high (low) segment gets lower (higher) price due to lower (higher) quality. If the firm switches from offering a standard product to offering two products, then the high (low) segment gets higher (lower) price due to higher (lower) quality.

Now we compare the prices with the corresponding prices in the unconstrained capacity case p_h^u and p_l^u, and find similar results as we did for qualities. That is, both consumer segments generally pay lower prices than the corresponding unconstrained prices due to lower quality. However, we also find that limited capacity can lead to higher prices for both high and low segments, as summarized in the following proposition.

PROPOSITION 2.9. *Under limited capacity, consumer segments generally receive lower prices than they do under unconstrained capacity. However,*

(i) The high segment receives higher price under limited capacity, if $(n_l K^S R^2 + n_h K^N)\frac{v_l}{n_h v_h} < K \leq \max[K^{ns}, K^{nl}]$*;*

(ii) The low segment receives higher price under limited capacity, if $K^S(1 - R) < K < K^{sl}$.

Note that the reasons behind the higher prices are different across different consumer segments. In particular, the high segment gets higher price under limited capacity due to the lack of cannibalization. This occurs when offering a niche product for only the high segment is optimal. In this case, the firm can implement first-degree price discrimination and extract all the surplus from the high segment without violating the self-selection constraints. In contrast, the low segment gets higher price under limited capacity mainly due to higher quality. Figure 2.4 graphically illustrates these results by depicting the price received by each segment as a function of K.

2.4.4 Consumer Welfare

In this section, we investigate the impact of limited capacity on consumer welfare which is defined by the difference between consumer utility and price of the product. In the context of product line design for segmented market, Moorthy (1984) and Moorthy and Png (1992) conclude that when capacity is not constrained, the firm's pricing mechanism extracts all the surplus from the low segment but leaves a surplus of $CW_h^u = (v_h - v_l)q_l^u$ to the high segment. When capacity is limited, we find that the low segment still gets no surplus while the high segment can enjoy a surplus of $CW_h = (v_h - v_l)q_l^*$ as long as the low segment is served (e.g., the firm either offers a standard or two products). In fact, the high segment enjoys a higher surplus when capacity is limited than it does when capacity is not limited as long as $q_l^* > q_l^u$, as the following proposition shows.

Proposition 2.10. *The high segment enjoys a higher surplus under limited capacity than it does under unconstrained capacity, if* $v_h < v_l + \frac{2bc(\sqrt{1+n_l/n_h}-1)}{a(n_h+n_l)}$ *and* $K^S(1-R) < K < K^{sl}$.

From Proposition 2.10, we know that the high segment can enjoy a higher surplus when capacity is limited if consumer segments are relatively close and capacity is relatively high. In this case, the firm is better served by offering a standard product rather than customizing for the high segment. To illustrate the impact of capacity on consumer welfare, we depict the ratio of CW_h/CW_h^u in Figure 2.5 for $v_h = 1.2$ and $v_h = 1.6$. From Figure 2.5, we find that the surplus for the high segment generally is non-decreasing in capacity unless the optimal strategy switches. Moreover, the high segment is

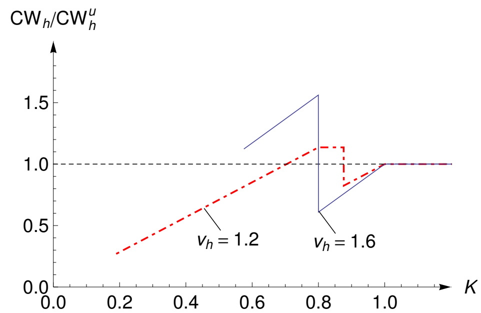


Figure 2.5: The Impact of Capacity on Consumer Welfare

Table 2.2: The Percentage of Naive Profit and Optimal Profit

		v_h				
		1.2	1.4	1.6	1.8	2.0
K	.1	0%	0%	0%	0%	0%
	.2	28%	26%	24%	23%	23%
	.4	52%	55%	49%	46%	45%
	.6	72%	72%	70%	69%	67%
	.8	89%	89%	88%	88%	88%

better-off (i.e., $CW_h/CW_h^u > 1$ as shown in Figure 2.5) when the Standard strategy is implemented and $q_l > q_l^u$.

2.4.5 Profit Loss for a Naive Firm

In this section, we want to examine the price of not considering capacity constraint in the decision-making process. Specifically, we examine a naive firm who ignores the impact of capacity and keeps the same product line design as in the unconstrained case. That is, the firm sets the quality at the unconstrained quality q_i^u where $i = h, l$. We assume that it serves each segment in the market with quantity proportional to their sizes. Therefore, the profit for a naive firm is $\Pi^I = \frac{(K-b)(n_h(v_h-v_l)^2+n_l v_l^2)}{2an_l v_l}$, where superscript I represents the naive firm. To see the implication of deviating from the optimal behavior, we calculate the profit ratio Π^I/Π^*. While analytical expressions are complicated, Table 2.2 is used to show the the results. Here, we set parameters such that $a = 1$, $b = .1$, $c = 1$, $n_h = .6$, $n_l = 1$, and $v_l = 1$. Also, we choose five levels of capacity $K = .1, .2, .4, .6, .8$ and five levels of valuation $v_h = 1.2, 1.4, 1.6, 1.8, 2.0$.

From the Table, we notice as expected that being ignorant would always result in a loss in profit. For given market, this loss increases as capacity decreases. For example, when $K = 0.8$ (the capacity is very close to the unconstrained capacity $\bar{K} = .9$), the profit of a naive firm is around 90% of corresponding optimal profit which indicates a 10% loss. As capacity decreases, the ignorant firm performs much worse. In particular, when $K = 0.2$, the resulted loss is more than 70%. More importantly, the rate of change for this ratio increases as capacity decreases. That is, for lower capacity, changing capacity would results in much more change in this ratio. Take $v_h = 1.2$ for example. When capacity decreases from $K = 0.8$ to $K = 0.1$, the rate of change in profit loss increases from 17% each 0.2 units of capacity to 28% each 0.1 unit of capacity. Therefore, managers should pay more attention to the product design decisions when the firm's capacity is relatively low.

2.5 Conclusions

In this chapter, we consider the effect of limited capacity on a monopoly firm's optimal product line design decisions. The firm operates in a market characterized by two consumer segments that differ in valuation and size. To serve this segmented market, the firm optimally designs its product line and correspondingly qualities, given limited capacity. Furthermore, its product line design affects both production cost and capacity consumption. On the cost side, product quality quadratically increases variable cost indicating an increasing cost to quality (Moorthy and Png 1992, Desai 2001). On the capacity side, the length of product line determines the fixed setup capacity such that offering two products requires b more units of capacity to setup than offering a single product does. Meanwhile, quality linearly increases variable capacity requirement, indicating a constant capacity to quality ratio. In this modeling context, we first conditionally solve the firm's design problem for each strategy and then we compare the corresponding profits to identify and characterize conditions on which insights from extant literature are challenged or even reversed. Then we discuss the impact of limited capacity on the firm's decisions.

Our analysis and discussion suggest that limited capacity can introduce *operations* cannibalization which intensifies as capacity decreases. This is in contrast to the *market* cannibalization introduced by differentiation in consumer valuations. While cannibalization always reduces the length of a product line (i.e., the number of product types), the exact source of cannibalization exhibits quite different effects on the firm's product line design decisions. For example, market cannibalization distorts the quality of the low-quality product. In contrast, operations cannibalization reduces the quality of each product. If two products are offered, then it reduces the qualities of both products at the same rate which leads to a constant width of product line (i.e., the differentiation in quality between two products).

We also find that offering a standard product for the whole market could be optimal when capacity is moderate and consumer segments are close in valuation. In this case, the firm prefers a shorter product line due to operations cannibalization, and it also prefers a larger sales volume because of weak market cannibalization. To a certain extent, operations constraint can be interpreted as an additional term in the objective function which represents the corresponding cost of certain resources. These operations cost generally introduces economy of scale because of lower unit cost. For example, Netessine and Taylor (2007) consider the cost of inventory and find offering a standard product is optimal when the corresponding cost parameter is relatively large and consumer valuations are close. In this chapter, we consider capacity and find similar results when capacity is relatively low. This is because the marginal return of capacity decreases in capacity, or in other words, lower capacity indicates higher shadow price for capacity.

Like any other model, our results rely on the modeling assumptions made. The most noteworthy with respect to capacity is that the firm consumes capacity in both fixed and variable way. We have shown that offering a standard product is never optimal when the setup capacity $b < \beta_1$. Note that when $v_h \to v_l$, $\beta_1 \to 0$. Hence, what's essential to this result is the assumption that $b > 0$. In other words, as long as a longer product line requires extra capacity to setup, then offering a standard product could be optimal when consumer segments are close to each other (i.e., $v_h \leq \widehat{v_h}$). However, a positive setup capacity is not required for us to understand the impact of limited capacity as an additional source of cannibalization. If $b = 0$, for example, then lower capacity favors a shorter product line with a niche product for the high segment only.

Also fundamental to our results is the assumption that variable capacity increases linearly in both quality and quantity. Given this modeling construct, our results show that the firm always prioritizes quantity over quality.

Consequently, the operations cannibalization introduced by limited capacity reduces the qualities of both products but does not affect the quantity of each product unless optimal product line strategy switches. We make this assumption for traceability and insights, but our insights are applicable to more general cases.

Chapter 3

Is Remanufacturing Environmentally Friendly?

3.1 Introduction

Remanufacturing is generally considered to be both profitable due to lower production cost (Caterpillar Press Release 2005) and environmentally beneficial due to reduced virgin material and waste (Guide 2000). Nevertheless, remanufacturing and its financial and environmental implications are complex. From the profitability perspective, for example, remanufacturing introduces a trade-off between current and future profits. Compared to a non-remanufacturer, a remanufacturer may increase sales of new products by reducing prices, sometimes even lower than marginal cost, so that more used products are available for remanufacturing. But, because not all customers are willing to pay as much for a remanufactured product as for a new one, the resulting cost saving associated with remanufacturing may not be enough to justify the reduced profit in the previous period.

Similarly, from the environmental perspective, remanufacturing can potentially have negative environmental consequences. This is true, for example, in ship remanufacturing where unprocessed scrap metals are oftentimes abandoned on the beach (CNN.com, 2008). Indeed, firms proliferated across many industries are guilty of greenwashing in the sense that they ride the "green wave" of environmental consumerism without necessarily considering whether or not their actions actually benefit the environment (Orange 2010). And exacerbating this effect, environmentally friendly products often entice consumers to increase their total consumption. As a March 2010 *Time Magazine* article titled *Less is Actually More* observes(Walsh 2010):

> ...studies indicate that people who install more-energy-efficient lights lose 5% to 12% of the expected savings by leaving them on longer ...

This suggests that consumption habits may compromise, if not totally offset, direct environmental benefits resulting from remanufacturing. Thus, the implicit presumption that remanufacturing benefits the environment warrants more rigorous examination.

In this chapter, we study a firm's design for remanufacturing problem and, perhaps more importantly, we examine the environmental consequences when remanufacturing is the optimal strategy. In particular, we investigate the conditions under which a firm makes its products remanufacturable. Accordingly, we develop a model to characterize the firm's optimal product portfolio of new and remanufactured products, and establish the corresponding optimal product design decisions. Then, we assess the environmental impact of these results. Consequently, we bridge the gap between profitability and environmental friendliness to ascertain whether or not the two are complementary, and in doing so, we identify key drivers that would make remanufacturing practices more environmentally friendly.

We approach these issues by modeling a firm's product design and remanufacturing decisions when serving a market with heterogenous consumers. On the product design front, the firm must choose to design either a non-remanufacturable product or to design a remanufacturable product; and the firm must also specify the corresponding quality. On the remanufacturing front, the firm must choose whether or not to remanufacture a given product if the product is designed to be remanufacturable. These choices impact both the manufacturer's cost structure and consumers' valuation. On the cost side, a remanufacturable product costs more to produce originally, but less to remanufacture, as compared to the cost of producing a non-remanufacturable product. On the consumer side, a remanufacturable product is valued more by consumers, but a remanufactured one is valued less, as compared to the valuation of a non-remanufacturable product. The firm thus must consider these trade-offs and optimally choose the design and corresponding quality and price.

To operationalize the firm's design for remanufacturing problem with a parsimonious model that captures remanufacturing fundamentals, we follow the lead of Ferrer and Swaminathan (2006) and Atasu et al. (2008) by formulating a two-stage analytic framework. At the beginning of stage 1, the firm first determines whether to design a remanufacturable product or a non-remanufacturable product and, correspondingly, establishes the quality of the chosen product. Then the firm sets the selling price for the product and sells an amount accordingly, as dictated by the specified consumer market's heterogeneity. Finally, to conclude stage 1, consumers who purchase the product extract its consumption value and then either discard the remains (which is the case if the product was designed to be non-remanufacturable) or recycle the remains (which is the case if the product was designed to be remanufacturable). The recycled amount, if applicable, thus establishes a supply constraint on the number of units that can be remanufactured for resale. Given that, at the beginning of stage 2, the firm's decision is to set its optimal product portfolio, that is, to determine how many units of new versus remanufactured products to produce and what associated prices to set accordingly for sale of each product type in stage 2.

This chapter specifically studies remanufacturing and its environmental consequences through a product design lens. Our modeling framework thus extends beyond the domain of product pricing into the domain of product design and pricing. Moreover, it sets the stage for assessing the environmental friendliness of profitable product design and pricing, and for exploring the consequences accordingly. As such, our model offers two benefits that constitute its primary contribution. The first benefit of our model is the incorporation of quality as an endogenous variable. Specifically, we formulate our model by explicitly building the firm's cost structure and consumers' valuation preferences on quality, which allows us to map the firm's optimal remanufacturing decisions onto quality space. As a result, we find that, everything else being equal, the firm would couple increased remanufacturing with higher quality. In addition, we find that quality significantly enriches the modeling interactions by providing a lever for manipulating the product mix of new versus remanufactured products offered in stage 2. For example, if quality were exogenous, then the firm could reap the cost benefits associated with remanufacturing in stage 2 only if it increased sales of new products in stage 1 by lowering price. However, with endogenous quality, the firm has the added opportunity of increasing product quality and extracting more profit from each product. Accordingly, we find that if the cost saving associated with remanufacturing is relatively low, the firm increases both remanufacturing and product quality and reduces the sales of new products in both stages so as to charge a higher margin. In contrast, however, if this cost saving associated with remanufacturing is relatively high and all returned products are remanufactured, the firm would increase quality by decreasing sales of both new and remanufactured products.

The second benefit of our model is the introduction of the notion of environmental damage, a quality-dependent analytical measure to quantitatively capture the environmental consequences of design for remanufacturing. This environmental measure provides a mechanism to assess the ecological footprint of product design in a remanufacturing context. Specifically, our measure focuses on the environmental impact associated with the extraction of virgin material and the discard of waste, but it can be extended to include the impact associated with transportation and consumption stages of a product life cycle as well. Moreover, this measure is robust in the sense that different weights can be assigned to the different stages of the product life cycle without altering the insights.

Perhaps most notable among these insights is that, in the context of our model, we find that environmental damage significantly increases when consumers are more environmentally conscious, that is, when consumers value the idea that a product *can be* remanufactured rather than the fact that the product *has been* remanufactured. This can be particularly detrimental to the environment if consumers value remanufacturable products on the one hand, but significantly devalue remanufactured products on the other hand. Intuitively, if consumers value the idea that a product can be remanufactured but not necessarily value remanufactured products, then the firm becomes reluctant to actually remanufacture any recycled units. Instead, it simply takes advantage of consumers' environmental consciousness by designing for remanufacturability, but avoids the potential cannibalization by not remanufacturing. This somewhat counterintuitive result reinforces the idea that consumption rather than production, per se, is the enemy of the environment. Thus, to paraphrase Orange (2010), the first step to a better environment is to reduce, not to recycle or to reuse.

In a similar vein, we also find that more cost efficient production technology (i.e., a lower production cost or a higher cost saving from remanufacturing) may not necessarily benefit the environment, despite increasing profit for the firm. In particular, the environment is worse off if production cost is sufficiently low. Intuitively, a lower production cost means a higher profit margin for new products, which in turn attracts the firm to provide a higher volume of new products, which thereby consumes more virgin resources and results in more discarded waste. Similarly, a higher cost saving from remanufacturing attracts the firm to design higher quality, which again ultimately results in more damage to the environment. Therefore, somewhat surprisingly, it is in the interest of the environment for production technologies not to be too efficient.

The remainder of this chapter is organized as follows. In §3.2, we review the literature and position this chapter accordingly. In §3.3, we specify and discuss our model primitives, and we formulate and solve the firm's resulting profit maximization problem by mapping out and cataloging different product design and remanufacturing strategies. Then in §3.4, we compare the different strategies to determine the firm's optimal decisions, and we explore implications accordingly. Section 3.5 defines the environmental damage measure that we use as an index to evaluate the impact on the environment resulting from the firm's optimal strategy. We discuss the scope and applicability of our model in §3.6, and we conclude the chapter in §3.7. Detailed proofs appear in Appendix.

3.2 Relation to Literature

Our research relates to two streams of literature. One stream is the literature on remanufacturing. Within this realm, there are two different foci. One focus is defined primarily by operational issues and thereby takes a cost-minimization approach to determine optimal system design. This approach assumes price, demand, and remanufacturability to be exogenous, and generally applies to issues such as logistics, production planning, and inventory control (Fleischmann et al. 1997, Toktay et al. 2000). The other focus is defined primarily by pricing issues and thereby takes a profit-maximization approach to addresses such issues as market segmentation (Debo et al. 2005, Ferrer and Swaminathan 2006, Atasu et al. 2008) and competition (Majumder and Groenevelt 2001, Ferrer and Swaminathan 2006, Ferguson and Toktay 2006). This profit-maximization approach captures richer interactions between price and quantity trade-offs. In particular, on the one hand, the firm has incentive to lower the first-stage new product price to boost sales in order to achieve cost savings realized by selling remanufactured products in the second stage. However, on the other hand, such remanufacturing would cannibalize new product sales in the second stage, thus creating a disincentive for remanufacturing. Considering these trade-offs, Majumder and Groenevelt (2001) analyze competition between an OEM and a local firm. They find that a social planner should provide incentives to the OEM to increase remanufacturing. Debo et al. (2005) study segmentation and remanufacturability in an infinite-horizon setting and find that both fixed and variable costs affect remanufacturing in a negative way. Ferrer and Swaminathan (2006, 2010) study both finite-horizon and infinite-horizon problems in a competition setting and conclude that remanufacturing decisions become stable after initial states. Ferguson and Toktay (2006) investigate remanufacturing as a strategy to deter market entry. Common to these models is the assumption that both new and remanufactured products are indistinguishable or that remanufactured products are discounted relative to new products. None of these models, however, define quality as endogenous. In contrast, our model, while similar to this second focus of remanufacturing literature, includes product quality as a decision in addition to price and quantity.

The second stream of related literature is that on product design, which we trace to Mussa and Rosen (1978) and to Moorthy (1984). This stream studies the optimal quality and pricing decisions of a product line that is differentiated by quality when serving consumers heterogenous in their willingness to pay. Results indicate that a low-quality product cannibalizes the sales of a high-quality product and, as a result, the firm responds by increasing the differentiation between the high- and low-quality products. Our problem is similar to this product line problem in that the new and remanufactured products in our model can be considered as the high- and low-quality products in the product line problem. However, our problem is different from the product line problem in two respects. The first difference is the remanufacturing supply constraint that restricts the number of remanufactured products to be no greater than the number of recycled products available. The second difference is consumer perception of quality. In our remanufacturing setting, both products have the same quality level. Thus, differentiation between products is due to the fact that consumers discount their valuation of remanufactured products because these products were used previous to being remanufactured.

Relative to these literatures, our framework underscores the fundamental interactions between three remanufacturing related factors, namely, supply constraint, cost saving, and cannibalization. The supply constraint exists because, in our remanufacturing setting, there is a natural upper limit on the number of recycled products that are available to remanufacture. Without this constraint, the second stage problem would resemble a product line problem. Cost saving is a direct driver of remanufacturing. To achieve cost saving in stage 2, the firm needs to incur higher variable cost in stage 1 to make its new product remanufacturable. Finally, remanufacturing introduces cannibalization into the second stage when both products are offered to the same market. Because remanufactured products are less expensive substitutes to the new products, they have the potential to reduce the firm's ability to extract surplus from consumers with high willingness to pay.

3.3 The Model

We specify the modeling assumptions that define the firm, the heterogenous consumers, and the decision making framework in §3.3.1. Then we develop and conditionally solve the firm's design for remanufacturing problem based first on the stipulation that the firm does not remanufacture any products in stage 2 (§3.3.2) and subsequently on the stipulation that the firm does remanufacture products in stage 2 (§3.3.3).

3.3.1 Modeling Assumptions

The Firm. We define the firm as a profit maximizer that can both *manufacture new* products and *remanufacture used* ones depending on its product design choices. These design choices include product quality and remanufacturability. We model product quality, denoted by q, as a one-dimensional measure representing all the components/attributes that consumers prefer. We model remanufacturability, denoted by k, such that $k = 0$ represents a *non-remanufacturable* product and $k = 1$ represents a *remanufacturable* product[1]. Once determined, the product design cannot be changed. Moreover, a remanufacturable product can be remanufactured at most once.

Given product quality q and remanufacturability k, the firm incurs a variable cost for each unit of its product produced[2]. Consistent with the product

[1]We will investigate the implications of relaxing this assumption so that $k \in [0, 1]$ is a continuous variable in Section 6.

[2]Although in some remanufacturing models the firm incurs a fixed cost associated with developing and choosing production technology, for tractability, we adopt the convention

design literature, the variable cost of production is quadratic in quality, but the magnitude of this variable cost depends on whether the unit produced is a new non-remanufacturable product, a new remanufacturable product, or a remanufactured product. Specifically, we use q^2 to model the variable cost per unit to produce a new *non-remanufacturable* product of quality q, we use $(1 + c_1)q^2$ to model the per unit variable cost to produce a new *remanufacturable* product (i.e., we use $c_1 q^2$ to denote the cost premium associated with producing a new remanufacturable unit over producing a new non-remanufacturable unit), and we use $(1-c_2)q^2$ to model the per unit variable cost to produce a *remanufactured* product (i.e., we use $c_2 q^2$ to denote the cost savings associated with remanufacturing a recycled unit over producing a new non-remanufacturable unit). Note that $c_1 \geq 0$, which means that producing a new product that *can* be remanufactured is more expensive than producing a new product that *cannot* be remanufactured, and that $0 < c_2 \leq 1$, which means that *remanufacturing* a recycled product is less expensive than *manufacturing* a new product. Our assumptions are consistent with Debo et al. (2005) in that our new product unit cost increases in k. Given this construct, to facilitate our presentation, we denote $g_n(k) = 1+c_1 k$ for $k = 0, 1$ as the production cost coefficient associated with a new product, which depends on whether or not the new product is remanufacturable, and we denote $g_r = 1 - c_2$ as the remanufacturing cost coefficient which is valid only when $k = 1$.

Consumers. Consumers differ vertically in their willingness to pay, and they differentiate whether a product is new and *non-remanufacturable*, new and *remanufacturable*, or used but *remanufactured*. Specifically, we model consumers' valuation for a new *non-remanufacturable* product of quality q as vq, where v follows a uniform distribution along $[0, 1]$[3]. Comparatively, we model consumers' valuation for a new *remanufacturable* product of quality q as $(1 + \theta)vq$ (i.e., we use θvq to denote consumers' valuation premium associated with purchasing a new remanufacturable product over a new non-

of Atasu et al. (2008) and normalize this fixed cost to zero. However, incorporating a fixed cost such as this in our model is straightforward.

[3]In this chapter, we assume a uniform distribution for consumer valuation of quality for several reasons: 1) it allows us to focus on the cannibalization between new and remanufactured products; 2) it enables us to derive closed form results; 3) it is a widely adopted assumption in remanufacturing literatures. Hence this assumption allows us to compare and contrast our results to those of others.

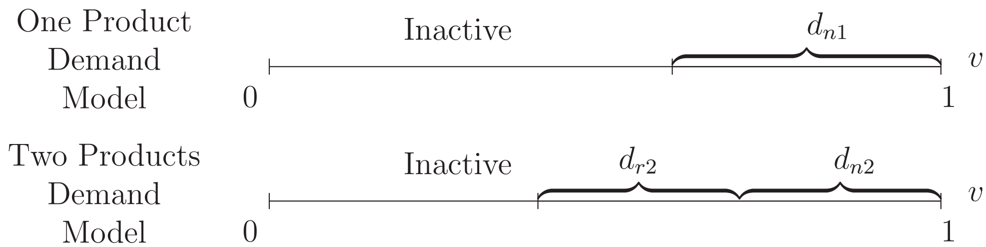


Figure 3.1: Heterogenous Consumers

remanufacturable one), and we model consumers' valuation for a *remanufactured* product as $(1-\alpha)vq$ (i.e., we use αvq to denote the valuation discount associated with purchasing a remanufactured product over a new non-remanufacturable one). Note that $\theta \geq 0$, which means that consumers value a new product that *can* be remanufactured more than one that *cannot* be remanufactured, and that $\alpha \geq 0$, which means that consumers value a used product that *has been* remanufactured less than a new one that *has not been* remanufactured. Analogous to $g_n(k)$ and g_r, we denote $f_n(k) = 1 + \theta k$ for $k = 0, 1$ and $f_r = 1 - \alpha$ to facilitate our presentation.

Consumers purchase to maximize their non-negative surplus, which is defined by the difference between valuation and price paid. Let p_n and p_r be the price of a new and a remanufactured product, respectively. The surplus derived by a consumer of type v from buying a new and a remanufactured product characterized by q and k is $f_n(k)vq - p_n$ and $f_r vq - p_r$, respectively. If only new products are available, then consumers choose to purchase the new product if $f_n(k)vq - p_n \geq 0$. If both new and remanufactured products are available, then consumers choose the new product over the remanufactured one if $f_n(k)vq - p_n \geq f_r vq - p_r$ and vice versa, but if consumers cannot derive nonnegative surplus from either product then they will remain inactive (i.e., they will purchase neither product), which occurs for consumers with low valuation as shown in Figure 3.1.

In the first stage, only new products can be sold regardless of product design. Thus the demand for new products in the first stage, d_{n1}, is

$$d_{n1} = 1 - \frac{p_{n1}}{f_n(k)q} \tag{3.1}$$

where p_{n1} is the new product price in the first stage, q is the quality, and

$k = 0, 1$ is the remanufacturability. If the product is designed to be non-remanufacturable (i.e., if $k = 0$), then in the second stage, again only new products can be sold. In this case, the corresponding second stage demand is similar to (3.1): $d_{n2} = 1 - \frac{p_{n2}}{f_n(k)q}$, where p_{n2} is the second stage new-product price. If the product is designed to be remanufacturable (i.e., if $k = 1$), then in the second stage the firm must choose whether or not to remanufacture. If the firm chooses not to remanufacture any units in the second stage, then only new products can be sold and the corresponding demand is again similar to (3.1). In contrast, if the firm chooses to remanufacture, then a fraction of consumers purchase new products, a fraction of consumers purchase remanufactured products, and the remainder of consumers purchase no products[4]. In this case, the corresponding second stage demands for new and remanufactured products, d_{n2} and d_{r2}, respectively, are

$$
\begin{aligned}
d_{n2} &= 1 - \frac{p_{n2} - p_{r2}}{(f_n(1) - f_r)q}, \qquad \text{and} \\
d_{r2} &= \frac{p_{n2} - p_{r2}}{(f_n(1) - f_r)q} - \frac{p_{r2}}{f_r q}
\end{aligned}
\tag{3.2}
$$

where p_{r2} is the remanufactured product price in the second stage.

The Decision Framework. Consistent with the related remanufacturing literature (Majumder and Groenevelt 2001, Ferrer and Swaminathan 2006, Ray et al. 2005, Atasu et al. 2008), we use a two-stage model to formulate the firm's design for remanufacturing problem. At the beginning of stage 1, the firm first chooses its product quality and remanufacturability. Then the firm sets a corresponding selling price and produces only new products. At the end of stage 1, these units either are discarded (which is the case if $k = 0$) or are collected through recycling (which is the case if $k = 1$)[5]. Finally, in stage 2, the firm sets the prices for new and (or) remanufactured products and produces the corresponding quantities subject to the remanufacturing supply constraint imposed by the new product sales in the first stage.

[4]Given that $\alpha \geq 0$ by assumption, consumers value remanufactured products less than they value new products. Thus, consumers who buy new products have higher v than those who buy remanufactured ones. This is depicted in Figure 3.1, which illustrates that d_{n2} is to the right of d_{r2}.

[5]Throughout our analysis, we assume 100% collection rate for the remanufacturable products at the end of stage 1. Similar to Atasu et al. (2008), we assume that the collection cost is linear in the quantity collected and is included in the remanufacturing cost $g_r q^2$. For the case in which the collection rate is less than 100%, our results are not significantly altered as long as the collection rate is not too small.

Given this, we next develop and solve the firm's profit maximization problem for given quality q to establish a strategy space for product design that depends on whether or not remanufacturing takes place. We first investigate the case in which there is no remanufacturing (i.e., the case in which $d_{r2} = 0$). We refer to this case as the *Non-Green* strategy. Then we study the case in which there is remanufacturing (i.e., the case in which $d_{r2} > 0$). We refer to this case as the *Green* strategy[6]. Later, in Section 4, we compare the profit of the optimal Green strategy to that of the optimal Non-Green strategy to establish conditions indicating when remanufacturing is optimal.

3.3.2 Non-Green Strategy

We define the Non-Green strategy as one in which only new products are sold in each period. In other words, by definition of this strategy, $d_{r2} = 0$. Nevertheless, at the design stage, two substrategies exist: either design the product to be non-remanufacturable ($k = 0$), which we refer to as the *Traditional* substrategy (T); or design the product to be remanufacturable ($k = 1$), which we refer to as the *Premium* substrategy (M). In either substrategy, the demand function is the same for both stage 1 and stage 2 and is given by (3.1). Accordingly, if the firm implements the Non-Green strategy, then $p_{n1} = p_{n2} = p$ and the total profit for the two stages is

$$\Pi^{NG}(p, q, k) = 2\left(1 - \frac{p}{f_n(k)q}\right)(p - g_n(k)q^2), \tag{3.3}$$

which is concave in p for given quality q and remanufacturability k. Optimizing (3.3) over p for given q and k, we have $p(q,k) = \frac{(f_n(k)+g_n(k)q)q}{2}$. Substituting this for p in (3.3), the resulting profit written as a function of q and k, is

$$\Pi^{NG}(q, k) = \frac{q(f_n(k) - g_n(k)q)^2}{2f_n(k)}. \tag{3.4}$$

Let $q^{NG}(k)$ denote the optimal product quality as a function of k given that the Non-Green strategy is implemented. Then, optimizing (3.4) over q yields

[6] Notice that in this chapter, our notion of "greenness" is defined by whether or not products are remanufactur<u>ed</u> as opposed to whether or not they are merely remanufactur<u>able</u>. In other words, we define "green" to mean not only that $k = 1$ but also that $d_{r2} > 0$.

$q^{NG}(k) = \frac{f_n(k)}{3g_n(k)}$ for $k = 0, 1$. This, from (3.4), implies that

$$\Pi^{NG}(k) \equiv \Pi^{NG}(q^{NG}(k), k) = \frac{2f_n(k)^2}{27g_n(k)} = \begin{cases} \frac{2}{27}, & \text{if } k = 0; \\ \frac{2(1+\theta)^2}{27(1+c_1)}, & \text{if } k = 1. \end{cases} \qquad (3.5)$$

Given (3.5), let $\Pi^T \equiv \Pi^{NG}(k = 0)$ denote the firm's profit if the Traditional substrategy is implemented, and let $\Pi^M \equiv \Pi^{NG}(k = 1)$ denote the firm's profit if the Premium substrategy is implemented. Then, comparing Π^T to Π^M leads directly to the following proposition.

PROPOSITION 3.1 (Optimal Non-Green Strategy). *Let* $(k^{NG*}, q^{NG*}, p^{NG*})$ *denote the optimal design and pricing decisions, given that* $d_{r2} = 0$*; and let* d^{NG*} *and* Π^{NG*} *denote the associated two-stage total demand and profit, respectively. Then, the Premium substrategy is optimal (i.e.,* $k^{NG*} = 1$*) if and only if* $1 + c_1 < (1 + \theta)^2$*. Accordingly,*

	k^{NG*}	q^{NG*}	p^{NG*}	d^{NG*}	Π^{NG*}
$1 + c_1 < (1 + \theta)^2$	1	$q^M = \frac{(1+\theta)}{3(1+c_1)}$	$\frac{2(1+\theta)^2}{9(1+c_1)}$	$\frac{2}{3}$	$\Pi^M = \frac{2(1+\theta)^2}{27(1+c_1)}$
$1 + c_1 \geq (1 + \theta)^2$	0	$q^T = \frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{3}$	$\Pi^T = \frac{2}{27}$

Recall that $1 + \theta$ represents a consumer's valuation coefficient for a remanufacturable product and that $1 + c_1$ represents the corresponding cost coefficient for the remanufacturable product. Thus, in essence, Proposition 3.1 indicates that if the marginal valuation for a remanufacturable product justifies the marginal cost of producing the product, then designing for remanufacturing is justified even though no units are actually remanufactured subsequently. In other words, even under the intention not to remanufacture, it still would be in the firm's best interest to design a remanufacturable product under the right circumstances. This indicates that the valuation premium θ is more representative of environmental consciousness, or the *idea* of a green world, more than it is representative of actual greenness, per se. In this sense, θ is akin to consumers "talking a green talk", but it is not indicative of whether or not consumers are willing to follow through by actually "walking the green walk".

Intuitively, because profit margin is quadratic in consumer valuation and linear in variable production cost, if $(1 + \theta)^2 > 1 + c_1$, then the valuation premium has a stronger positive effect on profit than the production cost has a negative effect. Thus, the increased production cost associated with remanufacturable products is justified. Interestingly, however, demand is not affected by remanufacturability. Instead, the market effects of remanufacturability are realized through an increased product quality coupled with a correspondingly increased price, which ultimately results in a higher profit margin but not a higher demand volume.

3.3.3 Green Strategy

We define the Green strategy as one in which remanufactured products are sold in the second stage. In other words, by definition of this strategy, $d_{r2} > 0$. Accordingly, given this strategy, the firm designs the new product to be remanufacturable (i.e., $k = 1$). In stage 2, then, the firm optimally determines the product portfolio of new and remanufactured products knowing that the remanufactured products will cannibalize the demand for new products. We therefore categorize the Green strategy into two substrategies depending on the firm's level of remanufacturing in stage 2. If all recycled products are remanufactured (i.e., if $d_{n1} = d_{r2} > 0$), then we say the firm implements the *Complete Remanufacturing* substrategy (CR). In contrast, if fewer than all recycled products are remanufactured (i.e., if $d_{n1} > d_{r2} > 0$), then we say the firm implements the *Partial Remanufacturing* substrategy (PR).

In either of these substrategies, the demand functions for stage 1 and stage 2 are given by (3.1) and (3.2), respectively. Accordingly, if the firm implements the Green strategy, then $k = 1$. Thus, to simplify notation, let $f_n \equiv f_n(1) = 1 + \theta$ and $g_n \equiv g_n(1) = 1 + c_1$. Then, the total profit for the two stages is

$$\begin{aligned}\Pi^G(p_{n1}, p_{n2}, p_{r2}, q) = d_{n1}(p_{n1} - g_n q^2) \\ + d_{n2}(p_{n2} - g_n q^2) + d_{r2}(p_{r2} - g_r q^2)\end{aligned} \tag{3.6}$$

Given (3.6), note that the Green strategy would be implemented only if $g_r/f_r < g_n/f_n$ because, otherwise, the Green strategy would be dominated by the Non-Green strategy of Section 3.2. (Please see Lemma 1 in Appendix for details.) Accordingly, in mapping out the Green strategy, we implicitly assume that $g_r/f_r < g_n/f_n$ to ensure that $d_{r2} > 0$. Optimizing (3.6) for given q, subject to the supply constraint $d_{r2} \leq d_{n1}$, thus yields Lemma B.2, which also is provided in Appendix. Lemma B.2 characterizes $(p^G_{n1}(q), p^G_{n2}(q), p^G_{r2}(q))$, the optimal two-stage pricing decisions as functions of q, given that the Green strategy is implemented, as well as $(d^G_{n1}(q), d^G_{n2}(q), d^G_{r2}(q))$, the corresponding optimal demands. This leads to the following proposition.

PROPOSITION 3.2. *If the Green strategy is implemented, then the optimal two-stage prices and demands, as functions of q, are such that:*

i) $\frac{\partial p^G_{n2}(q)}{\partial q} > \frac{\partial p^G_{n1}(q)}{\partial q} \geq \frac{\partial p^G_{r2}(q)}{\partial q} > 0$*;*

ii) $\frac{\partial d^G_{n2}(q)}{\partial q} < \frac{\partial d^G_{n1}(q)}{\partial q} < 0$*, whereas* $\frac{\partial d^G_{r2}(q)}{\partial q} > 0$ *for* $q < q_1$ *but* $\frac{\partial d^G_{r2}(q)}{\partial q} < 0$ *for* $q > q_1$*,*

where $q_1 = \frac{f_n f_r (f_n - f_r)}{f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n}$ *denotes the quality threshold above which* $d^G_{r2}(q) = d^G_{n1}(q)$*, but below which* $d^G_{r2}(q) < d^G_{n1}(q)$*.*

According to Proposition 3.2, the prices for both new and remanufactured products increase as quality increases, given that the Green strategy is implemented. Moreover, the stage 2 new product price increases faster than the stage 1 new product price, which increases faster than the stage 2 remanufactured product price. To understand this, consider that three factors — new product cost, cannibalization, and remanufacturing cost saving — influence prices. While new product cost positively influences all prices, the other two factors affect prices differently depending on the product type and the stage. In particular, the stage 2 new product price is positively affected by cannibalization, the stage 1 new product price is negatively affected by remanufacturing cost saving, and the stage 2 remanufactured product price is negatively affected by both cannibalization and remanufacturing cost saving. Therefore, intuitively, the firm has more incentive to increase its new product price in stage 2 than in stage 1 due to the cannibalization from the remanufactured products in stage 2.

As Proposition 3.2 also illustrates, if the Green strategy is implemented, then the sales volume of new products in both stage 1 and stage 2 decreases as quality increases. In contrast, the corresponding sales volume of remanufactured products first increases and then decreases in q. Intuitively, when quality is sufficiently low (specifically, when $q < q_1$), the firm does not remanufacture all recycled products from stage 1. In that case, an increase in quality means larger cost savings from remanufacturing, hence the firm remanufactures more. However, when quality is sufficiently high (specifically, when $q > q_1$), the firm remanufactures all recycled products from stage 1. In that case, an increase in quality not only affects the profit of remanufactured products in stage 2, but also that of new products in stage 1. And because the marginal value of remanufacturing a unit is smaller than the marginal loss of producing a unit of new product, the firm remanufactures less.

Given (3.6), let $\Pi^G(q) = \Pi^G(p_{n1}^G(q), p_{n2}^G(q), p_{r2}^G(q), q)$ denote the profit associated with the Green strategy, reduced to a function of q only. Lemma B.3, also provided in Appendix, establishes that $\Pi^G(q)$ is continuous and unimodal in q. Maximizing $\Pi^G(q)$ accordingly leads to the following proposition.

PROPOSITION 3.3 (Optimal Green Strategy). *Let $(q^{G*}, p_{n1}^{G*}, p_{n2}^{G*}, p_{r2}^{G*})$ denote the optimal design and pricing decisions given that $d_{r2} > 0$; and let d^{G*} and Π^{G*} denote the associated two-stage total demand and profit, respectively. Then by definition $k^{G*} = 1$, and*

$$q^{G*} = \begin{cases} q_S^G = \frac{f_n+f_r}{3(g_n+g_r)}, & 0 \le g_r \le g_{rS}; \\ q_C^G = \frac{f_n^2(g_n+g_r)+g_n(f_n^2-f_r^2)}{3(f_n g_n^2+f_r g_r^2+(f_n-f_r)(g_n+g_r)^2)}\left(2-\sqrt{1-\frac{3(f_n^2+f_nf_r-f_r^2)(f_rg_n-f_ng_r)^2}{(f_n^2(g_n+g_r)+g_n(f_n^2-f_r^2))^2}}\right), & g_{rS} < g_r \le g_{rC} \\ q_P^G = \frac{2f_nf_r(f_n-f_r)g_n}{3((f_rg_n-f_ng_r)^2+2f_r(f_n-f_r)g_n^2)}\left(2-\sqrt{1-\frac{3(f_rg_n-f_ng_r)^2}{2f_r(f_n-f_r)g_n^2}}\right), & g_{rC} < g_r \le \frac{f_rg_n}{f_n}; \end{cases}$$

where $g_{rS} = \frac{f_rg_n}{f_n} - \frac{(f_n^2-f_r^2)g_n}{f_n(3f_n-2f_r)}$ and $g_{rC} = \frac{f_rg_n}{f_n} - \frac{4f_rg_n(f_n-f_r)}{2f_n^2+3f_nf_r-3f_r^2}$. Accordingly, for $j = n1, n2, r2$:

	p_j^{G*}	d^{G*}	Π^{G*}
$0 \le g_r \le g_{rS}$	$p_j^G(q_S^G)$	$2d_{n1}^G(q_S^G)$	$\Pi^G(q_S^G)$
$g_{rS} < g_r \le g_{rC}$	$p_j^G(q_C^G)$	$2d_{n1}^G(q_C^G) + d_{n2}^G(q_C^G)$	$\Pi^G(q_C^G)$
$g_{rC} < g_r \le \frac{f_rg_n}{f_n}$	$p_j^G(q_P^G)$	$d_{n1}^G(q_P^G) + d_{n2}^G(q_P^G) + d_{r2}^G(q_P^G)$	$\Pi^G(q_P^G)$

Note that $g_{rS} < g_{rC} < \frac{f_rg_n}{f_n}$. Thus, Proposition 3.3 characterizes the optimal green quality based on the remanufacturing cost g_r, indicating that this optimal quality increases as g_r decreases, which is intuitive. Accordingly, from Proposition 3.2, the prices increase, and, correspondingly, the sales volumes of new products decrease while the sales volume of remanufactured products first increases and then decreases. Notice also from Proposition 3.3 that if the Green strategy is implemented, then g_{rC} establishes the critical remanufacturing cost threshold below which the Complete Remanufacturing substrategy is optimal ($d^*_{r2} = d^*_{n1}$), and above which the Partial Remanufacturing substrategy is optimal ($0 < d^*_{r2} < d^*_{n1}$). Similarly, g_{rS} establishes the critical remanufacturing cost threshold below which it is optimal not only to remanufacture all recycled products from stage 1 ($d^*_{r2} = d^*_{n1}$), but also to sell *only* remanufactured products in stage 2 ($d^*_{n2} = 0$).

3.4 Optimal Design for Remanufacturing

In this section, we compare the profit associated with the optimal Green strategy (Proposition 3.3) to the profit associated with the optimal Non-Green strategy (Proposition 3.1) to determine conditions for optimality. Specifically, we apply Propositions 3.3 and 3.1 to answer three questions: When is it optimal to *go green*? When is it optimal to go *completely* green? When is it optimal to design *for* green but *not* to go green?

3.4.1 When is it Optimal to Go Green?

In the context of our model, going green means that $d_{r2} > 0$ in an optimal solution. Thus, to answer this question, we directly compare the profit associated with the optimal Green strategy from Proposition 3.3 to that associated with the optimal Non-Green strategy from Proposition 3.1. This yields the following proposition.

Proposition 3.4 (Going Green). *If $g_r/f_r < g_n/f_n < f_n$, then it is optimal to go green; that is, it is optimal not only to design a remanufacturable product ($k^* = 1$), but also to sell remanufactured products ($d^*_{r2} > 0$).*

Proposition 3.4 provides a sufficient condition indicating when the Green strategy in Proposition 3.3 is optimal. Notice that this condition is twofold: First, for remanufacturing (i.e., the Green strategy) to be optimal, it must be worthwhile to produce a remanufacturable product in the first place (re: $g_n < f_n^2$). But that is not enough. Second, for remanufacturing to be optimal, it must also be worthwhile to actually remanufacture the remanufacturable products (re: $g_r/f_r < g_n/f_n$). The first half of this sufficient condition ($g_n < f_n^2$) is equivalent to that in Proposition 3.1, thus indicating when it is worthwhile to design for remanufacturability regardless of whether or not remanufacturing will actually occur. For insight into the second half of the sufficient condition, we note that the ratio g_i/f_i can be interpreted as a market cost efficiency ratio in the following sense: Consider a consumer of type v. If that consumer were to purchase a new product with quality q, which would cost $g_n q^2$ to produce, then the corresponding consumer valuation would be $f_n vq$. Thus, the cost per unit valuation of this new product is $g_n q^2/(f_n vq) = g_n q/(f_n v)$. Likewise, the cost per unit valuation of the corresponding remanufactured product is $g_r q/(f_r v)$. Therefore, Proposition 3.4 essentially indicates that it is optimal to go green if it is more cost efficient, as measured against market valuation, to *remanufacture* a remanufacturable unit of quality q than it is to *manufacture* a remanufacturable unit of quality q (i.e., if $g_r q/(f_r v) < g_n q/(f_n v)$ or $g_r/f_r < g_n/f_n$). Note that this condition is consistent with those in Atasu et al. (2008) and in Ferrer and Swaminathan (2010), both of which consider models of exogenous quality for the special case in which $f_n = g_n = 1$. Thus, Proposition 3.4 extends their results to our case of endogenous quality.

3.4.2 When is it Optimal to Go Completely Green?

In the context of our model, going completely green means that $0 < d_{r2} = d_{n1}$ in an optimal solution. Thus, to answer this question, we again compare the profit associated with the optimal Green strategy from Proposition 3.3 to that associated with the optimal Non-Green strategy from Proposition 3.1.

Proposition 3.5 (Going Completely Green). *If $g_n < f_n^2$ and $g_r \leq g_{rC} < g_n f_r/f_n$, then it is optimal to go completely green; that is, it is optimal not only to sell remanufactured products ($d_{r2}^* > 0$), but also to exhaust all available supply of recycled products in doing so ($d_{r2}^* = d_{n1}^*$).*

Proposition 3.5 provides a sufficient condition indicating when remanufacturing all available supply of recycled products is optimal. Notice that, again, this condition is two-fold. Consistent with Proposition 3.4, for complete remanufacturing to be optimal, not only must it be worthwhile to first produce remanufacturable products and then to sell remanufactured products (re: $g_n < f_n^2$ and $g_r < g_n f_r/f_n$), but also it must be worthwhile to remanufacture all of the recycled products (re: $g_r \leq g_{rC}$). Intuitively, for complete remanufacturing to be optimal, the market cost efficiency ratio of a remanufactured product must be sufficiently lower than that of a new product, namely $g_r/f_r \leq g_{rC}/f_r = g_n/f_n - \frac{4g_n(f_n - f_r)}{2f_n^2 + 3f_n f_r - 3f_r^2} < g_n/f_n$. If the market cost efficiency ratio of a remanufactured product were to exceed this threshold, then the potential cannibalization of new product sales would dominate the potential benefits of selling a marginal unit of the remanufactured products, in which case the firm would be better served by not going completely green.

3.4.3 When is it Optimal to Design for Green but Not to Go Green?

In the context of our model, designing for green but not going green means that $k = 1$ but $d_{r2} = 0$ in an optimal solution. Thus, to answer this question, first recall from Lemma B.1 (in Appendix) that $d_{r2}^* = 0$ if and only if $g_n/f_n \leq g_r/f_r$. Combining this with Proposition 3.1 then leads to the following.

PROPOSITION 3.6 (Designing for Green but Not Going Green). *If and only if $g_n/f_n \leq \min[f_n, g_r/f_r]$, then it is optimal to design for green but not to go green; that is, it is optimal to design a remanufacturable product ($k^* = 1$), but not to sell remanufactured products ($d_{r2}^* = 0$).*

Thus, consistent with Proposition 3.4, Proposition 3.6 indicates that, if the market cost efficiency ratio of a new remanufacturable product is lower than that of a remanufactured product (i.e., if $g_n/f_n \leq g_r/f_r$), then it is not worthwhile to remanufacture any units even though it is worthwhile to design remanufacturable products to begin with (i.e., even though $g_n/f_n < f_n$). Basically, under these conditions, optimality dictates cashing in on high profit margins in stage 1 by designing *for* green, but then avoiding cannibalization by *not* going green after all.

3.4.4 Discussion on the Optimal Quality and Demand

Note that, by combining Proposition 3.1 with Propositions 3.4–3.6, we can write the optimal quality (denoted by q^*), given the condition that $g_n/f_n \leq$

f_n, as follows:

$$q^* = \begin{cases} q^{G*}, & \text{if } g_r/f_r < g_n/f_n; \\ q^M = \frac{f_n}{3g_n}, & \text{if } g_r/f_r \geq g_n/f_n. \end{cases} \tag{3.7}$$

Thus, as a stepping stone to better understand the impact of endogenous quality in the context of our model, particularly with regard to its effect on the optimal remanufacturability (k^*), we graphically compare the optimal quality from (3.7) to $q^T = 1/3$, which from Proposition 3.1, denotes the quality that corresponds to the Traditional substrategy and, as such, represents the conditionally optimal quality given that $k = 0$. Specifically, in Figure 3.2, we graph the ratio q^*/q^T as a function of g_r. In a similar vein, we also graph in Figure 3.2 the ratio of the corresponding total demands $(d^*_{n1} + d^*_{n2} + d^*_{r2})/d^T$ as well as the ratios of the corresponding new product demands $(d^*_{n1} + d^*_{n2})/d^T$ and remanufactured product demands d^*_{r2}/d^T. In producing these graphs, we set $f_n = 1.2$, $f_r = 0.9$, and $g_n = 1.1$. These parameters serve as our illustrative and representative case for Figure 3.2, as well as for the figures that follow unless otherwise stated.

Examining first the graph of q^*/q^T, we find that, as the remanufacturing cost g_r increases, the optimal quality decreases until it reaches the minimal level at $g_r = g_n f_r/f_n$. At this point, $q^*/q^T = f_n/g_n$; thus, $q^* > q^T$ is assured if $f_n/g_n > 1$. Examining second the graphs of $(d^*_{n1} + d^*_{n2})/d^T$, d^*_{r2}/d^T and $(d^*_{n1} + d^*_{n2} + d^*_{r2})/d^T$, we find that a smaller volume of new products are sold when an optimal strategy is implemented than otherwise would be sold if $k = 0$, which is intuitive. However, a larger total volume of products are sold than otherwise would be sold if $k = 0$. This thus reflects the conventional wisdom that a firm reaches a larger consumer base when implementing a remanufacturing strategy.

3.5 Environmental Friendliness

In this section, we address the extent to which remanufacturable products are environmentally friendly, given that they are optimal to produce. In other words, we assess the environmental friendliness of $k^* = 1$. To this end, we first define an environmental measure that we refer to as *environmental damage*. We then apply this measure to assess the environmental damage

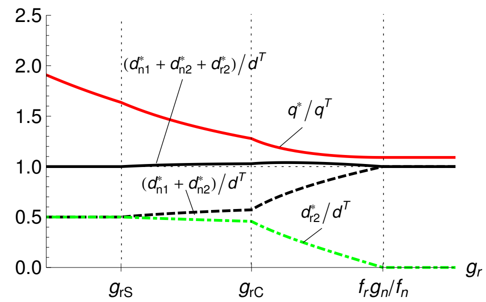


Figure 3.2: Illustration of Optimal Green Quality and Demand

associated with the optimal strategy (q^*), and we compare that to the environmental damage associated with the Traditional substrategy (q^T), which represents the conditionally optimal strategy given that $k = 0$. Specifically, we make comparisons to answer two questions: Is it necessarily environmentally friendly to design for green if the intention is not to go green? Is it even necessarily environmentally friendly to go completely green?

3.5.1 Definition of Environmental Damage

Our definition of environmental damage derives from the notion of carbon footprint. Carbon footprint refers to the total set of greenhouse gas emissions produced by an organization, event or product. By analogy, we define environmental damage as the total resources acquired from, and wastes discarded to, the environment during a product's life cycle. Specifically, we assume that the environmental impact of extracting the resources for, and disposing the remains of, a product of quality q is e_1q and e_2q, respectively, where e_1 and e_2 represent environmental damage coefficients. Accordingly, for given quality q and remanufacturability k, the environmental damage associated with stage 1 of our model is

$$\Delta_1 = d_{n1}e_1q + d_{n1}(1-k)e_2q,$$

where the first term $d_{n1}e_1q$ denotes the damage caused by the extraction of resources required to produce d_{n1} units of new product with quality q at the start of stage 1, and the second term $d_{n1}(1-k)e_2q$ denotes the environmental damage caused by waste disposal at the end of stage 1 if the d_{n1} units are not recycled (i.e., if $k = 0$). Analogously, the environmental damage associated with stage 2 of our model is

$$\Delta_2 = d_{n2}e_1q + (d_{n1} - d_{r2})ke_2q + (d_{n2} + d_{r2})e_2q,$$

where the first term $d_{n2}e_1q$ denotes the environmental damage caused by the extraction of resources required to produce d_{n2} units of new product with quality q at the start of stage 2, the second term $(d_{n1} - d_{r2})ke_2q$ denotes the environmental damage caused by waste disposal at the start of stage 2 if used units are recycled at the end of stage 1 (i.e., if $k = 1$) but not remanufactured in stage 2, and the third term $(d_{n2} + d_{r2})e_2q$ denotes the environmental damage caused by waste disposal at the end of stage 2. Thus, the total environmental damage for the two stages, which we denote $\Delta(q)$, is

$$\Delta(q) = \Delta_1 + \Delta_2 = (d_{n1} + d_{n2})(e_1 + e_2)q \tag{3.8}$$

where (3.8) reflects the fact that $(1 - k)d_{r2} = 0$ because $k = 0$ requires that $d_{r2} = 0$ by definition.

Notice therefore that the environmental damage measure reduces to a function that is independent of remanufacturing quantity d_{r2} and remanufacturability k. This is because, in the context of our model, remanufacturing does not require the extraction of virgin resources beyond those originally extracted to produce the units when they were new. Moreover, the waste resulting from the disposal of remanufactured products in stage 2 is offset completely by the waste avoided by not having to dispose the remanufacturable products recycled for remanufacturing in stage 1. Thus, as (3.8) illustrates, what ultimately impacts the environment are two factors: (i) *what* is produced (which is represented by product quality q), and (ii) *how much new* is produced (which is represented by new product demand $d_{n1} + d_{n2}$). This makes sense because, in essence, that is what is drawn from and eventually returned back to the earth. Note that in this sense, the cannibalization of new products by remanufactured products in stage 2 is a good thing from the standpoint of environmental friendliness. Note also that we write environmental damage $\Delta(q)$ as a function of q because $(d_{n1} + d_{n2})$ ultimately is a function of q as per Propositions 3.1 and 3.3.

Given this measure of environmental damage defined by (3.8), we are particularly interested in comparing $\Delta(q^*|k^* = 1, d^*_{r2} = 0)$ to $\Delta(q^T)$ as well as comparing $\Delta(q^*|k^* = 1, d_{r2}^* = d_{n1}^*)$ to $\Delta(q^T)$. The first comparison assesses the environmental damage associated with implementing an optimal strategy, given that it is optimal to design a remanufacturable product but not to remanufacture that product, relative to the environmental damage that otherwise would result if $k = 0$. Accordingly, this comparison addresses our first question of environmental friendliness: Is it environmentally friendly to design for green if the intention is not to go green? Similarly, the second comparison assesses the environmental damage associated with implementing an optimal strategy given that it is optimal to remanufacture completely, relative to the environmental damage that otherwise would result if $k = 0$. Accordingly, this comparison addresses our second question of environmental friendliness: Is it even necessarily environmentally friendly to go completely green?

3.5.2 Is it Environmentally Friendly to Design for Green but Not to Go Green?

Recall that, in the context of our model, designing for green but not going green means that $k = 1$ but $d_{r2} = 0$ in an optimal solution. Thus, to answer this question, we leverage Proposition 3.6 to obtain the following.

PROPOSITION 3.7. *If and only if $f_n/g_n > \max[1, f_r/g_r]$, the optimal strategy is to design for green but not to go green ($k^* = 1$, $d_{r2}^* = 0$), but this strategy is not environmentally friendly ($\Delta(q^*|k^* = 1, d_{r2}^* = 0) > \Delta(q^T)$).*

Proposition 3.7 is interesting because it illustrates how the environment ultimately pays the price when consumers overvalue the *idea* of remanufacturability (f_n) relative to how much they value actual remanufacturing (f_r). In particular, Proposition 3.7 demonstrates a necessary and sufficient condition indicating when it is optimal for the firm to design a remanufacturable product ($k^* = 1$) that it has no intention of remanufacturing ($d_{r2}^* = 0$), although the environment would be better served if the firm simply designed a non-remanufacturable product instead ($k = 0$). In this case, the firm's decision not to remanufacture coupled with the increased quality associated with a remanufacturable design results in increased environmental damage.

This result is an example of how good market intentions may produce unintentional outcomes. If environmentally conscious consumers express their concerns for the environment through a higher willingness to pay for products that can be remanufactured, then this preference attracts the firm to design a remanufacturable product. However, that same consumer preference will hurt the environment if it is coupled with a low willingness to pay for products that have been remanufactured. Indeed, it is precisely this type of inconsistency in consumers' preferences that results in the unintended environmental consequences. This insight thus underscores a qualification for consumers truly to be green. In particular, to be green, it is not sufficient for consumers simply to value the idea of remanufacturing; they must also value products that actually are remanufactured. In other words, it is not enough to "talk green"; to be green, consumers must also "walk green".

3.5.3 Is it Environmentally Friendly to Go Completely Green?

In Section 5.2, we found that it is not necessarily environmentally friendly to design for green if the intention is not actually to go green. But that begs the question at the other extreme: What if it is optimal to go completely green? Would even that necessarily be environmentally friendly? To address this question, recall that, in the context of our model, going completely green means that $0 < d_{r2} = d_{n1}$ in an optimal solution.

PROPOSITION 3.8. *If* $g_r \leq g_{rC}$ *and* $g_n/f_n < 1 - \frac{f_r^2(3f_n - f_r)}{(2f_n - f_r)(f_n + f_r)^2} < 1$*, then the optimal strategy is to go completely green* ($k^* = 1$, $d^*_{r2} = d^*_{n1}$)*, but that strategy is not environmentally friendly* ($\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1}) > \Delta(q^T)$)*.*

Proposition 3.8 demonstrates that, indeed, it is not necessarily environmentally friendly to go green even if it means going completely green. Surprisingly, but not entirely unexpectedly, this would be the case if the production technology is especially cost efficient (in the sense that g_n/f_n is relatively low). Intuitively, if the market cost efficiency ratio of a new remanufacturable product is low (specifically, if $g_n/f_n < 1 - f_r^2(3f_n - f_r)/((2f_n - f_r)(f_n + f_r)^2)$), then it is optimal not only to remanufacture completely when $g_r \leq g_{rC}$, but also to set quality relatively high. As a result, the associated environmental damage ends up greater than it otherwise would if $k = 0$.

This suggests that cost efficiency potentially affects the environment differently than it affects the firm. To the firm, cost efficiency translates into

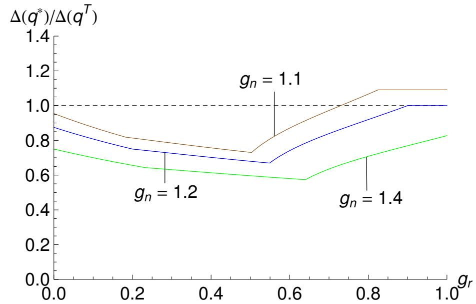


Figure 3.3: Illustration of Environmental Damage for Varying Production Cost Parameters

increased profit. To the environment, however, cost efficiency translates into higher quality which in turn leads to greater environmental damage. Therefore, although it is in the best interest of the firm to pursue such efficiencies, it is not particularly in the best interest of the environmentalist for the firm to achieve such pursuits.

Proposition 3.7 and 3.8 illustrate that cost efficiency of new remanufacturable products can lead to worse environmental consequences. This effect is manifested in higher product quality when remanufacturing cost g_r is relatively large in which case $d^*_{r2} = 0$ as in Proposition 3.7, or when g_r is relatively small in which case $d^*_{r2} = d^*_{n1}$ as in Proposition 3.8. While the former requires moderate cost efficiency with production technology, the latter demands much more higher cost efficiency. Hence, the latter is less likely to happen than the former. In other words, to remanufacture is generally more environmentally friendly than not to remanufacture given that the product is remanufacturable.

3.5.4 Sensitivity Analysis for Environmentally Friendliness

Next, in the form of a sensitivity analysis, we investigate the impact of production costs (g_n and g_r) and consumer valuations (f_n and f_r) on the environmental friendliness of an optimal policy. Specifically, we fix two of the four parameters while varying the other two, and we compare $\Delta(q^*)$, the environmental damage associated with the optimal strategy, to $\Delta(q^T)$, the environmental damage associated with the conditionally optimally strategy given that $k = 0$.

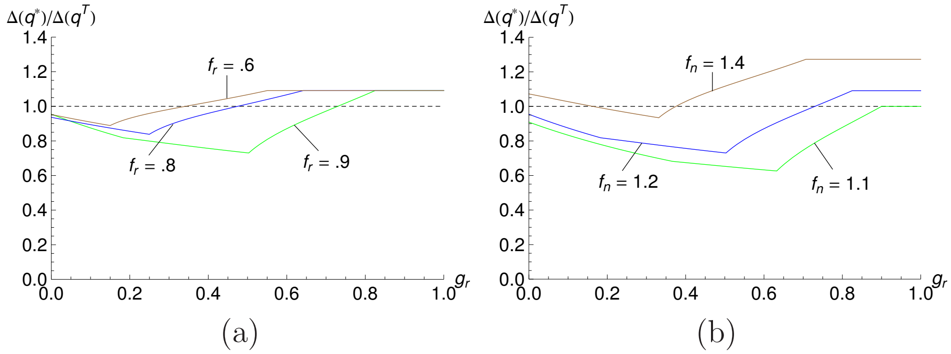


Figure 3.4: Illustration of Environmental Damage for Representative Valuation Parameters

Figure 3.3 shows graphs of $\Delta(q^*)/\Delta(q^T)$ as a function of remanufacturing cost g_r for three different new product production costs $g_n = 1.1, 1.2, 1.4$. Notice that a reduction in the remanufacturing cost yields a decrease in environmental damage if g_r is relatively high, but it yields an increase in environmental damage if g_r is relatively low. Intuitively, Figure 3.3 reflects that complete remanufacturing is not optimal for relatively high g_r but it is optimal for relatively low g_r. Hence, reducing the remanufacturing cost when $g_r > g_{rC}$ increases the volume of remanufacturing, which increases cannibalization of new products (smaller d_{n2}), thus reducing the damage on the environment. However, further reductions in g_r, when g_r is already low, do not affect cannibalization because all available units already are being remanufactured. Rather, such reductions in g_r result in higher quality, thus increasing the damage on the environment. In fact, this increase in environmental damage could potentially render complete remanufacturing less environmentally friendly.

Figure 3.3 also illustrates that environmental damage increases as g_n decreases. This observation reflects two effects: First, a lower g_n means a higher profit margin for new remanufacturable products. Second, a lower g_n also means a smaller relative cost difference between new and remanufactured products, $g_n - g_r$. These two effects thus combine to make remanufacturing less attractive. Hence, the firm sells a larger volume of new products and a smaller volume of remanufactured ones. As a result, cannibalization decreases, thus increasing the damage on the environment.

In contrast, as illustrated by Figure 3.4, which provides graphs of $\Delta(q^*)/\Delta(q^T)$ as a function of g_r for three values of f_r ($f_r = 0.6, 0.8, 0.9$) and for three values of f_n ($f_n = 1.1, 1.2, 1.4$), environmental damage decreases as consumers' willingness to pay for remanufactured products (f_r) increases, but it increases as consumers' willingness to pay for new products (f_n) increases. Intuitively, as Figure 3.4 (a) reflects, the firm is more reluctant to remanufacture for lower f_r. Thus, reductions in f_r decrease cannibalization, thus increasing the damage on the environment. Similarly, as Figure 3.4 (b) reflects, a higher f_n translates not only into a greater reluctance to remanufacture but also into a desire for higher quality. Hence, increases in f_n decrease cannibalization and increase quality, thus creating a reinforcing effect increasing the damage on the environment.

These results once again support insights from Proposition 3.7 indicating that inconsistency in consumers' valuations between what *can be* remanufactured relative to what *has been* remanufactured contributes to environmental damage. Basically, the larger is the difference, $(f_n - f_r)$, whether it is due to increases in f_n or to decreases in f_r, the larger is the environmental damage that results from an optimal solution. In other words, although valuing remanufactured products leads to smaller environmental damage, valuing only the notion of remanufacturability leads to greater environmental damage.

This is indicative of the notion that consumption, as opposed to production, is a primary driver of environmental damage. Accordingly, to paraphrase Orange (2010), the key to green is first to *reduce* consumption, and only then to *recycle* and to *reuse* that which has been consumed. This is also indicative of firms not particularly operating in the interests of the environment. Indeed, as the *Wall Street Journal* article titled "The Case Against Corporate Social Responsibility" argued in September 2010, "the concept of Corporate Social Responsibility is flawed. Companies do good [socially] only if they can do well [financially]." Restated in the language of our model, when a product is remanufacturable, firms may not necessarily remanufacture, and even if they do, they may not remanufacture completely.

3.6 Conclusion

In this chapter, we have studied remanufacturing and its environmental consequences through a product design lens. In this context, a firm has the option to design a non-remanufacturable or a remanufacturable product and to specify a corresponding quality, and these design choices affect both the production costs and the consumer valuations associated with the product. On the cost side, remanufacturable products cost more to produce originally, but less to remanufacture, than non-remanufacturable products cost to produce. Analogously, on the consumer side, remanufacturable products are valued more, but remanufactured products are valued less, than non-remanufacturable products are valued. Given this, we investigate the environmental consequences of remanufacturing by first defining a measure of environmental damage that, ultimately, is a function of what is produced and how much is produced, and then applying that measure to assess the environmental damage associated with the firm's optimal strategy relative to the environmental damage associated with the firm's otherwise optimal strategy if a non-remanufacturable product were designed and produced. In doing so, we bridge the gap between profit maximization and environmental friendliness to ascertain the extent to which the two are complementary and to identify key factors of compatibility when they are not complementary.

Our results indicate that consumer preferences and production technology potentially affect environmental friendliness differently than they affect profit maximization. We find, for example, that consumer greenness and production efficiency generally lead to a more profitable firm, which is consistent with the remanufacturing literature (e.g., Debo et al. 2005), but they do not necessarily translate to a more environmentally friendly firm. Specifically, we find that if green consumers overvalue the idea that a product *can be* remanufactured relative to how much they value a product that *has been* remanufactured, then the firm's optimal mix of new versus remanufactured products will not necessarily achieve the level of cannibalization required to benefit the environment in the form of waste disposal reductions. Consequently, under such circumstances, consumer greenness translates into increased damage on the environment. In a related vein, we find that although production efficiencies translate into healthier bottom lines, they also translate into higher quality products that, in turn, translate into increased damage on the environment.

Fundamental to our analysis is the modeling assumption that consumers differentiate between remanufacturable products, non-remanufacturable products, and remanufactured products specifically such that they value remanufactured products (weakly) less than non-remanufacturable products and they value non-remanufacturable products (weakly) less than remanufacturable products. In other words, fundamental to our modeling construct is the stipulation that $f_r \leq 1 \leq f_n$. Nevertheless, whereas our stipulation that consumers value remanufacturable products more than non-remanufacturable products ($f_n \geq 1$) is essential to our analysis, at least to the extent that we also assume remanufacturable products cost more than remanufacturable products to produce, *ceteris paribus*, our companion stipulation that consumers value remanufactured products less than non-remanufacturable products ($f_r \leq 1$) is, as it turns out, superfluous. We make this modeling assumption to parallel our presumption that the cost to remanufacture is less than the cost to produce originally, but in the end what drives our results is not the condition $f_r \leq 1$ but rather the condition that $f_r \leq f_n$. Thus, even if consumers valued remanufactured products more than they valued new non-remanufacturable products, our analysis would apply as long as they didn't value remanufactured products more than they valued new remanufacturable products.

In contrast, if consumers were to value remanufactur*ed* products more than they valued remanufactur*able* products, that is, if consumers were to view new products as *inferior* substitutes for remanufactured products, then the demand model of Figure 3.1 would change at its core because consumers with higher valuation of quality would prefer remanufactured products and those with lower valuation of quality would prefer new products. In other words, if $f_r > f_n$, then the relative positions of d_{n2} and d_{r2} in Figure 3.1 would reverse so that d_{r2} would be depicted to the right of d_{n2} rather than vice versa. Ultimately, such a recharacterization would mean that remanufactured products not only would benefit from lower production costs (in the sense that $g_r/f_r < g_r/f_n < g_n/f_n$ would be true) but also would benefit from higher prices (in the sense that $p^*_{r2} > p^*_{n2}$ would result). Under these circumstances, the analysis therefore would imply simply that $d^*_{n2} = 0$ unconditionally.

Also fundamental to our analysis are the definitions of virgin resource extraction and waste disposal as measures that are directly proportional to quality. Nevertheless, similar to the specifications of consumer valuations, this proportionality assumption is required only to the extent that environmental damage is defined to be non-decreasing in quality and in quantity. Because our assessments of environmental friendliness begin with the Premium substrategy, in which case it is optimal to design for green but not to go green, and because this substrategy results in the same sales volume as otherwise would result in the Traditional substrategy (re: Proposition 3.1), which serves as the benchmark for environmental friendliness, it is the comparison between the optimal quality and the conditionally optimal quality given that $k = 0$ that is fundamental to our results. Thus, if virgin resource extraction and waste disposal were defined more generally to be non-decreasing functions of quality, then we would expect similar qualitative assessments of environmental friendliness as those determined in §3.5 to result.

Note, however, that these qualitative assessments very well could be affected if environmental damage were decreasing over some quality levels. As an illustrative example of such a potential, consider Subramanian et al. (2009) who develop a repeated purchase model in which environmental resources are consumed not only during production (by the firm) but also during use (by consumers). Thus, in this context, consumers face a trade-off between a during-use cost, on the one hand, and a product replacement cost on the other hand. If such a during-use environmental effect were incorporated into our modeling context, then an additional term would be required to represent the amount of environmental damage incurred during use, which potentially could be decreasing in quality. Therefore, exploration of such an effect constitutes a viable direction for future research.

Chapter 4

Environmental Friendliness: Scope and Applicability

In §3, we have studied remanufacturing and its environmental consequences through a product design lens. To this purpose, we developed a two-stage model in which the manufacturer has the option to design a non-remanufacturable or a remanufacturable product and to specify a corresponding quality. Our results indicate that consumer preferences and production technology potentially affect environmental friendliness differently than they affect profit maximization. We find, for example, that consumer greenness and production cost efficiency generally translate to a more profitable firm, but they do not necessarily lead to a more environmentally friendly firm.

In this chapter, we further explore the scope and applicability of the model and results in §3. Specifically, we consider several extensions to, and variations of, our modeling assumptions and settings, and we discuss their implications. In §4.1, we study the value of educating consumers. In §§4.2–4.3, we explore the impact of a continuous remanufacturability and an exogenous quality on the optimal decisions and on associated environmental impact. In §4.4, we investigate the effect of an infinite-horizon model. Building on this model, we discuss the effects of collection rate, as well as the perspectives of both an environmental planner and a global planner in §§4.5–4.6.

4.1 The Value of Consumer Education

Recall from §3.5 that consumers who are environmentally conscious in the sense that they value the idea that a product *can be* remanufactured, but are not necessarily green in the sense that they do not particularly value a product that *has been* remanufactured could lead to unintentional consequences in the form of unwarranted environmental damage. Thus, as a natural extension, we explore in this section the potential value of educating consumers to be less environmentally conscious, per se, and to be more green. Specifically,

Table 4.1: Marginal Effect of Consumer Valuation on Environmental Damage

		θ				
		0.1	0.2	0.3	0.4	0.5
α	0.1	0.12, 0.08	0.07, 0.02	0.21, 0.19	0.15, 0.13	0.13, 0.08
	0.2	0.08, 0.02	0.17, 0.14	0.13, 0.09	0.12, 0.07	0.11, 0.06
	0.3	0.15, 0.11	0.13, 0.07	0.12, 0.06	0.11, 0.05	0.11, 0.04
	0.4	0.12, 0.06	0.11, 0.05	0.11, 0.04	0.10, 0.04	0.09, (0.09)
	0.5	0.12, 0.04	0.11, 0.04	0.09, 0.04	0.09, (0.09)	0.09, (0.09)

we focus on the question of what would be more environmentally beneficial, a decrease in f_n (i.e., a decrease in θ) or an increase in f_r (i.e., a decrease in α). To this end, we numerically evaluate changes to environmental damage relative to changes in θ and in α, given that the optimal policy is implemented. Specifically, we set $c_1 = 0.2$, $c_2 = 0.5$, and $e_1 + e_2 = 1$, and we compute $\frac{\partial \Delta(q^*)}{\partial \theta}$ and $\frac{\partial \Delta(q^*)}{\partial \alpha}$ for values of θ and α such that $(\theta, \alpha) \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$.

Table 4.1 provides a matrix of results where each entry presents ($\frac{\partial \Delta(q^*)}{\partial \theta}$, $\frac{\partial \Delta(q^*)}{\partial \alpha}$) for specified values of θ and α, with parenthesis indicating a negative value. According to Table 4.1, increases in θ result in increases to environmental damage regardless of the value of α. Similarly, increases in α result in increases in environmental damage for most values of θ. However, increases in α yield decreases to environmental damage if both θ and α are large. Thus, as a general rule, there is value (to the environmentalist) in educating consumers to be less environmentally conscious (smaller θ) and more environmentally green (smaller α). In other words, it is in the environment's better interest to have consumers not necessarily value so highly that which *can be* remanufactured, but rather to value highly that which *has been* remanufactured. Moreover, notice that $\frac{\partial \Delta(q^*)}{\partial \theta} > \frac{\partial \Delta(q^*)}{\partial \alpha}$. Thus, if the choice must be made between educating consumers against the ills of overvaluing that which *can* be remanufactured and the ills of undervaluing that which *has been* remanufactured, the environment would be better served by investing in the former.

4.2 Remanufacturability

In §3, we defined remanufacturability (k) to be a binary variable. In this section, we relax this stipulation and assume that k is a continuous variable such that $k \in [0, 1]$. In this new context, however, the interpretation of k requires refinement. If $k \in [0, 1]$, then rather than representing whether or not a product can be remanufactured (as it does in this chapter), k would represent the *fraction* of a product that can be remanufactured. As such, this concept is related to, but is different from, both the notion of collection rate (Majumder and Groenevelt 2001, Ferrer and Swaminathan 2006, 2010) and the notion of yield ratio (Ferguson and Toktay 2006, Ferguson and Koenigsberg 2007). To illustrate this difference, consider a cell phone as an example. If the cell phone were defined to have two components, a shell and a battery, then the notion of collection rate would refer to the ratio of how many used cell phones are recycled after use relative to how many are sold, and the notion of core yield would refer to how many of the cell phones (as complete units) that are recycled have enough residual value remaining to justify remanufacturing. In contrast, a fractional k in our modeling context would refer to how much of a given cell phone (just the shell, just the battery, both, or neither) is remanufacturable.

If, indeed, k were redefined in our model such that $k \in [0, 1]$, then most of our analysis leading to Propositions 3.1 – 3.3 would remain unaltered, except for the analysis to determine k^*. Because we found that analysis not to be tractable analytically, we implemented a numerical procedure to establish the optimal k for an extensive array of input parameters. From this analysis, which is provided in Appendix C.2, we found that k^* is a boundary point solution such that $k^* = 0$ or $k^* = 1$ even if k is defined on $[0, 1]$. Thus we infer that, qualitatively, the results and interpretations from Chapter 3 continue to hold if k were defined as a continuous design variable.

4.3 Exogenous Quality

In §3.5.2, we established that the environmental damage associated with a remanufacturable product, when such a design is optimal, could be larger than that associated with a non-remanufacturable product, depending on the circumstances. Moreover, under such circumstances, a primary reason for the increased environmental damage is that a remanufacturable product leads to a higher quality design than does a non-remanufacturable product. Thus, in this section, to test the extent to which this result depends on the

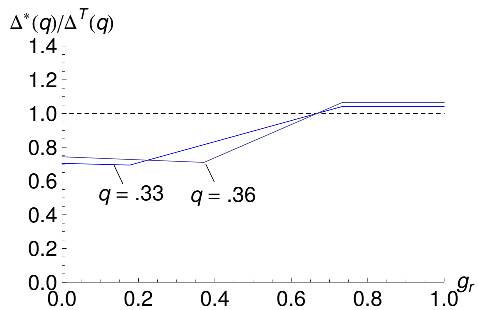


Figure 4.1: Illustration of Environmental Damage for Exogenous Quality

endogeneity of quality in our model, we revisit the comparison of environmental damage behind Proposition 3.7 but this time we assume exogenous quality instead. Accordingly, for the purpose of this section, let $\Delta^*(q)$ denote the environmental damage associated with the optimal two-stage pricing and remanufacturability strategy for given q, and let $\Delta^T(q)$ denote the environmental damage associated with the conditionally optimal pricing policy, given that $k = 0$, for given q.

PROPOSITION 4.1. *If* $g_n/f_n < 1$ *and* $g_r > g_n - (f_n - f_r)(1 - g_n/f_n)$, *then* $\Delta^*(q) > \Delta^T(q)$.

As Proposition 4.1 demonstrates, even for an exogenous quality, it is not necessarily environmentally friendly to go green. In particular, going green leads to larger environmental damage if the remanufacturing cost is sufficiently high (specifically, if $g_r > g_n - (f_n - f_r)(1 - g_n/f_n)$). In this case, Proposition 4.1 dictates that the firm design remanufacturable products but not remanufacture enough of them to achieve the level of cannibalization needed for the environment to benefit. In fact, under these circumstances, the firm ends up selling a higher volume of new products than it otherwise would had the product been non-remanufacturable.

To graphically illustrate the implications of exogenous quality, we plot, in Figure 4.1, graphs of environmental damage for $q = 1/3$ (which corresponds to the conditionally optimal quality when $k = 0$) and $q = f_n/3g_n$ (which corresponds to the conditionally optimal quality when $k = 1$ but $d_{r2} = 0$). We find that higher quality generally results in higher environmental damage if g_r is either large or small. However, we also find that higher quality leads to lower environmental damage if g_r is moderate. Intuitively, if g_r is moderate, then higher quality means that the firm remanufactures a higher

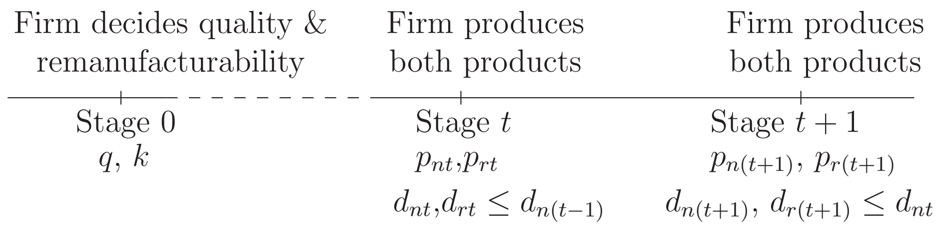


Figure 4.2: Infinite-horizon Model Specification

percentage of recycled units due to the correspondingly smaller volume of new product sales. This increases cannibalization, thus decreasing damage on the environment.

4.4 Infinite Horizon

In §3, a two-stage model is used to exemplify the cannibalization effect between new and remanufactured products. While a two-stage model is parsimonious and useful for developing insights into the problem context studied in §3, an infinite horizon model would be equally parsimonious for developing similar insights into the effects of longer problem horizons.

The Decision Framework. We use a infinite-horizon model as shown in Figure 4.2 to formulate the firm's profit-maximization problem. First, at the design stage, stage 0, the firm determines product quality and the level of remanufacturability. These two design dimensions remain unchanged throughout the horizon. Then in the t-th stage where $t \in \mathbb{R}^+$, the firm sets the prices for *new* and (or) *remanufactured* products and produces the corresponding quantities. At the end of the period, used units are recycled through reverse logistic network as in Figure 4.2. And this amount of recycled product establishes the supply constraint for the $t+1$-th stage.

The key here is the interaction between the supply constraint imposed by new products sales from the last period, and cannibalization arising from selling both new and remanufactured products. Unlike a two-period model which requires the firm to sell only new products in the first period and allows the firm to sell only remanufactured products in the second, an infinite-horizon model requires the firm to produce new products every period to maintain a steady state. Moreover, (Ferrer and Swaminathan 2006, Ray et al. 2005) shows that in an infinite-horizon model, it is optimal for the firm to set the same price and to sell the same volume of products in each period. In other words, the pricing decision in an infinite-horizon model is one-time (Atasu et al. 2008). Hence, we drop the subscript t in our model formulation.

Consumer Demand. Consumers purchase to maximize their non-negative surplus, which is defined by the difference between valuation and price paid. Let p_n and p_r be the price of a new and a remanufactured product, respectively. Then the demand functions for new and remanufactured products are similar to those in §3.3, but with different subscript. Specifically, if only new products are available, then the corresponding demand for new products is represented by (4.1).

$$d_n = 1 - \frac{p_n}{f_n(k)q} \tag{4.1}$$

If the product is designed to be remanufacturable (i.e., if $k = 1$), then the demand for new and remanufactured products, d_n and d_r, respectively, are

$$d_n = 1 - \frac{p_n - p_r}{(f_n - f_r)q}, \quad \text{and} \quad d_r = \frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q} \tag{4.2}$$

Note that d_n and d_r represent the demand for new and remanufactured products in a stable state hence no subscript is used to represent the time period.

Profit. If the firm designs a non-remanufacturable product (i.e., if $k = 0$), then it only sells new products in each period. Accordingly, the demand is represented by (4.1) and the corresponding profit is

$$\Pi(p_n, q, k) = d_n(p_n - g_n q^2) \tag{4.3}$$

If the firm designs a remanufacturable product (i.e., if $k = 1$), then the pricing decisions for new and remanufactured products would be stationary over time (Atasu et al. 2008), the corresponding remanufacturing problem boils down to

$$\max_{p_n, p_r} \Pi = d_n(p_n - g_n q^2) + d_r(p_r - g_n q^2) \tag{4.4}$$

$$s.t. \quad d_r \leq d_n \tag{4.5}$$

By applying the same solution procedure as in §3, we analogously solve (4.4). (Detailed derivations are provided in Appendix C.2.)

Accordingly, Figure 4.3 shows the graphs of the optimal quality (q^{I*}), as

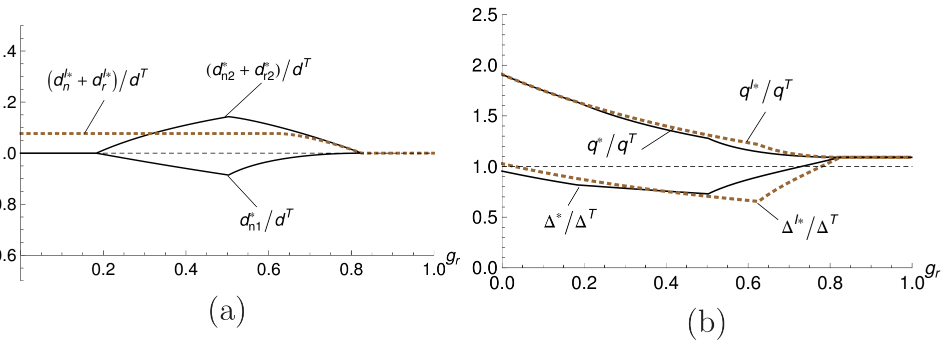


Figure 4.3: Comparison between Two-Stage and Infinite-Horizon Quality, Demand, and Environmental Damage

well as the corresponding total demand for a given period ($d_n^{I*} + d_r^{I*}$) and the associated environmental damage (Δ^{I*}), all as functions of the remanufacturing cost g_r. For comparison, we likewise graph in Figure 4.3 the optimal quality from the two-stage formulation (q^*), as well as the corresponding stage 1 demand (d_{n1}^*), stage 2 demand ($d_{n2}^* + d_{r2}^*$), and the associated environmental damage (Δ^*). Note that the two-stage results are normalized such that they represent the average values for a single stage.

As Figure 4.3(a) illustrates, the infinite-horizon model implies a greater volume of products sold in each period than does the two-stage model in stage 1 (i.e., $d_n^{I*} + d_r^{I*} > d_{n1}^*$). This is because two types of products (new and remanufactured) are sold in the infinite-horizon model compared to only one type of product (only new) in stage 1 of the two-stage model. Similarly, the infinite-horizon model implies a larger volume of products sold per period than does stage 2 of the two-stage model (i.e., $d_n^{I*} + d_r^{I*} > d_{n2}^* + d_{r2}^*$) if g_r is sufficiently small because two types of products (new and remanufactured) are sold in the infinite-horizon model compared to only one type of product (only remanufactured) in stage 2 of the two-stage model. However, the infinite-horizon model implies a smaller volume of products sold per period than does stage 2 of the two-stage model if g_r is sufficiently large. This is because, like in the infinite-horizon model, two types of products (new and remanufactured) are sold in stage 2 of the two-stage model, but comparatively, there is less cannibalization *of remanufactured products by new products* due to the fact that new products in stage 2 of the two-period model need not be collected for later use.

As Figure 4.3(b) illustrates, an infinite-horizon model dictates a higher quality than does the two-stage model. This is because the new units in stage 2 of the two-stage model will not be remanufactured, thus the cost saving potential of these units cannot be realized. Hence, the two-stage model yields a lower quality so as to reduce this waste of cost saving potential. Figure 4.3(b) also shows that the infinite-horizon model does not fundamentally alter the pattern in which environmental damage changes with regards to remanufacturing cost. However, the infinite-horizon model does yield slightly higher or lower environmental damage as does the two-stage model in §3. If the remanufacturing cost is relatively small, on one hand, the infinite-horizon model dictates a higher sales volume of new products as well as a higher quality than does the two-stage model, leading to more environmental damage. If the remanufacturing cost is relatively large, on the other hand, the infinite-horizon model dictates a much smaller sales volume of new products due to stronger cannibalization, as reduces the environmental damage. This reduction in environmental damage outweighs the effect of a slightly higher quality. Hence, the total environmental damage is smaller.

4.5 Collection Rate

Product return has been considered as a "dead weight" of a firm's operation rather than as a source of value creation. It was not until the last decades or two that this situation started to change. Instead of passive handling return after it happens, firms have started to consider the reverse supply chain in their decision-making process. For example, end-of-use products can be reintroduced as remanufactured products by recovering the residual value of the functioning components from the returned products. In related literature on product return, there are four types of product returns depending on the nature: commercial, warranty, end-of-use, and end-of-life. Each type of return can be treated so that it will create value for the firm.

As Guide et al. (2003) pointed out, the profitability of remanufacturing depends critically on product acquisition management. One would expect that the more used products are recycled, the more remanufactured products can be sold. Thus increases in collection rate could potentially generate more economic gains and causing less environmental damage. For example, in September 2010, *Chemistry & Industry* features an article stating that the carbon footprint of recycling PET (a type of packaging plastics) is lower than landfill when the recycling rate is higher than 45%. The economic and environmental benefits of recycling more used units may remain true even if the reverse logistics entails a fixed cost, and even more so if recycling is mandatary due to related legislation as the case in many nations in the European Union.

In this section, we focus on the end-of-use product returns, that is, products still contain significant amount of value and can be remanufactured to original functionality. Geyer et al. (2007) study the impact of durability and product life-cycle on the profitability of remanufacturing by minimizing the total cost of collection, sorting, remanufacturing, and disposal. Guide et al. (2003) recognize the variation of conditions in returned products and propose a price-sensitive return framework to determine the optimal acquisition price for each condition of used products. Both research assume that remanufacturing cost for each condition class is exogenous while the product return and total demand depends on the acquisition and retail price respectively. They find that decrease in acquisition price in one conation class would increase the return from other condition class. Galbreth and Blackburn (2010) also consider the variation of condition in returned products but consider demand and the distribution of used products condition exogenous. They assume that unit remanufacturing cost decreases as more returned products are acquired and then trade-off the decreasing remanufacturing cost vs increasing the collection and inspection cost. Savaskan et al (2004) and Savaskan and Van Wassenhove (2006) study the different structure of reverse supply chain and their impact on the optimal collection rate. Their results suggest that in decentralized setting, collecting through retailer yields a higher collection rate than collecting through manufacturer or a third-party. All these models assume no differentiation between new and remanufactured products hence no cannibalization. The collection rate mainly affects the operational side of the question. Webster and Mitra (2007) and Mitra and Webster (2008) investigate the impact of subsidy and take-back law on product returns and consequently on the profitability of remanufacturing.

The objective in this section is to investigate the effect of collection rate on a monopolist's design for remanufacturing problem and examine its environmental consequences. We study the interactions between product collection rate and an endogenous quality, and study how these interactions would af-

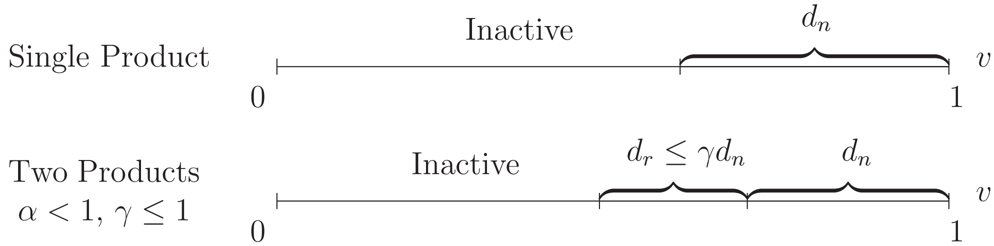


Figure 4.4: Illustration of Consumer Demands in Infinite-Horizon Model

fect the firm's quality design and remanufacturing decision. In particular, we study the conditions on which the firm makes its products remanufacturable given that only a certain fraction of used products is recycled at the end of each period. If so, how do the optimal product quality and optimal product mix between new and remanufactured products change according to the collection rate? We also examine the effect of collection rate on the environment and identify the key drivers that make remanufacturing more environmentally friendly.

4.5.1 Model and Solution

Here, we first introduce some necessary assumptions and then define the firm's design for remanufacturing problems. Each problem is then solved separately.

Collection Rate. At the end of each period, used products are collected back to the firm through reverse supply chain. In this section, we model the reverse logistics with an exogenous collection rate, denoted by γ, which represents the percentage of used products that can be recycled at the end of each period. This setting represents the scenario where the firm has no control over the reverse logistics, and consumers randomly decide whether to recycle a used product as shown in Figure 4.4. Consistent with §4.4, we assume that the recycling of each used unit can only occur in the same period as the purchase and consumption of the new product. Otherwise, the unit is lost and will never be recycled.

Profit. The introduction of collection rate does not affect the expressions for profit directly. If the firm designs a non-remanufacturable product (i.e., if $k = 0$), then the corresponding profit is (4.3). If the firm designs a remanufacturable product (i.e., if $k = 1$), then the corresponding profit is (4.4).

The key distinction between the current section and §4.4 is that the supply constraint (4.5) changes to

$$d_r \leq \gamma d_n. \tag{4.6}$$

This difference has two effects. On one hand, recycling a fraction of used products means the cost saving potential in those products that are not collected can not be realized, which makes remanufacturing less attractive. On the other hand, fewer remanufactured products in the market will lead to weaker cannibalization between the two products, which makes remanufacturing more attractive.

Following the same solution procedure as in §4.4, we can solve the firm's remanufacturing problem under an exogenous collection rate. Due to the fact that the non-remanufacturing problem is fundamentally the same with or without collection rate, the solution to the Traditional strategy remains the same in §3.3 and §4.4.

For the remanufacturing problem (4.4), Lemma C.1 in Appendix C.1 demonstrates that for given q, the firm implements the Premium strategy if $g_n/f_n \leq g_r/f_r$, and implements the Green strategy if $g_n/f_n > g_r/f_r$. Note that g_i/f_i represents the cost efficiency of product $i = n, r$. Accordingly, the firm chooses not to remanufacture if the new remanufacturable product is more cost efficient, and vice versa. In the former case, $d_r = 0$, hence (4.4) becomes $\Pi^M(q) = \frac{f_n q (f_n - g_n q)^2}{4 f_n}$, which has the same structure as (4.3).

In the latter case, Lemma C.2 characterizes $(p_n^G(q), p_r^G(q))$, the optimal pricing decisions as a function of q, as well as $(d_n^G(q), d_r^G(q))$, the corresponding demands. We find that the firm remanufactures more recycled units as product quality increases. As a result, the new product sales volume decreases in quality due to cannibalization from remanufactured products. A closer look reveals that as quality increases, the firm sells fewer new products in each period to increase the price and thus marginal return of both products. Let $\Pi^G(q) = \Pi^G(p_n^G(q), p_r^G(q), q)$ denote the profit associated with the Green strategy as a function of q. Then Lemma C.3, also provided in Appendix C.1, establishes that $\Pi^G(q)$ is continuous and unimodal in q. Maximizing $\Pi^M(q)$ and $\Pi^G(q)$ accordingly leads to the following Proposition.

Proposition 4.2 (Optimal Remanufacturing Strategy). *Let q^{R*} denote the optimal quality if the Remanufacturing strategy is implemented. Then by* *definition,* $k = 1$*, and*

$$q^{R*} = \begin{cases} q_C^G = \frac{f_n+\gamma f_r}{3(g_n+\gamma g_r)}, & 0 \le g_r \le g_{rC}; \\ q_P^G = \frac{f_r(f_n-f_r)g_n\left(2-\sqrt{1-3(f_r g_n - f_n g_r)^2/[f_r(f_n-f_r)g_n^2]}\right)}{3(f_r g_n(g_n-g_r)-g_r(f_r g_n - f_n g_r))}, & g_{rC} < g_r \le \frac{f_r g_n}{f_n}; \\ q^M = \frac{f_n}{3g_n}, & \frac{f_r g_n}{f_n} < g_r \le 1. \end{cases}$$

where $g_{rC} = \frac{(1+\gamma)(f_n+\gamma f_r)-3\gamma(f_n-f_r)}{(f_n+\gamma f_r)^2+3\gamma^2 f_r(f_n-f_r)} f_r g_n$.

Proposition 4.2 characterizes the optimal product quality in the remanufacturing cost g_r when the firm designs a remanufacturable product. A couple of observations are worth mentioning. First, considering collection rate in the design for remanufacturing problem does not fundamentally change the structure of the optimal product quality. That is, the optimal quality increases as remanufacturing cost decreases. Second, collection rate only affects product quality when complete remanufacturing is optimal. This is intuitive because the supply constraint is not binding under partial or no remanufacturing.

4.5.2 Results and Discussion

In this subsection, we first compare the optimal profit when a non-remanufacturable product is designed (i.e., $k = 0$) to the optimal profit when a remanufacturable product is designed (i.e., $k = 1$). This comparison helps us understand the effect of collection rate on the profitability of remanufacturing. Then, we compare the environmental damage associated with optimal decision.

Profitability

First, comparing the conditional optimal solution when $k = 0$ to that when $k = 1$ yields the following proposition.

PROPOSITION 4.3. *Given that* $g_r/f_r < g_n/f_n < f_n$*, then it is optimal for the firm to remanufacture. Accordingly, the collection rate affects the firm's remanufacturing strategy such that*

i) If $\hat{\gamma} < \gamma \le 1$*, then it is optimal to partially remanufacture;*

ii) If $\gamma \le \min[\hat{\gamma}, 1]$*, then it is optimal to completely remanufacture.*

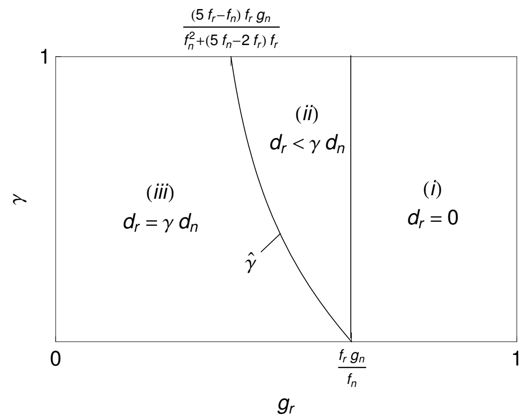


Figure 4.5: Illustration of Optimal Strategy

where

$$\hat{\gamma} = \frac{(f_n - f_r)\left(g_n(1 + \sqrt{1 - \frac{3(f_r g_n - f_n g_r)^2}{(f_n - f_r) f_r g_n^2}}) - 2g_r\right)}{2(f_n - f_r)g_r - (f_r g_n - f_n g_r)} - 1$$

Proposition 4.3 summarizes the effects of an exogenous collection rate on the firm's optimal remanufacturing decisions. These results are two-folded. First, collection rate has no effect on the firm's optimal remanufacturability decision. As we recall, whether or not to design a remanufacturable product is independent from collection rate, and depends on the comparison of cost efficiency between a non-remanufacturable product and a remanufacturable one. In a similar vein, collection rate produces no effect on whether or not a used product will be remanufactured. However, when the firm does remanufacture, collection rate may affect the firm's optimal decision. In particular, there exist a threshold collection rate, $\hat{\gamma}$, such that when the collection rate is higher than this threshold, the firm will partially remanufacture. If the collection rate is lower than this threshold, then the quantity of used products collected is smaller than the amount of used products the firm would like to remanufacture. As a result, the firm will remanufacture all available used products.

Note that $\hat{\gamma}$ is a decreasing function in g_r. When remanufacturing cost is relatively low (i.e., $g_r \leq \frac{(5f_r - f_n)g_n f_r}{f_n^2 + (5f_5 - f_r)f_r}$), we have $\hat{\gamma} \geq 1$, which means that the firm will always remanufacture all used products. When remanufacturing cost is relatively high (i.e., $g_r \geq g_n f_r / f_n$), we have $\hat{\gamma} \geq 0$, which indicates that the firm will remanufacture nothing.

Figure 4.5 graphically represents the firm's optimal remanufacturing decision as in Proposition 4.2. We set parameters at $\theta = 0.2$, $\alpha = .2$, $c_1 = .1$ and keep the same values throughout this chapter unless otherwise stated. When the remanufacturing cost is relatively high compared to new product cost, as in region (i), it is not worthwhile to sell remanufactured products as $g_r/f_r > g_n/f_n$. In this case, collection rate has no effect because the firm designs a remanufacturable product with quality q^M but does not remanufacture. As g_r reduces, remanufactured product becomes more and more cost efficient and the firm starts to sell remanufactured products to reap the benefits of cost savings. Hence, the effect of collection rate starts to emerge. In particular, the firm remanufacture a fraction of recycled units at higher collection rate (as in region (ii)) and remanufactures all recycled units at lower collection rate (as in region (iii)). In this case, the remanufactured products are not sufficiently cost efficient, i.e., $\frac{(5f_r - f_n)g_n}{f_n^2 + (5f_5 - f_r)f_r} < g_r/f_r < g_n/f_n$. The firm will not remanufacture more than $\hat{\gamma} d_n^*$ units of available recycled units due to potential cannibalization. In this context, $\hat{\gamma}$ represents the percentage threshold above which partial remanufacturing is optimal and below which complete remanufacturing is optimal. As g_r further decreases, the remanufactured products become sufficiently cost efficiency, i.e., $g_r/f_r < \frac{(5f_r - f_n)g_n}{f_n^2 + (5f_5 - f_r)f_r}$ such that complete remanufacturing is always optimal.

We know from previous analysis, collection rate affects the firm's remanufacturing decisions if low remanufacturing cost is coupled with low collection rate. On one hand, low remanufacturing cost guarantees that remanufactured products are more cost efficient than new products so that the firm finds it worthwhile to sell remanufactured products. On the other hand, low collection rate dictates that the firm cannot remanufacture more than it would otherwise do if more recycled units were available. Given Proposition 4.2, we have the following results about the optimal product design and product mix, as well as the associated environmental damage.

PROPOSITION 4.4. *Given that $g_n/f_n < f_n$, then collection rate γ affects the firm's quality and quantity decisions only when $g_r \leq g_{rC}(\gamma)$. Accordingly,*

(i) $\frac{\partial q^*}{\partial \gamma} > 0$;

(ii) $\frac{\partial d_n^*}{\partial \gamma} < 0$, $\frac{\partial d_r^*}{\partial \gamma} > 0$.

Proposition 4.4 demonstrates that if remanufacturing cost is sufficiently low (i.e., $g_r \leq g_{rC}(\gamma)$), then higher collection rate translates into higher optimal quality and lower (higher) new (remanufactured) products sales volume. In this case, the firm completely remanufactures all recycled units. When collection rate increases, the firm can, *ceteris paribus*, realize more cost saving potential by doing two things. One is to increase product quality which increases the cost saving potential for each product. The other is to remanufacture more used products. This increase in remanufacturing volume intensifies cannibalization, hence the firm reduces new products sale volume to raise the prices.

Environmentally Friendliness

Here, we compare the environmental damage associated with the optimal profit when a non-remanufacturable product is designed (i.e., $k = 0$) to that associated with the optimal profit when a remanufacturable product is designed (i.e., $k = 1$). This yields the following proposition.

PROPOSITION 4.5. *Given that $g_r/f_r < g_n/f_n < f_n$, then it is optimal for the firm to remanufacture, and*

i) $\frac{\partial \Delta(q^*(\gamma))}{\partial \gamma} < 0$, *if* $0 \leq \gamma \leq \min[\hat{\gamma}, 1]$*;*

ii) $\frac{\partial \Delta(q^*(\gamma))}{\partial \gamma} = 0$, *otherwise.*

Proposition 4.5 summarizes the effect of product collection rate on the environmental damage. Increasing collection rate will decrease environmental damage. This result is intuitive. We know from Proposition 4.4 that increases in collection rate mean more bigger volume of remanufactured products and smaller volume of new products. The corresponding strong cannibalization reduces the environmental damage. However, this effect is true only when collection rate is relatively low. In particular, if collection rate is higher than the threshold $\hat{\gamma}$, then increases in collection rate yields no effect on the environment.

To better illustrate these results, Figure 4.6 shows the graphs of environmental damage ($\Delta(q^*)/\Delta(q^T)$) as a function of g_r for collection rate $\gamma = 0.25, 0.5, 1$. Notice that for a given remanufacturing cost, decreases in γ increase the associated environmental damage, which is consistent with

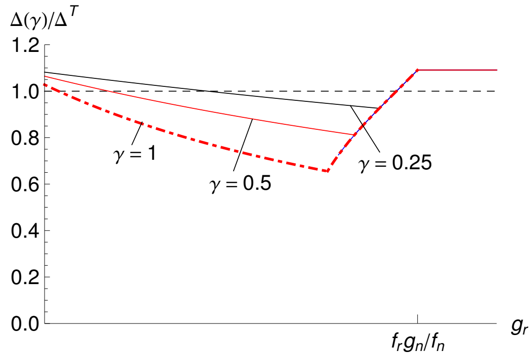


Figure 4.6: Illustration of Environmental Damage for Various Collection Rate

Proposition 4.4. However, we also notice that decreases in γ may have no effect on the environment. This result occurs when remanufacturing cost is relatively large in which case the firm would not remanufacture more than a certain amount of recycled units regardless of the collection rate. Therefore, it is not necessarily beneficial to the environment to collect more used units.

4.6 Environmental Design

In today's world, more and more firms brand themselves as environmentally friendly. They claim to be operating according to double or even triple bottom line. Are these firms suppose to behave differently as opposed to those who maximize profit? In this section, to provide a benchmark for comparison, we consider the design for remanufacturing problem for two additional types of decision makers — an environmental planner and a global planner. In this context, we define an environmental planner's objective to be to maximize profit net of the associated environmental damage. That is, we define the environmental planner as one who would adopt a double bottom line (2BL) perspective by maximizing $\Phi = \Pi - \beta\Delta$, where $\beta > 0$ is a cost coefficient to translate environmental damage into monetary terms. Analogously, we define a global planner as one who would adopt a triple bottom line (3BL) perspective by maximizing $\Theta = \Pi - \beta\Delta + \Omega$, where Ω is the corresponding consumer welfare. In a sense, we represent the interests of three stakeholders: the money (Π), the people (Ω), as well as the environment ($-\beta\Delta$).

Consistent with the §3, we represent the total environmental damage in each period with $\Delta = (e_1+e_2)d_n q$, where coefficient e_1 and e_2 representing the environmental impact of extracting raw materials and discarding waste. $\beta > 0$ is the coefficient that transform environmental impact to monetary term. In a sense, βe_1 and βe_2 can be considered as the shadow price for extracting raw material and discarding waste of unit quality. Let $e = \beta(e_1 + e_2)$ denote the corresponding marginal environmental cost in quality.

Decision makers differ from each other by assigning different weights to the interest of each stakeholder in their objectives. In particular, the firm assigns 100% weight to profit; the environmental planner assigns equal weights (50%) to profit and to environmental consequence; while the global planner assigns equal weights (33%) to profit, to consumer welfare and to environmental consequence. Here, our interests are limited to these three cases. However, one can always explore the implications of different weights assignment. As a case in point, a social planner assigns equal weights to profit and consumer welfare.

In the rest of this section, we first define and then solve the design for remanufacturing problem for the environmental planner and global planner, respectively. Then, we compare and contrast these solutions with those of a firm. These comparisons help us develop insights into the effects of considering multiple stakeholders on the optimal remanufacturing strategies and the associated environmental impact.

4.6.1 Environmental Planner

If the environmental planner implements the Traditional strategy, then the corresponding objective function is analogous to (4.3) and denoted by $\Phi(p, q, k) = d_n(p_n - g_n q^2 - eq)$, where d_n is again represented by (4.1). Following the same solution procedure as in §4.3, we obtain the corresponding the optimal quality denoted with superscript $\prime$ is

$$q^{T\prime} = \frac{1-e}{3}; \quad d_n^{T\prime} = \frac{1-e}{3} \tag{4.7}$$

In order to make the Traditional strategy meaningful, namely $q^{T\prime} > 0$, we assume that $e < 1$, i.e., the cost of environmental damage should not be too high. Notice that $q^{T\prime} < q^T$ and $d_n^{T\prime} < d_n^T$. In other other words, if a non-remanufacturable product is designed, then the environmental planner always designs a lower quality and sells a smaller quantity than the firm does. This is intuitive in the sense that environmental damage increases in both quality and new product sales volume. Considering the environmental damage in the objective function thus reduces the attractiveness of designing a high quality or selling a high volume.

If the environmental planner designs a remanufacturable product, then the corresponding profit function is analogous to (4.4) and denoted by

$$\max_{p_n, p_r} \Phi(q) = d_n(p_n - g_n q^2 - eq) + d_r(p_r - g_r q^2) \tag{4.8}$$

where d_n and d_r are represented by (4.2). In equation (4.8), the first term denotes the double-bottom-line contribution from selling new products and the second term denotes the double-bottom-line contribution from selling remanufactured products. The environmental planner hence optimize (4.8) subject to supply constraint (4.6). Again, we develop the solution, provided in Appendix C.1, to the environmental planner's problem analogous to the procedure followed in §4.3.

4.6.2 Global Planner

In this section, we analyze the problem faced by a global planner who carries both social and environmental responsibilities. Here the global planner to an environmentally friendly firm is the same as social planner to a profit-maximizing firm. The key difference is that global planner takes into consideration of consumer welfare (Ω) in addition to firm's profit (Π) and environmental impact (Δ). The corresponding objective function for the global planner (denoted by Θ) is $\Theta = \Phi + \Omega$.

Analogous to (4.3), the global planner's objective function, if the Traditional strategy is implemented (i.e., $k = 0$), is $\Theta(p, q) = (p - q^2 - eq)d_n + \frac{q-p}{2} d_n$, where the first term $(p - q^2 - eq)d_n$ denotes the double-bottom-line contribution and the second term $\frac{q-p}{2} d_n$ denotes the corresponding consumer welfare. Next analogous to (4.8), the global planner's objective function if a remanufacturable product is designed (i.e., $k = 1$) is

$$\Theta(p_n, p_r, q) = d_n(p_n - g_n q^2 - eq) + d_n \frac{f_n q + f_n(1-d_n)q - 2p_n}{2} + d_r(p_r - g_r q^2) + d_r \frac{f_r(1-d_n)q - p_r}{2} \quad (4.9)$$

where the first two terms denote the triple-bottom-line contribution from selling new products, and the second two terms denote the triple-bottom-line contribution from selling remanufactured products. Following the same solution procedure as in §4.3, we can again derive the optimal solution for global planner, denoted with superscript $''$. Detailed solutions are listed in Appendix C.1.

4.6.3 Discussion

Now, we compare the optimal decisions of the environmental (global) planner to that of the firm. Then, we discussion the implications of these decisions on environmental damage and consumer welfare.

Total Welfare

To answer the question on welfare, we compare the welfare associated with the remanufacturing scenario (i.e., $k = 1$) with that associated with non-remanufacturing scenario (i.e., $k = 0$) similar to our discussion for the firm in §4.3. Let (q', k') denote the environmental (global) planner's optimal design decisions.

PROPOSITION 4.6. *Given that* $g_n \leq \frac{(f_n-e)^2}{(1-e)^2}$, *then it is optimal for the environmental (global) planner to remanufacture. Accordingly, the collection rate affects the environmental (global) planner's remanufacturing strategy such that*

i) If $\frac{f_r g_n}{f_n}\frac{(f_n+2e)}{(f_n-e)} \leq g_r \leq 1$, *then it is optimal to design for remanufacturing but not to remanufacture, and correspondingly* $q' = q^{M\prime} = \frac{f_n-e}{3g_n}$;

ii) If $g_r < \frac{(f_n+2e)f_r g_n}{f_n(f_n-e)}$, *when* $\hat{\gamma}' \leq \gamma \leq 1$, *then it is optimal to remanufacture but not to exhaust all recycled units, and correspondingly* $q' = q_P^{G\prime}$;

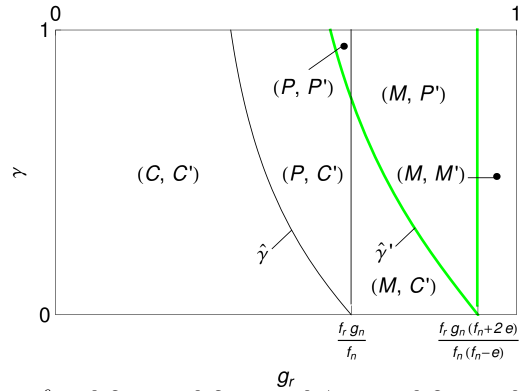
iii) when $\gamma \leq \min(\hat{\gamma}', 1]$, *then it is optimal not only to remanufacture but also to completely remanufacture, and correspondingly* $q' = q_C^{G\prime} = \frac{(f_n+\gamma f_r)-ef_r(1+\gamma)}{3(g_n+\gamma g_r)}$.

where $\hat{\gamma}'$ and $q_P^{G'}$ are defined in Appendix C.1.

Proposition 4.6 describes how the optimal strategy for environmental and global planner changes in γ. Similar to Proposition 4.3, we find that collection rate exhibits similar effects on the optimal remanufacturing strategy, that is collection rate produces no effects on whether or not to remanufacture but does affect how much to remanufacture. Note that $\hat{\gamma}' \to \hat{\gamma}$ as $e \to 0$, that is, the consideration of environmental damage in the decision making process (i.e., for the environmental planner) does not fundamentally change the structure of the optimal solution as opposed to not considering environmental damage (i.e., for the firm).

However, changing the decision rules from single-bottom line to double- or even triple-bottom line does make remanufacturing more likely. This effect is demonstrated in three ways. First, it is more likely for an environmental planner to design a remanufacturable product than it is for a firm (i.e., $f_n^2 < \frac{(f_n-e)^2}{(1-e)^2}$). Hence for given the same set of parameters, the environmental planner is more likely to design for remanufacturing than is the firm. Second, if the product is designed to be remanufacturable, then it is more likely for an environmental planner to sell remanufactured products than it is for a firm. Note here that the minimum remanufacturing cost above which the decision maker will not remanufacture is higher for the environmental planner than for the firm, i.e., $\frac{(f_n+2e)f_r g_n}{f_n(f_n-e)} > \frac{f_r g_n}{f_n}$. Finally, if selling remanufactured products is attractive, then it is more likely for an environmental planner to remanufacture all recycled units than it is for a firm. This is demonstrated as $g'_{rC}(\gamma) > g_{rC}(\gamma)$ for given γ. In other words, the remanufacturing cost threshold (minimum level of cost saving), below (above) which complete remanufacturing is optimal, is higher (lower) for the environmental planner than for the firm.

To better compare and contrast the impact of decision rules, Figure 4.7 establishes the optimal remanufacturing strategy space in γ and g_r for different decision makers. The two letters in each parenthesis correspond to the optimal strategy according to single bottom line or double-bottom line. For instance, (P, C') corresponds to the case in which the firm does partial remanufacturing while the environmental planner does complete remanufacturing. From Figure 4.7, we observe that the environmental planner always implements "greener", or at least no worse, strategic decisions than the firm



Note: $\theta = 0.2$, $\alpha = 0.2$; $c_1 = 0.1$; $c_2 = 0.3$; $e = 0.15$;

Figure 4.7: Illustration of Optimal Remanufacturing Strategy for Different Decision Rules

does, which is intuitive. Here, we mention two regions of interests. The first region is (M, M') in which even the environmental planner designs for remanufacturing but does not actually sell any remanufactured products. In this case, the remanufacturing cost is too high (i.e., $g_r > \frac{(f_n+2e)f_r g_n}{f_n(f_n-e)}$), thus it is not worthwhile to sell remanufactured products. Notice that as consumers value new products more (f_n increases), this region expands to the left ($\frac{(f_n+2e)f_r g_n}{f_n(f_n-e)}$ decreases), suggesting that the environmental planner is more likely to design but not to remanufacture. The other region is (P, P') in which even the environmental planner implements partial remanufacturing. In this case, the remanufacturing cost is moderate hence the decision makers are not willing to implement complete remanufacturing due to the potential cannibalization.

Notice that as the environmental factor e increases, both $\hat{\gamma}'$ and $\frac{(f_n+2e)f_r g_n}{f_n(f_n-e)}$ increase, or move to the right in Figure 4.7. This suggests that e, or more specifically β, can be thought of as a possible policy lever, such as a tax penalty for example, to push firms toward more environmentally friendly decisions.

Comparing the optimal product design for an environmental planner and a global planner, we have the following results on the optimal quality and demand that maximize welfare.

Proposition 4.7. *The global planner's objective function value is always twice that of the environmental planner, i.e.,* $\Theta(q) = 2\Phi(q)$ *for* $\forall q$. *Let* q' *(*q''*) and* d'_i *(*d''_i*) denote the optimal quality and demand for product* $i = n, r$ *that maximize the environmental (global) planner's objective function* $\Phi(\Theta)$.

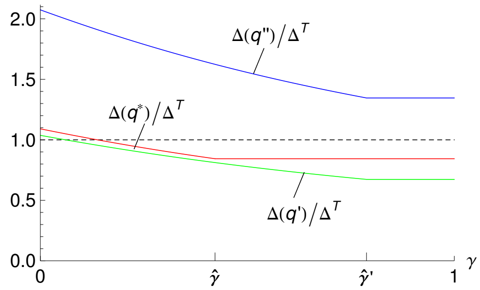


Figure 4.8: Illustration of Δ Under Different Bottom Line ($\theta = 0.2$,$\alpha = .1$; $c_1 = .1$; $c_2 = 0.3$; and $e = .03$;)

Accordingly,

(i) $q' = q''$*;*

(ii) $2d_i' = d_i''$*,* $i = n, r$*.*

Proposition 4.7 characterizes the intrinsic relationship between the environmental planner and global planner's design for remanufacturing problem. Most interestingly, considering consumer welfare in the decision-making process yields no effect in the choice of optimal quality. However, the global planner sells both products at lower prices such that the demand for each product is doubled relative to that for the environmental planner. As a result, more consumers are enjoying surplus. In fact, the contribution from the third bottom line, consumer welfare, equals to the contribution from the double-bottom line, sum of profit and environmental damage, i.e., $\Omega(q') = \Theta(q') - \Omega(q')$.

Environmental Damage

Our previous discussion indicates that different decision rules do not fundamentally change the structure of the optimal solution to the design for remanufacturing problem. Consequently, given a decision rule, higher collection rate should reduce, or at least not increase, environmental consequences of remanufacturing. This results is exhibited in Figure 4.8 which depicts the associated environmental damage for 1BL ($\Delta(q^*)$), 2BL ($\Delta(q')$) and 3BL ($\Delta(q'')$) respectively, as compared to the environmental damage associated with the non-remanufacturing solution ($\Delta(q^T)$).

Examining first the curve for each decision rule, we find that the environmental damage first decreases and then remains constant regardless of the decision rules. Take the environmental planner for example. If the collection rate is low (i.e., $\gamma \leq \hat{\gamma}'$), then increases in collection rate translate to lower environmental damage. In this case, the environmental planner remanufactures all recycled units. Hence higher collection rate means more used units are remanufactured in each period. As a result, fewer new products are sold, and a stronger cannibalization leads to smaller environmental damage. If the collection rate is relatively high (i.e., $\gamma \leq \hat{\gamma}'$), however, increases in collection rate do not affect the environment. In this case, the environmental planner remanufactures a fixed amount of recycled products to reduce cannibalization. Higher collection rate does not increase remanufacturing but means more used units are discarded after being collected. As a result, the environment cannot benefit from higher collection rate. Note that the set of parameters in Figure 4.8 represents the scenario in which the environmental factor e is not significant. Accordingly, both the firm and the environmental planner find it optimal to implement partial remanufacturing when collection rate is close to 100% (as region (P, P') in Figure 4.7).

Examining second the environmental damage for given collection rate, we notice that 2BL always causes lower environmental damage than 1BL does. Recall that environmental damage is a function of both quality and new products sales volume. The environmental planner always sells a smaller volume of products and generally designs lower quality than the firm does, hence the associated environmental damage is lower. If a higher quality is designed (which could occur when remanufacturing cost is moderate and collection rate is high), then the environmental planner will sell much smaller volume of new products. And this reducing effect from smaller new products sales outweighs the increasing effect from high quality, thus again leading to lower environmental damage.

However, we also observe that 3BL causes more environmental damage than 1BL or 2BL. We know from Proposition 4.7 that the global planner designs the same product quality as the environmental planner but sells twice as much new products as well as remanufactured products. In this case, the increasing effect from large volume dominates the potentially reducing effect from lower quality. As a result, the environment is always worse-off when 3BL is adopted as the decision rule. This again suggests that it is consumption rather than production that is the fundamental driver for more environmental damage.

Consumer Welfare

Consumer welfare is defined as the difference between the utility and price associated with purchasing a product. Accordingly, an individual consumer derive smaller consumer welfare with lower product quality or higher price, or both. Figure 4.9 (a) shows the consumer welfare for 1BL ($\Omega(q^*)$), 2BL ($\Omega(q')$), and 3BL ($\Omega(q'')$), respectively, relative to that for the Traditional case ($\Omega(q^T)$). First, we find that as collection rate increases, consumer welfare first decreases and then remains the same. We know from Proposition 4.4 that increases in collection rate mean higher quality, higher prices and lower sales volumes. While the lower sales volumes mean that fewer consumers can enjoy positive welfare, higher quality and prices mean that each consumer can enjoy a lower consumer welfare. Hence the total consumer welfare for the market decreases. When collection rate is higher enough, increases in collection rate do not affect quality design nor remanufacturing decisions. Hence, the total consumer welfare for the market remains the same.

Second, we find that 3BL leads to the highest amount of consumer welfare followed by 1BL and 2BL. 3BL yields the highest amount of consumer welfare, which is intuitive because the global planner considers consumer welfare in the decision making process and sells products to much more consumers. In contrast, 2BL yields the lowest amount of consumer welfare. Compared to the global planner, the environmental planner sells products to much fewer consumers and charges higher prices. Compared to the firm, the environmental planner generally designs lower quality and sells to fewer consumers, and both factors lead to lower consumer welfare.

Our discussion shows that higher sales volumes and the consequent higher consumer welfare under 3BL perspective lead to higher environmental damage. Then a natural follow-up question is how does 3BL perform when environmental damage is normalized by consumer welfare. Accordingly, Figure 4.9 (b) illustrates the environmental damage associated with unit level of surplus for 1BL ($\Delta(q^*)/\Omega(q^*)$), 2BL ($\Delta(q')/\Omega(q')$), and 3BL ($\Delta(q'')/\Omega(q'')$), respectively, normalized by that for the Traditional case ($\Delta(q^T)/\Omega(q^T)$). We notice that remanufacturing for all decision rules performs better than the

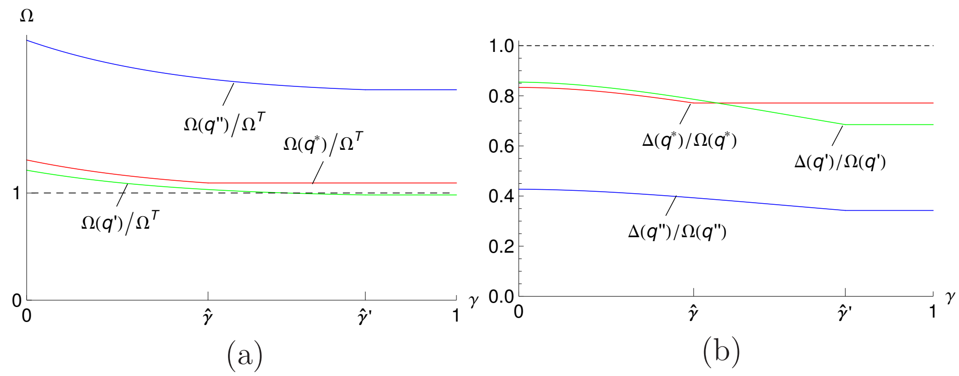


Figure 4.9: Illustration of Consumer Welfare ($\theta = 0.1$,$\alpha = .1$; $c_1 = .1$; $c_2 = 0.2$; and $e = .03$;)

non-remanufacturing. In particular, 2BL performs worse than 1BL when collection rate is low and does better when collection rate is high. We know from previous discussion that 2BL yields lower environmental damage as well as lower consumer welfare. When collection rate is low, both the firm and the environmental planner implement complete remanufacturing indicating the same level of cannibalization. In addition, the environmental planner designs a lower quality and sells smaller volume than the firm does. Consequently, switching from 1BL to 2BL leads to a larger reduction in consumer welfare than in environmental damage. When collection rate is high, however, the environmental planner sells a larger volume of remanufactured products than the firm does, which translates into stronger cannibalization. Consequently, switching from 1BL to 2BL leads to a smaller reduction in consumer welfare than in environmental damage. Moreover, 3BL causes the least amount of environmental damage normalized by consumer welfare. This result is to be expected because the global planner induces much larger consumer welfare than the firm as well as the environmental planner.

These results have important implications for the policy makers. If the policy makers care more about the environment, then they should minimize the total environmental impact. This objective indicates lower product quality and lower sales volume. On the other hand, if the concern is on both the consumers and the environment, then they should minimize the environmental damage per unit of consumer surplus. However, this objective leads to bigger environmental impact and the environment is worse-off.

Public Policy Instruments

From the public policy maker's perspective, a profit-maximizing firm does not serve the interests of the environment nor the consumers. To improve this situation, various policy instruments can be employed. Here, we discuss two common practices that are designed for the firm: 1) taxation on each unit of environmental damage incurred, denoted by b, and 2) subsidy for each used product remanufactured, denoted by s. Specifically, we study what policy or combination of policies are necessary for the firm to behave as an environmental planner or a global planner.

First, by setting $b = \beta$, taxation alone suffices for the firm to consider environmental damage in its product design process. In essence, taxation functions as additional cost to each unit of new products. This additional cost makes the firm design lower quality and remanufacture more. However, subsidy alone will not achieve the intended objective but drive the firm towards the opposite direction. This is because subsidy, in effect, reduces the remanufacturing cost and increases the cost saving from remanufacturing. To a certain extent, high subsidy means smaller g_r. In response, the firm would increase the optimal quality and consequently cause more environmental damage.

Second, for the firm to operate as a global planner, both taxation and subsidy instruments are required. On one hand, imposing a tax of β on each unit of environmental damage incurred will force the firm to behave as an environmental planner as discussed above. On the other hand, the difference in profit between the environmental planner and global planner, $\Pi'(q') - \Pi''(q')$, must be compensated through subsidies. The following proposition summarizes our discussion.

Proposition 4.8. *Public policy instruments can provide incentives for a profit-maximizing firm to behave as an environmental planner or even a global planner. First, taxation, rather than subsidy, alone is effective to transform the firm to an environmental planner. Second, both taxation and subsidy are required to transform the firm to an environmental planner. Specifically, impose a tax of* $b = \beta$ *on each unit of environmental damage and provide a* *subsidy s for each remanufactured product, where*

$$s = \begin{cases} \frac{(f_n+f_r\gamma-e)(f_n+f_r\gamma-4e)}{18(g_n+g_r\gamma)}, & \gamma \leq \min[\hat{\gamma}', 1]; \\ q' \frac{(f_n-f_r-(g_n-g_r)q'-e)(f_n-f_r-(g_n-g_r)q'-3e)+(f_n-f_r)(f_r-g_r q')^2}{2((f_r g_n - f_n g_r)q' + e f_2)}, & \hat{\gamma}' < \gamma \leq 1. \end{cases}$$

where q' is specified in Proposition 4.6.

4.7 Conclusion Remark

In this chapter, we first examine the applicability of the results in §3, and then extend the model to study the impact of collection rate and different decision rules on the design for remanufacturing problem.

In §4.1, we explore the value of educating consumers regarding their evaluations towards new and remanufactured products. We find that it is environmentally friendly to reduce consumers' evaluation for new products and to improve their evaluation for remanufactured products. §§4.2-4.4 examine the robustness of our results in §3 by relaxing our modeling assumptions to represent a continuous remanufacturability k, a exogenous quality, and a infinite-horizon model. The discussion suggests that our results in §3 are quite robust. For example, if remanufacturability is continuous, then our computational analysis demonstrates that the firm would still set the optimal remanufacturability at either $k^* = 0$ or $k^* = 1$. If product quality is not a decision, then the firm's decision to design for remanufacture could still hurt the environment when the level of remanufacturing does not bring out strong enough cannibalization. And, if the planning horizon changes from two periods to infinite, then the firm would slightly increases its product quality, producing similar impact on the environment.

In §4.5, we explore the impact of collection rate to the firm's design for remanufacturing problem. In contrast with §3, we consider an exogenous collection rate which introduces new tradeoffs to the problem. On the down side, low collection rate creates a more stringent supply constraint. The cost saving potential in used products that are not recycled cannot be realized. On the up side, low collection rate would reduce cannibalization of new products from remanufactured ones, which could potentially increase the profitability of remanufacturing. Considering this new tradeoff, we use a infinite-horizon model to study the impact of collection on the profitability and consequently the environmental impact of remanufacturing.

Our analysis show that collection rate does not affect whether or not to remanufacture, but affects how much to remanufacture. As a result, when remanufacturing is sufficiently cost efficient, increases in collection rate generally increase profitability and reduce environment consequences. Higher collection rate translates to larger volume of remanufactured products hence more cost savings; in the meantime, it also translates to stronger cannibalization hence smaller volume of new products. Consequently, the environment is better-off. However, this is not necessarily true. In particular, if remanufacturing cost is moderate, then there exists a threshold collection rate above which changes in collection rate do not affect the corresponding design nor remanufacturing decisions. This threshold provides another explanation why it is not necessarily beneficial to invest in reverse logistics so as to improve the collection rate.

In §4.6, we revisit the design for remanufacturing problem from the perspectives of different stakeholders by considering the cost of environment damage and consumer welfare in the objective function. In doing so, we understand how the interest of one stakeholder may affect the interests of others. Our results indicate that considering the interests of different stakeholders does not change the structure of the optimal remanufacturing solutions but can potentially lead to unintended outcomes. For example, considering the environmental damage in the objective function in addition to profit will benefit the environment due to lower quality and stronger cannibalization in each period. Surprisingly but not unexpectedly, however, including consumer welfare in the objective could hurt the environment even more. Although the optimal quality design remains the same, considering consumer welfare would double the volumes of both new and remanufactured products, thus doubling the environmental damage. This result once again underscores our conclusion that it is consumption rather than production that hurts the environment.

An interesting implication of our discussion on public instruments reveals that imposing tax on each unit of environmental damage can force a decision maker to design lower quality and to sell smaller volume of products. The higher is this tax, the smaller is the associated environmental impact. However, subsidy may not function as effective in reducing environmental damage as tax does. In fact, subsidy may cause the firm to design high quality which could potentially lead to more environmental damage.

Chapter 5

Conclusion

5.1 Summary and Results

In this dissertation, we study the interactions among three stakeholders—a firm, market, and environment—in the context of product design as illustrated in Figure 5.1. In the center of this triangular is the product(s) characterized by quality, a single dimensional vertical measure. This product offering (quality, price, and quantity) affects the interests of all three stakeholders. The firm is characterized by its production technology, which affects its cost structure such as variable cost (Moorthy 1984, Moorthy and Png 1992), fixed cost (Krishnan and Zhu 2006), inventory cost (Netessine and Taylor 2007), as well as operational constraints such as common components (Kim and Chhajed 2000, Heese and Swaminathan 2006, Desai et al. 2001), flexibility (Shao 2007), and lead time (Chayet et al. 2011). Given this, the firm decides product quality and price to maximize its profit, which is denoted by the difference between revenue and cost.

The market is characterized by consumer differentiation, either horizontal (e.g., taste of color) or vertical (r.e., valuation of quality) which in turn affects the demand. Horizontal differentiation generally assumes exogenous quality and studies search cost (e.g., Kuksov 2004), product positioning (e.g., Tyagi 2000), personalized pricing (e.g., Choudhary et al. 2005, Liu and Zhang 2006), and customization (e.g., Dewan et al 2003, Syam et al 2005). In contrast, vertical differentiation generally considers endogenous quality and studies topics such as segmentation (e.g., Moorthy 1984, Moorthy and Png 1992, Desai 2001), valuation change (e.g., Kim and Chhajed 2000). Given this differentiation and product offering, consumers purchase the product which maximizes their non-negative surplus, which is denoted by the difference between utility and price.

Compared to the firm and the market, the environment makes no deci-

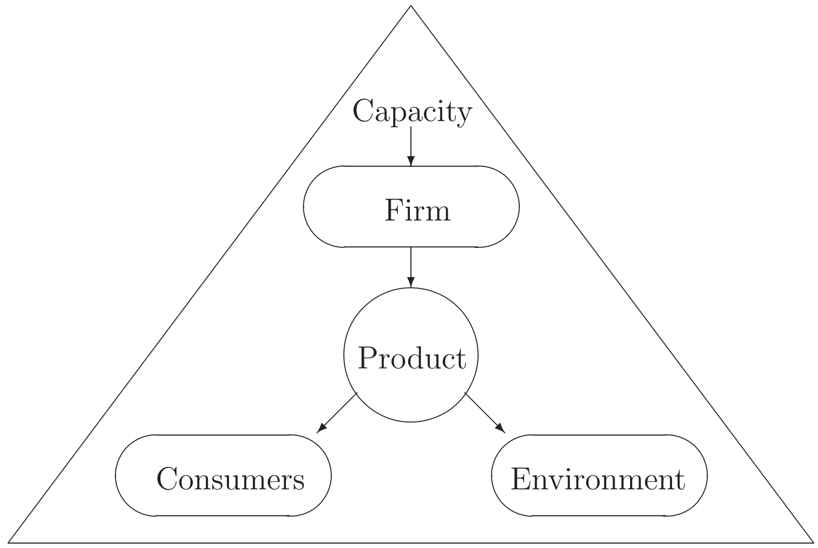


Figure 5.1: Stakeholder Triangular

sions but suffers the potential consequences. In this dissertation, we characterize the environment from two perspectives, resource extraction and waste disposal, which both work against the environment. Given this, the environment's interest is to minimize the environmental consequences associated with decisions made by the firm and consumers.

In the first essay (§2), we consider the interaction between the firm and the market, and investigate the effect of back-end operational constraint such as limited capacity on the firm's optimal product line design decision. The firm serves a segmented market characterized by two consumer segments that differ in valuation and size. Its product line design affects both production cost and capacity consumption. On the cost side, product quality quadratically increases variable cost indicating an increasing cost to quality, which is consistent with existing literature (e.g., Moorthy and Png 1992, Desai 2001). On the capacity side, the length of product line determines the fixed setup capacity such that offering two products requires b more units of capacity to setup than offering a single product does. Meanwhile, variable capacity consumption linearly increases in product quality, indicating a constant capacity to quality ratio.

Given this modeling context, we find that operational constraints such as limited capacity can introduce cannibalization from the operations side. This *operations* cannibalization, which increases as the corresponding resource decreases, is as opposed to the *market* cannibalization introduced by heterogeneity in consumer valuations. While both types of cannibalization reduce the length of a product line (r.e., the number of product types), the exact type of cannibalization exhibits quite different effects on the firm's product line design decisions. For example, market cannibalization distorts the quality of the low-quality product, increasing the width of the product line (r.e., the quality differentiation between two products). In contrast, operations cannibalization reduces the quality of each product and, if two products are offered, it does so by maintaining a constant width of product line. Another result is that limited capacity can potentially make it optimal to offer a standard product for the whole market. When capacity is limited, how the firm consumes capacity matters. In particular, if a longer product line requires larger setup capacity (higher cost of variety), then offering a standard product results in lower unit capacity requirement due to economy of scale. To a certain extent, operations constraints can be interpreted as additional shadow costs to the objective function. If these factors bring about economy of scale, then they would produce similar effects on the optimal product line strategy, making standardization potentially optimal. As cases in point, consider the inventory cost in Netessine and Taylor (2007) and the fixed cost in Krishnan and Zhu (2006).

In the second essay (§3), we again explore the dynamics between the firm and the market but focus on remanufacturing and its environmental consequences in the context product design. Specifically, the firm's design choices—remanufacturability and quality—affect both the production costs and the consumer valuations associated with each type of product. Given this, we investigate the environmental consequences of remanufacturing by measuring environmental damage as a function of both quality and quantity. We find that consumer greenness and production cost efficiency generally lead to a more profitable firm, which is consistent with the remanufacturing literature (e.g., Debo et al. 2005). But, our results also indicate that higher profitability does not necessarily translate into more environmentally friendliness. Moreover, consumer preferences and production technology potentially affect the environment differently than they do the profit.

In our model, we differentiate consumers' valuations for remanufacturable products from those for remanufactured ones. This differentiation enables us to pinpoint the essential characteristics of what makes green consumers "green". Specifically, we find that if green consumers overvalue the idea that a product *can be* remanufactured relative to how much they value a product that *has been* remanufactured, then the firm will not remanufacture enough used products so as to achieve the required level of cannibalization to benefit the environment. Consequently, under such circumstances, green consumerism may even translate into increased damage on the environment. In a related vein, we find that although production cost efficiencies translate into healthier bottom lines, they also translate into higher quality products that, in turn, translate into increased damage on the environment.

As an extension to the second essay, the third essay (§4) explores the effect of an exogenous collection rate on the optimal design for remanufacturability and its impact on the environment. We find that, only a fraction of used products are collected at the end of each period, new challenges are imposed on the design for remanufacturability problem. On the one hand, a low collection rate means that the cost saving potential in used products that are not collected cannot be realized, hence reducing the profitability of remanufacturing. On the other hand, a low collection rate creates a more stringent supply of used products, hence fewer products can be remanufactured. This reduces the cannibalization of new products from remanufactured ones, which could potentially increase the profitability of remanufacturing. Given this trade-off, our results confirm that an increase in the collection rate generally benefits both the firm and the environment. A higher collection rate translates into a larger sales volume of remanufactured products and, hence, a higher profit; it also translates into stronger cannibalization thus reducing the sales volume of new products, which, in turn, translates into lower environmental damage. However, there exists a threshold above which increases in collection rate produce no effects on either profit or environmental impact.

Also in §4, we compare and contrast the firm's optimal design for remanufacturability results to those that would maximize social and environmental welfare. We find that, when compared to the social and environmental planner's optimum, the firm over produces in the sense that it increases both product quality and sales volume.

5.2 Future Research

Like any other modeling research, our results rely on the modeling assumptions we have made. The most essential assumption is that quality can be defined as a single dimensional vertical measure that represents all more-is-better attributes of a product. Other assumptions are important to derive analytical results, but they are not critical to the insights. For example in §2, we assume that the firm consumes capacity in both fixed and variable ways. While the fixed capacity consumption is essential for offering a standard product to be optimal, it does not alter the overall effect of limited capacity. Similarly, the implications of capacity remain fundamentally the same as long as variable capacity consumption increases in quality faster than it does in quantity. For another example, in §§3–4, we assume that environmental damage associated with a product increases linearly in quality. Our discussions show that our results hold as long as environmental damage is defined to be a non-decreasing function of quality.

However, considering some other factors in our model could potentially change our results. Here, we discuss two major directions for future research. First, this dissertation assumes that environmental damage is mainly reflected by resource extraction and waste disposition. Consumption, however, may also be a major contributor to the environmental damage during a product's life cycle. Considering consumption could lead to two complications. On the one hand, higher quality could potentially decrease the associated environmental damage during consumption due to better technology or higher efficiency (i.e., Subramanian et al. 2009). On the other hand, remanufactured products will contribute to the environmental damage measure. If such a during-use environmental effect were incorporated into our modeling context, then an additional term would be required to represent the amount of environmental damage incurred during use, which could potentially alter the relationship between quality and environmental impact. Therefore, exploration of such an effect constitutes a viable direction for future research.

Second, this dissertation explores the product design problem faced by a monopoly firm, and provides a picture of how operational issues affect the monopolist's product design decisions and how these decisions interact with the environment. Another interesting factor that we have not yet considered is competition. Especially in the remanufacturing industry, a firm may face competition from a third-party remanufacturer who intercepts and remanufactures a fraction of used products and sells remanufactured products to the same market (e.g., Majumder and Groenevelt 2001). It may also face competition from another manufacturer who competes with new rather than remanufactured products (Atasu et al. 2008). In these cases, the firm may want to thwart a competitor by designing a non-remanufacturable product or, if the product is designed to be remanufacturable, by remanufacturing more used products. Although related literature has studied competition in various settings, the effect of endogenous quality has not yet been examined.

Appendix A

Appendix for Chapter 2

Due to the similarities between single product models, we prove Lemma 2.1 and Proposition 2.1 for the Niche strategy.

Proof of Lemma 2.1. The lagrange function corresponding to (3.3) is

$$\max_{x,q} \mathcal{L} = x(qv_h - cq^2) + \lambda(K - axq) + \mu(n_h - x) \tag{A.1}$$

The KKT conditions are

$$\frac{\partial L}{\partial q} = x(v_h - a\lambda - 2cq)$$
$$\frac{\partial L}{\partial x} = q(v_h - a\lambda - cq) - \mu$$

And the corresponding solutions are

$$q = \frac{(v_h - a\lambda)}{2c};$$

To prove Lemma 2.1, we first discuss the quantity condition. If $\mu > 0$, then complementary condition dictates $x = n_h$. If $\mu = 0$, then $\frac{\partial L}{\partial q} = \frac{(v_h - a\lambda)^2}{4c} > 0$. Thus, the firm should product as much as possible, or $x = n_h$.

Now we discuss the capacity condition. If $\lambda > 0$, then $K = an_h q = \frac{an_h(v_h - a\lambda)}{2c} < \frac{an_h v_h}{2c}$. If $\lambda = 0$, then $K \geq an_h q = \frac{an_h v_h}{2c}$. Thus, the capacity constraint is binding when $K \leq \frac{an_h v_h}{2c}$. □

Proof of Proposition 2.1. From Lemma 2.1, we know that for the Niche strategy, $x^* = n_h$. If $K \leq K^N$, then capacity constraint is binding. In other words, $q^* = \frac{K}{an_h}$. If $K > K^N$, $q^* = \frac{v_h}{2c}$. Similarly, for the Standard strategy, $x^* = (n_h + n_l)$, $q^* = \frac{K}{a(n_h+n_l)}$ if $K \leq K^S$ and $q^* = \frac{v_l}{2c}$ if $K > K^S$. □

Proof of Lemma 2.2. To prove our results, we again use lagrangian method.

The lagrange function corresponding to (2.14) is

$$\mathcal{L}(x_h, x_l, q_h, q_l) = x_h(q_h v_h - c q_h^2 - q_l(v_h - v_l)) + x_l(v_l q_l - c q_l^2) \qquad \text{(A.2)}$$
$$+ \lambda(K - b - a(x_h q_h + x_l q_l)) + \mu_h(n_h - x_h) + \mu_l(n_h + n_l - x_h - x_l)$$

where λ, μ_h and μ_l are lagrangian coefficients for conditions (2.13), (2.3), and (2.6) respectively. The corresponding KKT conditions are

$$\frac{\partial L}{\partial q_h} = x_h(v_h - a\lambda - 2cq_h) = 0$$
$$\frac{\partial L}{\partial q_l} = x_l(v_l - a\lambda - 2cq_l) - x_h(v_h - v_l) = 0$$
$$\frac{\partial L}{\partial x_h} = q_h(v_h - a\lambda - cq_h) - q_l(v_h - v_l) - \mu_h - \mu_l = 0$$
$$\frac{\partial L}{\partial x_l} = q_l(v_l - a\lambda - cq_l) - \mu_l = 0$$

From the KKT conditions, we have

$$q_h = \frac{(v_h - a\lambda)}{2c}; \quad q_l = \frac{v_l - a\lambda}{2c} - \frac{x_h(v_h - v_l)}{2cx_l}$$

First, we prove that quantity condition (2.6) is always binding. We consider two cases: If $\mu_l > 0$, then this is true due to complementary slackness. If $\mu_l = 0$, then $\frac{\partial L}{\partial x_l} = \frac{1}{4c}((v_l - a\lambda)^2 - x_h^2(v_h - v_l)^2/x_l^2) > 0$ because the Product-Line strategy dictates $q_l = \frac{v_l - a\lambda}{2c} - \frac{x_h(v_h - v_l)}{2cx_l} > 0$. Thus, condition (2.6) is always binding.

Now, we consider quantity condition (2.3). From previous proof, we know that $\mu_l = \frac{1}{4c}((v_l - a\lambda)^2 - x_h^2(v_h - v_l)^2/x_l^2) > 0$. Hence, $\frac{\partial L}{\partial x_h} = (q_h - q_l)(\frac{(v_h - v_l)}{2} + \frac{x_h(v_h - v_l)}{2x_l} - \mu_h$. We again consider two cases: if $\mu_h > 0$, then condition (2.3) is binding due to complementary slackness. If $\mu_h = 0$, then $\frac{\partial L}{\partial x_h} > 0$, thus condition (2.3) is binding.

Now we prove that capacity constraint (2.13) is binding. If $\lambda > 0$, then

$$K = b + a(n_h \frac{(v_h - a\lambda)}{2c} + n_l(\frac{v_l - a\lambda}{2c} - \frac{n_h(v_h - v_l)}{2cn_l}))$$
$$= b + \frac{a(n_h + n_l)}{2c}(v_l - a\lambda)$$
$$\iff 0 \leq a\lambda = b + \frac{a(n_h + n_l)v_l}{2c} - K$$

Thus, (2.13) is binding if $K < b + \frac{a(n_h+n_l)v_l}{2c}$. □

Proof of Proposition 2.2. From Lemma 2.2, we know that at optimality $x_h^* = n_h$, $x_l^* = n_l$, and $\lambda = b/a + \frac{(n_h+n_l)v_l}{2c}$. Correspondingly, the optimal qualities are

$$q_h^* = \frac{(K-b)}{a(n_h+n_l)} + \frac{(v_h - v_l)}{2c}; \quad q_l^* = \frac{(K-b)}{a(n_h+n_l)} - \frac{n_h(v_h-v_l)}{2cn_l}.$$

$q_l^* > 0$ requires $K > K^L = \bar{K} - K^S(1-R)$. □

Proof of Proposition 2.3. We prove this proposition by comparing the profit associated with each strategy. First, we compare the profits of niche and standard strategies. From Proposition 2.1, we know that profit function $\Pi^N(K)$ and $\Pi^N(K)$ are both defined on two domains, and $K^N \geq K^S$ requires $v_h \geq \frac{(n_h+n_l)}{n_h}v_l$. Hence, we consider four cases based on the available capacity K. Case 1: If $K > \max[K^N, K^S]$, then, $\Pi^N(K) - \Pi^S(K) = \frac{n_h v_h^2}{4c} - \frac{(n_h+n_l)v_l^2}{4c} \geq 0$ which requires $v_h \geq \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$. Case 2: If $K < \min[K^N, K^S]$, then

$$\begin{aligned} \Pi^N(K) - \Pi^S(K) &= \frac{K(an_h v_h - cK)}{a^2 n_h} - \frac{K(a(n_h+n_l)v_l - cK)}{a^2(n_h+n_l)} \geq 0 \\ &\Rightarrow K \geq 2RK^S \end{aligned}$$

For this case to be valid, we have $v_h \leq v_h^{ns} = \frac{2(n_h+n_l)v_l}{(2n_h+n_l)}$. Case 3: If $K^N < K \leq K^S$, then $\Pi^N(K) - \Pi^S(K) = \frac{n_h v_h^2}{4c} - \frac{K(a(n_h+n_l)v_l - cK)}{a^2(n_h+n_l)} \geq 0$ requires $K < K^S(1 - \sqrt{1 - \frac{n_h v_h^2}{(n_h+n_l)v_l^2}})$. For this case to be valid, we again have $v_h^{ns} < v_h \leq \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$. Case 4: If $K^S < K \leq K^N$, then $\Pi^N(K) \geq \Pi^S(K)$ is always true. Correspondingly, we define the isoprofit line K^{ns} such that the Standard strategy is more profitable when $K > K^{ns}$.

$$K^{ns} = \begin{cases} 2RK^S, & v_h \leq v_h^{ns}; \\ K^S(1 - \sqrt{1 - \frac{n_h v_h^2}{(n_h+n_l)v_l^2}}), & v_h^{ns} < v_h < \sqrt{\frac{(n_h+n_l)}{n_h}}v_l; \end{cases}$$

Second, we compare the profit of Product-Line strategy with that of the Standard strategy. We consider two cases. Case 1: If $K < K^S$, then $\Pi^L(K) - \Pi^S(K) = \frac{n_h v_h^2}{4c} + \frac{n_l v_l^2}{4c}(1-R) - \frac{c(\bar{K}-K)^2}{a^2(n_h+n_l)} - \frac{K(a(n_h+n_l)v_l - cK)}{a^2(n_h+n_l)} \geq 0$ requires $K \geq \bar{K} - \frac{a^2 n_h(n_h+n_l)^2(v_h-v_l)^2 + 4b^2c^2n_l}{8bc^2n_l}$. For this case to be valid, we need $v_h > v_h^S =$

$v_l(1+\frac{b\sqrt{n_l}}{K^S\sqrt{n_h}})$. Case 2: If $K^S < K < \bar{K}$, then $\Pi^L(K)-\Pi^S(K) = \frac{n_h v_h^2}{4c} + \frac{n_l v_l^2}{4c}(1-R) - \frac{c(\bar{K}-K)^2}{a^2(n_h+n_l)} - \frac{(n_h+n_l)v_l^2}{4c} \geq 0$ requires $K \geq \bar{K} - K^S\frac{(v_h-v_l)\sqrt{n_h}}{v_l\sqrt{n_l}}$. For this case to be valid, we need $v_h \leq v_h^S$. Correspondingly, we define the isoprofit line K^{sl} such that the Product-Line is more profitable when $K > K^{sl}$.

$$K^{sl} = \begin{cases} \bar{K} - \frac{RK^S\sqrt{n_l}}{\sqrt{n_h}}, & v_h \leq v_l + \frac{2bc\sqrt{n_l/n_h}}{a(n_h+n_l)}; \\ \bar{K} - (\frac{b}{2} + \frac{n_l(RK^S)^2}{2bn_h}), & v_l + \frac{2bc\sqrt{n_l/n_h}}{a(n_h+n_l)} < v_h \leq \frac{(n_h+n_l)v_l}{n_h}; \end{cases}$$

Third, we compare the profit of Product-Line strategy with that of the Niche strategy. We again consider two cases. Case 1: If $K < K^N$, then $\Pi^L(K) - \Pi^N(K) = \frac{n_l v_l^2}{4c}(1-R) - \frac{c(\bar{K}-K)^2}{a^2(n_h+n_l)} + \frac{c(K^N-K)^2}{a^2 n_h} \geq 0$ requires $K \geq RK^S - \frac{bn_h}{n_l}(1+\sqrt{\frac{(n_h+n_l)}{n_h} + K^S(1-R)\frac{2n_l}{bn_h}})$. Correspondingly, we require $v_h \leq v_h^N = \frac{(n_h+n_l)v_l}{n_h} - \frac{2bc(n_l-\sqrt{(n_h+n_l)n_l})}{an_h^2}$. Case 2: If $K > K^N$, then then $\Pi^L(K) - \Pi^N(K) = \frac{n_l v_l^2}{4c}(1-R) - \frac{c(\bar{K}-K)^2}{a^2(n_h+n_l)} \geq 0$ requires $K \geq \bar{K} - K^S(1-R)\frac{\sqrt{n_l}}{\sqrt{(n_h+n_l)}}$. For this case to be valid, we need $v_h^N < v_h \leq \frac{(n_h+n_l)v_l}{n_h}$. Corresponding, we define the isoprofit line K^{nl} such that the Product-Line is more profitable when $K > K^{nl}$.

$$K^{nl} = \begin{cases} K^S R - \frac{bn_h}{n_l}(1+\sqrt{\frac{(n_h+n_l)}{n_h} + K^S(1-R)\frac{2n_l}{bn_h}}) & v_h \leq v_h^N; \\ \bar{K} - K^S(1-R)\frac{\sqrt{n_l}}{\sqrt{(n_h+n_l)}}, & v_h^N < v_h \leq \frac{(n_h+n_l)v_l}{n_h}; \end{cases}$$

Therefore, offering two products is optimal when $K > \max K^{sl}, K^{nl}$, offering a niche product is optimal when $K \leq \min K^{ns}, K^{nl}$, and offering a standard is optimal when $K^{ns} < K < K^{sl}$. □

Proof of Proposition 2.4. To prove the first part, it suffices to show that $K^{ns} < K < K^{sl}$ when $v_l \leq v_h < \widehat{v_h}$. From Proposition 2.3, we know that K^{ns} increases in v_h and K^{sl} decreases in v_h. Then it suffices to show that $K^{ns} = K^{sl}$ at $\widehat{v_h}$, which is true by definition. To identify $\widehat{v_h}$, we first show that $v_h^S \leq \widehat{v_h}$ by contradiction. Assume $v_h^S > \widehat{v_h}$. Then $K^{ns}(v_h^S) < K^{ns}(\widehat{v_h}) = K^{ns}(\widehat{v_h}) <= K^{ns}v_h^S$ which is impossible. Therefore, $v_h^S \leq \widehat{v_h}$ is always true. For given setup capacity b, if $\widehat{v_h} \leq v_h^{ns}$, then $K^{ns} = K^{sl}$ implies that $\widehat{v_h} = v_l + \frac{2bv_l}{K^S}(-1+\sqrt{1+\frac{n_l}{4n_h}(1+\frac{2K^S}{b})})$. This requires $b \leq \widehat{b} = K^S\frac{(\sqrt{n_l(n_l+n_h)}-n_l)}{2n_h+n_l}$. If $\widehat{v_h} > v_h^{ns}$, then $K^{ns} = K^{sl}$ implies that $\widehat{v_h} =$

$v_l + \frac{bv_l}{K^S}\sqrt{\frac{n_l}{(n_h+n_l)}}(-1+\sqrt{\frac{n_l}{n_h}(-1+\frac{2K^S}{b}\sqrt{\frac{(n_h+n_l)}{n_l}})})$. This requires that $b > \widehat{b}$.

To prove the second part, it suffices to show that $\max[K^{sl}, K^{nl}] \geq \widehat{K}$. From the proof for Proposition 2.3, we know that K^{sl} decreases in v_h. Hence for any $v_h \leq \widehat{v_h}$, we have $K^{sl}(v_h) \geq K^{sl}(\widehat{v_h})$ which equals to $\widehat{K}$ by definition. Similarly, we know that K^{nl} increases in v_h. Hence for any $\widehat{v_h} < v_h \leq \frac{n_h+n_l}{n_h}v_l$, we have $K^{nl}(v_h) \geq K^{sl}(\widehat{v_h})$ which equals to $\widehat{K}$ by definition. □

Proof of Proposition 2.5. Now we prove that offering a standard product strategy is optimal only when $b \geq \beta_1$. It suffice to show that for given valuation v_h, it satisfies $v_h \leq \widehat{v_h}$ when $b \geq \beta_1$. Consider two cases: If $v_h \leq v_h^{ns}$, then $v_h \leq \widehat{v_h}$ implies $b \geq \beta_1 = K^S(1-2R)\left(\sqrt{1+\frac{n_l R^2}{n_h(1-2R)^2}}-1\right)$. If If $v_h \leq v_h^{ns}$, then $v_h \leq \widehat{v_h}$ implies $b \geq \beta_1 = K^S\frac{\sqrt{n_l}}{\sqrt{(n_h+n_l)}}\left(1-R-\sqrt{1-R\frac{(v_h+v_l)}{v_l}}\right)$. □

Proof of Proposition 2.6. To prove that the firm idles capacity before switching its optimal strategy, we show that the iso-profit capacity level K^{nl} and K^{ns} are higher than K^N, and that K^{sl} is higher than K^S. We consider three cases based on v_h. First, assume $\frac{2(n_h+n_l)v_l}{(2n_h+n_l)} < v_h < \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$. If setup capacity for offering two products $b \leq \beta_1$, then the optimal strategy is either Niche or Product-Line. Idling capacity means $K^{nl} > K^N$, which implies that $b > \beta_2 = \frac{an_l v_l(\sqrt{(n_h+n_l)/n_l}-1)}{2c}(1-R)$. If setup capacity for offering two products $b > \beta_1$, then the optimal strategy changes from Niche to Standard and to Product-Line as capacity increases. For the firm to idle capacity before switching from Niche to Standard strategy, we need $K^{ns} > K^N$ which is always true as $v_h > \frac{2(n_h+n_l)v_l}{(2n_h+n_l)}$. For the firm to idle before switching from Standard to Product-Line, we need $K^{sl} > K^S$ which implies that $b > \beta_3$.

Second, assume $v_h \leq \frac{2(n_h+n_l)v_l}{(2n_h+n_l)}$. We have $K^{ns} \leq K^N$ which indicates no capacity idling when the optimal strategy switches from Niche to Standard. Capacity idling occurs only when the optimal strategy switches from Standard to Product-Line. This requires $K^{sl} > K^S$ which is true when setup capacity is large enough, $b > \beta_3$.

Third, assume $v_h \geq \sqrt{\frac{(n_h+n_l)}{n_h}}v_l$. Standard strategy is always dominated by Niche strategy. Capacity idling occurs only when the optimal strategy switches from Niche to Product-Line. This requires $K^{nl} > K^N$ which is true when $b > \beta_2$. □

Proof of Proposition 2.7. When $\beta_2 < b \leq \beta_1$ or $b > \beta_3$, the firm idles capacity before switching from offering a single product to offering a product line. Given this, it is easy to verify that $K^N + b > K^{nl}$ and $K^S + b > K^{sl}$. When $b > \beta_1$, the firm idles capacity before switching from offering a niche product to offering a standard product. Given this, it is easy to verify that $K^N + b = K^N + \beta_1 > K^{nl}(\beta_1) = K^{ns}$. □

Proof of Proposition 2.8. First, for the high segment. As of Propositions 2.1 and 2.2, $q_h^* \leq \frac{v_h}{2c} = q_h^u$ is always true. Next, for the low segment. $q_l^* > q_l^u = \frac{v_l}{2c}(1-R)$ can only be true when offering a standard product is optimal, which means, as of Proposition 2.3, $K < K^{sl}$. In this case, $q_l^* = \frac{K}{a(n_h+n_l)} > \frac{v_l}{2c}(1-R)$ requires $K > K^S(1-R)$. Thus, we conclude the proof. □

Proof of Proposition 2.9. First, for the high segment. As of Propositions 2.1 and 2.2, $p_h^* > q_h^u$ can only be true when offering a niche product is optimal, which means, as of Proposition 2.3, $K < \max[K^{ns}, K^{nl}]$. In this case, $p_h^* = \frac{v_h K}{a n_h} > \frac{v_h^2}{2c} - (v_h - v_l)\frac{v_l}{2c}(1-R) = p_h^u$ requires $K > (n_l K^S R^2 + n_h K^N)\frac{v_l}{n_h v_h}$. Thus the high segment receives a higher price under limited capacity if $(n_l K^S R^2 + n_h K^N)\frac{v_l}{n_h v_h} < K < \max[K^{ns}, K^{nl}]$.

Next, for the low segment. $p_l^* > p_l^u$ is equivalent to $q_l^* > q_l^u$ which, from Proposition 2.9, requires $K^S(1-R) < K < K^{sl}$. Thus, we conclude the proof. □

Proof of Proposition 2.10. $CW_h > CW_h^u$ is equivalent to $q_l^* > q_l^u$ which, from Proposition 2.9, requires $K^S(1-R) < K < K^{sl}$. For this to be valid, we need $v_h < v_l + \frac{2bc(\sqrt{1+n_l/n_h}-1)}{a(n_h+n_l)}$. □

Appendix B

Appendix for Chapter 3

As building blocks for proving Propositions 3.2–3.8, we first establish three lemmas. The proofs of Lemmas B.1 and B.2 are combined and follow the statement of Lemma B.2. The proof of Lemma B.3 directly follows its statement. For Lemmas B.1–B.3, let $p_{ij}^G(q), d_{ij}^G(q)$ denote the optimal Green strategy price and corresponding demand as a function of q for product $i = n, r$ in stage $j = 1, 2$, respectively. Moreover, let $q_1 = \frac{f_n f_r (f_n - f_r)}{f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n}$ denote the quality threshold above which $d_{r2}^G(q) = d_{n1}^G(q)$, $q_2 = \frac{(f_n^2 - f_r^2)}{f_n g_n - f_r g_r}$ denote the quality threshold above which $d_{r2}^G(q) = d_{n1}^G(q)$ and $d_{n2}^G(q) = 0$, and $q_3 = \frac{(f_n + f_r)}{(g_n + g_r)}$ denote the quality threshold above which $d_{r2}^G(q) = d_{n1}^G(q) = d_{n2}^G(q) = 0$.

LEMMA B.1. *If $k = 1$, then for given $q < q_3$, $d_{r2}(q) > 0$ if and only if $g_r/f_r < g_n/f_n$.*

LEMMA B.2. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $q^{G*} < q_3$ and, for $i = n, r$ and $j = 1, 2$, $p_{ij}^G(q), d_{ij}^G(q)$ are as follows:*

(i) for $0 < q < q_1$,

$$d_{n1}^G(q) = \frac{f_n - g_n q}{2f_n}; \quad d_{n2}^G(q) = \frac{f_n - f_r - (g_n - g_r)q}{2(f_n - f_r)};$$
$$d_{r2}^G(q) = \frac{(f_r g_n - f_n g_r)q}{2f_r(f_n - f_r)};$$
$$p_{n1}^G(q) = p_{n2}^G(q) = \frac{(f_n + g_n q)q}{2}; \quad p_{r2}^G(q) = \frac{q(f_r + g_r q)}{2}; \tag{B.1}$$

(ii) for $q_1 \leq q < q_2$,

$$d^G_{n1}(q) = d^G_{r2}(q) = \frac{f_n(f_n - g_n q) + (f_r g_n - f_n g_r)q}{2(f_n^2 + f_r(f_n - f_r))};$$
$$d^G_{n2}(q) = \frac{f_n(f_n - g_n q) - f_r(f_r - g_r q)}{2(f_n^2 + f_r(f_n - f_r))};$$
$$p^G_{n1}(q) = \frac{(f_n(f_n + g_n q) + 2f_r(f_n - f_r) - (f_r g_n - f_n g_r)q)f_n q}{2(f_n^2 + f_r(f_n - f_r))};$$
$$p^G_{n2}(q) = \frac{(f_n + g_n q)q}{2};$$
$$p^G_{r2}(q) = \frac{(f_n(f_n + g_n q) - (f_n - f_r)(f_n - f_r - (g_n + g_r)q))f_r q}{2(f_n^2 + f_r(f_n - f_r))} \tag{B.2}$$

(iii) for $q_2 \leq q < q_3$,

$$d^G_{n1}(q) = d^G_{r2}(q) = \frac{f_n + f_r - (g_n + g_r)q}{2(f_n + f_r)}; \quad d^G_{n2}(q) = 0;$$
$$p^G_{n1}(q) = \frac{f_n}{f_r} p^G_{r2}(q) = \frac{f_n q(f_n + f_r + (g_n + g_r)q)}{2(f_n + f_r)}; p^G_{n2}(q) = p^G_{r2}(q) + (f_n - f_r)q \tag{B.3}$$

Proof of Lemmas B.1–B.2: Here we prove our results using the lagrangian method. The corresponding lagrangean of profit (3.6) for given q is

$$\begin{aligned}\mathcal{L}(p_{n1}, p_{n2}, p_{r2}) &= (p_{n1} - g_n q^2 + \lambda)\left(1 - \frac{p_{n1}}{f_n q}\right) \\ &+ \left(1 - \frac{p_{n2} - p_{r2}}{(f_n - f_r)q}\right)(p_{n2} - g_n q^2 + \mu) \\ &+ \left(\frac{p_{n2} - p_{r2}}{(f_n - f_r)q} - \frac{p_{r2}}{f_r q}\right)(p_{r2} - g_r q^2 - \lambda + \eta)\end{aligned}$$

where λ, μ and η are lagrangian coefficients for conditions $d_{r2} \leq d_{n1}$, $d_{n2} \geq 0$, and $d_{r2} \geq 0$ respectively. The corresponding KKT conditions are

$$\frac{\partial \mathcal{L}}{\partial p_{n1}} = 1 - \frac{p_{n1}}{f_n q} - \frac{(p_{n1} - g_n q^2 - \lambda)}{f_n q} = 0$$
$$\frac{\partial \mathcal{L}}{\partial p_{n2}} = 1 - \frac{p_{n2} - p_{r2}}{(f_n - f_r)q} - \frac{(p_{n2} - g_n q^2 + \mu)}{(f_n - f_r)q} + \frac{(p_{r2} - g_r q^2 - \lambda + \eta)}{(f_n - f_r)q} = 0$$
$$\frac{\partial \mathcal{L}}{\partial p_{r2}} = \frac{(p_{n2} - g_n q^2 + \mu)}{(f_n - f_r)q} + \frac{p_{n2} - p_{r2}}{(f_n - f_r)q} - \frac{p_{r2}}{f_r q} - \frac{f_n(p_{r2} - g_r q^2 - \lambda + \eta)}{f_r(f_n - f_r)q} = 0$$

as well as the corresponding orthogonal conditions. The corresponding solutions are

$$p_{n1}(q)=\frac{(f_n+g_nq)q-\lambda}{2};\quad d_{n1}(q)=\frac{(f_n-g_nq)q+\lambda}{2f_nq};$$
$$p_{n2}(q)=\frac{(f_n+g_nq)q-\mu}{2};\quad d_{n2}(q)=\frac{(f_n-f_r)q-(g_n-g_r)q^2+\mu+\lambda-\eta}{2(f_n-f_r)q};$$
$$p_{r2}(q)=\frac{(f_r+g_rq)q+\lambda-\eta}{2};\quad d_{r2}(q)=\frac{(f_rg_n-f_ng_r)q^2-f_n\lambda-f_r\mu+f_n\eta}{2f_r(f_n-f_r)q}$$

To prove Lemma B.1, we first prove sufficient and then necessary conditions. To that end, we first assume $d_{r2}(q)>0$ which implies $\eta=0$. Thus

$$0<d_{r2}(q)=\frac{(f_rg_n-f_ng_r)q^2-f_n\lambda-f_r\mu}{2f_r(f_n-f_r)q}\Rightarrow f_rg_n-f_ng_r>\frac{f_n\lambda+f_r\mu}{q^2}\geq 0$$

which completes the sufficient condition.

Next, assume $f_rg_n>f_ng_r$, which implies $q_1<\frac{(f_n-f_r)}{(g_n-g_r)}<q_2<\frac{f_n}{g_n}<q_3$. We consider four cases based on the multipliers λ and μ: (1) if $\lambda=\mu=0$, then $d_{r2}(q)>\frac{f_n\eta}{2f_r(f_n-f_r)q}\geq 0$; (2) $\lambda=0$ and $\mu>0$, then $d_{n2}(q)=0$, which implies $d_{r2}(q)\geq\frac{(f_r-g_rq)}{2f_r}>\frac{(f_n-g_nq)}{2f_n}=d_{n1}(q)\geq 0$; (3) if $\lambda>0$ and $\mu=0$, then $d_{n1}(q)=d_{r2}(q)$, which implies $\eta=\frac{((f_rg_n-f_ng_r)f_n+f_rg_n(f_n-f_r))q}{f_n^2}(q_1-q)+\frac{\lambda(f_n^2+f_n(f_n-f_r))}{f_n^2}$. Hence, $d_{n2}(q)\geq 0$ implies $0<\lambda\leq(f_n-f_r)(f_n-g_nq)q$, or $(f_n-g_nq)>0$. Thus, $d_{n1}(q)=d_{r2}(q)>\frac{(f_n-g_nq)}{2f_n}>0$; (4) if $\lambda>0$ and $\mu>0$, then

$$0<\lambda=\frac{(f_rg_n-f_ng_r)q^2+f_n\eta}{(f_n+f_r)};\quad 0<\mu=\frac{q(f_ng_n-f_rg_r)(q-q_2)+f_r\eta}{(f_n+f_r)}$$

and, consequently, $d_{n1}(q)=d_{r2}(q)=\frac{(g_n+g_r)q(q_3-q)+\eta}{2(f_n+f_r)}>0$, thereby completing the proof of Lemma B.1.

To prove Lemma B.2, note that $d_{r2}(q)>0$ (by definition); thus, $\eta=0$ and $f_rg_n>f_ng_r$ (by Lemma B.1). Accordingly,

$$0\leq d_{n1}(q)-d_{r2}(q)=\frac{q_1-q}{2q_1}+\frac{(f_n^2+f_nf_r-f_r^2)\lambda}{2f_nf_r(f_n-f_r)q}+\frac{\mu}{2(f_n-f_r)q}\qquad\text{(B.4)}$$

Notice from (B.4) that $d_{n1}(q)-d_{r2}(q)$ is decreasing in q. There are four possibilities. **Case I:** If $0<q<q_1$, then $q<\frac{f_n-f_r}{g_n-g_r}<\frac{f_n}{g_n}$. Accordingly, $d_{n1}(q)-d_{r2}(q)>\frac{(f_n^2+f_nf_r-f_r^2)\lambda}{2f_nf_r(f_n-f_r)q}+\frac{\mu}{2(f_n-f_r)q}\geq 0$, and $d_{n2}(q)>\frac{\mu+\lambda}{2(f_n-f_r)q}\geq 0$.

These inequalities imply that $\lambda = 0$ and $\mu = 0$, respectively. Thus, the KKT conditions above directly imply (B.1).

Case II: If $q_1 \le q < q_2$, then

$$\lambda = \frac{2f_nf_r(f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}(d_{n1}(q) - d_{r2}(q) + \frac{q - q_1}{2q_1}) - \frac{f_nf_r\mu}{f_n^2 + f_r(f_n - f_r)},$$

$$\Rightarrow d_{n2}(q) = \frac{f_nf_r(d_{n1}(q) - d_{r2}(q))}{(f_n^2 + f_r(f_n - f_r))} + \frac{(f_ng_n - f_rg_r)(q_2 - q)}{2(f_n^2 + f_r(f_n - f_r))} + \frac{(f_n + f_r)\mu}{2(f_n^2 + f_r(f_n - f_r))q} > \frac{(f_n + f_r)\mu}{2(f_n^2 + f_r(f_n - f_r))q} \ge 0$$

Thus, $\mu = 0$ and correspondingly, $\lambda = \frac{2f_nf_r(f_n-f_r)q}{(f_n^2+f_r(f_n-f_r))}(d_{n1}(q) - d_{r2}(q) + \frac{q-q_1}{2q_1})$. Notice that if $d_{n1}(q) > d_{r2}(q)$, then $\lambda > 0$ which is a contradiction. Therefore, $d_{n1}(q) = d_{r2}(q)$ and $\lambda = \frac{2f_nf_r(f_n-f_r)q}{(f_n^2+f_r(f_n-f_r))}\frac{q-q_1}{2q_1}$. Correspondingly, the KKT conditions above directly imply (B.2).

Case III: If $q_2 \le q < q_3$, then $q > \frac{f_n-f_r}{g_n-g_r}$. Assume $\lambda = 0$. If $\lambda = 0$, then on the one hand, (B.4) becomes $0 \le d_{n1}(q) - d_{r2}(q) = \frac{q_1-q}{2q_1} + \frac{\mu}{2(f_n-f_r)q}$, which implies $\mu > 0$. But if $\lambda = 0$, then on the other hand, $d_{n2}(q) = \frac{(g_n-g_r)}{2(f_n-f_r)}\left(\frac{f_n-f_r}{g_n-g_r} - q\right) + \frac{\mu}{2(f_n-f_r)q} > 0$ which, in turn, implies $\mu = 0$, a contradiction. Thus $\lambda > 0$, which implies $d_{n1}(q) = d_{r2}(q)$. Correspondingly,

$$0 < \lambda = \frac{f_nf_r(f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}\frac{(q - q_1)}{q_1} - \frac{f_nf_r\mu}{f_n^2 + f_r(f_n - f_r)}$$

$$\mu = \frac{2(f_n^2 + f_r(f_n - f_r))qd_{n2}(q) + (f_ng_n - f_rg_r)(q - q_2)q}{(f_n + f_r)}$$

Notice that if $d_{n2}(q) > 0$, then $\mu > 0$, a contradiction. Therefore, $d_{n2}(q) = 0$, and thus, $\mu = \frac{(f_ng_n-f_rg_r)(q-q_2)q}{f_n+f_r} > 0$. Correspondingly, the KKT conditions above directly imply (B.3).

Case IV: If $q \ge q_3$, then $q > \frac{f_n}{g_n}$. Accordingly, $0 \le d_{n1}(q) < \frac{\lambda}{2f_nq}$. This implies that $\lambda > 0$ and $d_{n1}(q) = d_{r2}(q)$. Similar to Case III, we also have $\mu > 0$. Thus the KKT conditions again reduce to (B.3). But notice that, given (B.3), $q \ge q_3$ implies that $d_{n1}(q) = d_{n2}(q) = d_{r2}(q) = 0$. Thus, $q^{G*} < q_3$. □

LEMMA B.3. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $\Pi^G(q)$ is unimodal in q over $q \in [0, q_3)$.*

Proof of Lemma B.3: From Lemma B.2 and equation (3.6), $\Pi^G(q)$ reduces to

$$\Pi^G(q) = \begin{cases} \Pi_P(q) = \frac{2q(f_n - g_n q)^2}{4f_n} + \frac{(f_r g_n - f_n g_r)^2 q^3}{4f_n f_r (f_n - f_r)} & q < q_1 \\ \Pi_C(q) = \frac{(f_n - g_n q)^2 q}{4f_n} + \frac{q(f_n(f_n - g_n q) + (f_r g_n - f_n g_r) q)^2}{4f_n (f_n^2 + f_r (f_n - f_r))} & q_1 \le q < q_2 \\ \Pi_S(q) = \frac{q(f_n + f_r - (g_n + g_r) q)^2}{4(f_n + f_r)} & q_2 \le q < q_3 \end{cases} \tag{B.5}$$

To prove Lemma B.3, we first establish that $\Pi^G(q)$ is continuous and differentiable over $[0, q_3)$. Then $\Pi^G(q)$ is unimodal over $q \in [0, q_3)$ if $\Pi_i(q)$ for $i = P, C, S$ is such that $\Pi_P(q)$ is unimodal over $(0, q_1)$, $\Pi_C(q)$ is unimodal over $[q_1, q_2)$, and $\Pi_S(q)$ is unimodal over $[q_2, q_3)$.

Accordingly, first we prove continuity and differentiability.

$$\lim_{q \to q_1^-} \Pi^G(q) = \lim_{q \to q_1^+} \Pi^G(q) = \frac{f_n^2 f_r (f_r g_n - f_n g_r)^2 (2f_n - f_r)(f_n^2 - f_r^2)}{4(f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n)^3}$$
$$\lim_{q \to q_2^-} \Pi^G(q) = \lim_{q \to q_2^+} \Pi^G(q) = \frac{f_n (f_n - f_r)(f_n + f_r)^2 (f_r g_n - f_n g_r)^2}{4(f_n g_n - f_r g_r)^3}$$

and

$$\lim_{q \to q_1^-} \frac{\partial \Pi^G(q)}{\partial q} = \lim_{q \to q_1^-} \frac{\partial \Pi^G(q)}{\partial q}$$
$$= \frac{3 f_n f_r (f_n - f_r)(f_r g_n - f_n g_r)^2}{4(f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n)^2}$$
$$+ \frac{f_n^2 (f_n g_r (f_n^2 g_r - f_r^2 g_n) - f_r^2 g_n (f_n (g_n + g_r) - 2 f_r g_n))}{2(f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n)^2}$$
$$\lim_{q \to q_2^-} \frac{\partial \Pi^G(q)}{\partial q} = \lim_{q \to q_2^+} \frac{\partial \Pi^G(q)}{\partial q}$$
$$= \frac{(f_n + f_r)\left(3(f_r g_n - f_n g_r)^2 + 2 g_n g_r (f_n - f_r)^2 - 2 f_n f_r (g_n - g_r)^2\right)}{4(f_n g_n - f_r g_r)^2}$$

Next, we establish that $\Pi_P(q)$ is unimodal over $(0, q_1)$. To that end, we define the following notations:

$$\gamma = \frac{f_r}{f_n}; \beta = \frac{g_r}{g_n}; \quad a_i = i + \frac{(\gamma - \beta)^i}{\gamma(1 - \gamma)} > i; \Rightarrow a_i' = -\frac{i(\gamma - \beta)^{i-1}}{\gamma(1 - \gamma)}, \quad i = 1, 2$$

Accordingly, we have $q_1 = \frac{f_n}{g_n}\frac{\gamma(1-\gamma)}{\gamma(1-\gamma)+\gamma-\beta}$, which implies $\frac{f_n}{g_n} = a_1 q_1$.

$$\Pi_P(q) \propto \frac{2f_n^2}{g_n^2}q - 4\frac{f_n}{g_n}q^2 + q^3 a_2$$
$$\Rightarrow \frac{\partial \Pi_P(q)}{\partial q} \propto \frac{2f_n^2}{g_n^2} - 8\frac{f_n}{g_n}q + 3a_2 q^2 = 3a_2(q-\hat{q}^-)(q-\hat{q}^+)$$

where $\hat{q}^{\pm} = \frac{a_1(4\pm\sqrt{16-6a_2})}{3a_2}q_1$. Notice, if $3a_2 \geq 8$, then $\frac{\partial \Pi_P(q)}{\partial q} > 0$ over $[0, q_1)$, in which case $\Pi_P(q)$ is unimodal. Thus, consider if $3a_2 < 8$. Let $H = \sqrt{16-6a_2}$, and $\hat{q}^- = \frac{a_1(4-H)}{3a_2}q_1 < \hat{q}^+ = \frac{a_1(4+H)}{3a_2}q_1$. Then to show that $\Pi_P(q)$ is unimodal, it suffices to show that $\hat{q}^+ \geq q_1$. To that end, if $\hat{q}^- \geq q_1$, then $\hat{q}^+ > \hat{q}^- \geq q_1$. Otherwise, if $\hat{q}^- < q_1$, then $H > \frac{4a_1-3a_2}{a_1}$, or $\beta > \gamma\frac{(3\gamma-1)(2-\gamma)}{2+3\gamma(1-\gamma)}$. In this case, we have $\hat{q}^+ \geq q_1 \iff K(\beta) = \frac{a_1}{a_2}\frac{4+H}{3} \geq 1$. Thus, to complete the proof that $\Pi_P(q)$ is unimodal, it suffices to show that $\min K(\beta) \geq 1$. We consider two cases. First, if $4a_1 \geq 3a_2$, then $K(\beta) \geq \frac{4a_1}{3a_2} \geq 1$. Next, if $4a_1 < 3a_2$, then it suffices to show that (1) $K(\beta)$ is unimodal in β, and (2) $\min[K(\beta_{\max}), K(\beta_{\min})] \geq 1$. To that end, we establish (2) first: $K(\beta_{\max}) = K(\gamma) = 1$; if $\gamma \geq 1/3$, then $K(\beta_{\min}) = K(\gamma\frac{(3\gamma-1)(2-\gamma)}{2+3\gamma(1-\gamma)}) = 1$; and if $\gamma < 1/3$, then

$$K(\beta_{\min}) = K(0) \geq 1 \iff \lim_{\beta\to 0} H^2 \geq \lim_{\beta\to 0}(\frac{3a_2}{a_1} - 4)^2$$
$$\iff \lim_{\beta\to 0} 8a_1 - 2a_1^2 - 3a_2 \geq 0 \iff \lim_{\beta\to 0} a_1(5-2a_1) = a_1\frac{1-3\gamma}{1-\gamma} \geq 0$$

Finally, we establish that $K(\beta)$ is, indeed, unimodal in β, thereby completing the proof that $\Pi_P(q)$ is unimodal over $[0, q_1)$. Let $Z = K\frac{a_2}{a_1}$. Hence, $Z' = Z(\frac{K'}{K} + X)$ and $Z'' = \frac{(Z')^2}{Z} + Z(\frac{K''}{K} - (\frac{K'}{K})^2 + X')$, where $X = \frac{a_2'}{a_2} - \frac{a_1'}{a_1}$. Correspondingly, we have

$$3Z - 4 = H = \sqrt{16-6a_2} > 0 \Rightarrow Z'(3Z-4) = -a_2' > 0$$
$$\Rightarrow Z''(3Z-4) + 3(Z')^2 = -a_2''$$
$$\Rightarrow \frac{K''}{K}\bigg|_{K'=0} = -\frac{a_2'' + Z(3Z-4)X' + 2(3Z-4)\frac{(Z')^2}{Z}}{Z(3Z-4)} < 0$$

where the inequality follows because $a_2'' = \frac{2}{\gamma(1-\gamma)} > 0$ and $X' = \frac{a_2''}{a_2} - (\frac{a_2'}{a_2} + \frac{a_1'}{a_1})X > 0$ due to $-(\frac{a_2'}{a_2} + \frac{a_1'}{a_1}) > 0$ and $X|_{K'=0} > 0$. This implies that $K(\beta)$ is unimodal, thereby completing the proof that $\Pi_P(q)$ is unimodal in q over

$(0, q_1)$.

Next, we follow a similar analysis to establish that $\Pi_C(q)$ is unimodal in q over $[q_1, q_2)$. Let

$$b_i = 1 + \frac{(1-\gamma+\beta)^i}{1+\gamma(1-\gamma)}; \quad b_i' = \frac{i(1-\gamma+\beta)^{(i-1)}}{1+\gamma(1-\gamma)} \quad i = 0, 1, 2$$

Note that $b_0 > b_1 > b_2$. Accordingly, we have $q_2 = \frac{f_n}{g_n}\frac{1-\gamma^2}{1-\beta\gamma}$, which implies $\frac{f_n}{g_n} = \frac{1-\beta\gamma}{1-\gamma^2}q_2$.

$$\Pi_C(q) \propto \frac{b_0 f_n^2}{g_n^2}q - 2b_1\frac{f_n}{g_n}q^2 + b_2 q^3$$
$$\Rightarrow \frac{\partial \Pi_C(q)}{\partial q} \propto \frac{b_0 f_n^2}{g_n^2} - 4b_1\frac{f_n}{g_n}q + 3b_2 q^2 = 3b_2(q-\hat{q}^-)(q-\hat{q}^+)$$

where $\hat{q}^{\pm} \equiv \frac{1-\beta\gamma}{1-\gamma^2}\frac{2b_1 - \sqrt{4b_1^2 - 3b_0 b_2}}{3b_2}q_2$. Notice that if $4b_1^2 \leq 3b_0 b_2$, then $\frac{\partial \Pi_C(q)}{\partial q} > 0$ over $[q_1, q_2)$, in which case $\Pi_C(q)$ is unimodal. Thus, consider if $4b_1^2 > 3b_0 b_2$. Let $H = \sqrt{4b_1^2 - 3b_0 b_2} > 0$. Then, to show that $\Pi_C(q)$ is unimodal, it suffices to show that $\hat{q}^+ \geq q_2$. To that end, if $\hat{q}^- \geq q_2$, then $\hat{q}^+ > \hat{q}^- \geq q_2$. Otherwise, if $\hat{q}^- < q_2$, then $H > 2b_1 - 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, or $\beta > \frac{3\gamma-2}{3-2\gamma}$. In this case, we have $\hat{q}^+ \geq q_2 \Longleftrightarrow K(\beta) = \frac{1-\beta\gamma}{1-\gamma^2}\frac{b_0}{(2b_1 - H)} \geq 1$. Thus, to complete the proof that $\Pi_C(q)$ is unimodal, it suffices to show that $\min K(\beta) \geq 1$. We consider two cases. First, if $2b_1 \geq 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, then $K(\beta) \geq \frac{1-\beta\gamma}{1-\gamma^2}\frac{2b_1}{3b_2} \geq 1$. Next, if $2b_1 < 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, then it suffices to show that (1) $K(\beta)$ is unimodal in β, and (2) $\min[K(\beta_{\max}), K(\beta_{\min})] \geq 1$. To that end, we establish (2) first: $K(\beta_{\max}) = K(\gamma) = 1$;if $\gamma \geq 2/3$, then $K(\beta_{\min}) = K\left(\frac{3\gamma-2}{3-2\gamma}\right) = 1 + \frac{2(4\gamma(1-\gamma)+(3\gamma-2))}{10-17\gamma+8\gamma^2} \geq 1$; and if $\gamma < 2/3$, then

$$K(\beta_{\min}) = K(0) \geq 1 \Longleftrightarrow H^2 \geq (3b_2(1-\gamma^2) - 2b_1)^2$$
$$\Longleftrightarrow b_0 \leq (1-\gamma^2)(4b_1 - 3(1-\gamma^2)b_2) \Longleftrightarrow \gamma(1+\gamma)(3\gamma-2) \leq 0 \Longleftrightarrow \gamma \leq 2/3$$

Finally, we establish that $K(\beta)$ is unimodal in β, thereby completing the proof that $\Pi_C(q)$ is unimodal over $[q_1, q_2)$. Let $Z \equiv \frac{(1-\gamma^2)K}{(1-\beta\gamma)}$. Hence $\frac{Z'}{Z} =$

$\frac{K'}{K} + \frac{\gamma}{(1-\beta\gamma)}$. Correspondingly, we have

$$\begin{aligned}
& b_2 Z - 2b_1 = H = \sqrt{4b_1^2 - 3b_0 b_2} > 0 \\
\Rightarrow & \frac{Z'}{Z} = \frac{2-(1-\gamma+\beta)Z}{H} b_1' \Rightarrow \frac{K'}{K} = \frac{2(1+\gamma)-(1+\beta)Z}{(1-\beta\gamma)(b_2 Z - 2b_1)} \\
\Rightarrow & \left.\frac{K''}{K}\right|_{K'=0} = -\frac{(1+\gamma)Z}{(1-\beta\gamma)^2(b_2 Z - 2b_1)} < 0
\end{aligned}$$

This implies that $K(\beta)$ is unimodal, thereby completing the proof that $\Pi_C(q)$ is unimodal over $[q_1, q_2)$.

Next, to complete the proof of Lemma B.2, we establish that $\Pi_S(q)$ is unimodal in q over $[q_2, q_3)$. Accordingly, we have $q_3 = \frac{f_n}{g_n}\frac{(1+\gamma)}{(1+\beta)}$, which implies $\frac{f_n}{g_n} = \frac{(1+\beta)}{(1+\gamma)} q_3$.

$$\begin{aligned}
\Pi_S(q) &\propto (1+\gamma)\frac{f_n^2}{g_n^2} q - 2(1+\beta)\frac{f_n}{g_n} q^2 + \frac{(1+\beta)^2}{(1+\gamma)} q^3 \\
\Rightarrow \frac{\partial \Pi_S(q)}{\partial q} &\propto (1+\gamma)\frac{f_n^2}{g_n^2} - 4(1+\beta)\frac{f_n}{g_n} q + 3\frac{(1+\beta)^2}{(1+\gamma)} q^2 \\
&= 3\frac{(1+\beta)^2}{(1+\gamma)} \left(\frac{q_3}{3} - q\right)(q_3 - q)
\end{aligned}$$

Thus $\frac{\partial \Pi_S(q)}{\partial q} > 0$ for $q < \frac{q_3}{3}$, and $\frac{\partial \Pi_S(q)}{\partial q} < 0$ for $\frac{q_3}{3} < q < q_3$, which implies that $\Pi_S(q)$ is unimodal in q over $[q_2, q_3)$, thereby completing the proof for Lemma 3. □

Proof of Proposition 3.2: From Lemma B.1, we know that $f_r g_n > f_n g_r$, and from Lemma B.2, we know that prices and demands are defined over three segments: $(0, q_1)$, $[q_1, q_2)$, and $[q_2, q_3)$. We first establish the continuity of both price and demand, and then we analyze the impact of q for each segment. For the prices, we have for $j = 1, 2$

$$\begin{aligned}
\lim_{q \to q_1^-} p_{nj}^G(q) = \lim_{q \to q_1^+} p_{nj}^G(q) &= \frac{f_n^3 f_r (f_n - f_r)((f_n - f_r)g_n + f_n(g_n - g_r))}{2(f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n)^2} \\
\lim_{q \to q_2^-} p_{r2}^G(q) = \lim_{q \to q_2^+} p_{r2}^G(q) &= \frac{f_n f_r^2 (f_n - f_r)(2 f_n g_n (f_n - f_r) + f_r(f_r g_n - f_n g_r))}{2(f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n)^2}
\end{aligned}$$

Accordingly, for the demand, we have

$$\lim_{q\to q_1^-} d_{n1}^G(q) = \lim_{q\to q_1^+} d_{n1}^G(q) = \frac{f_n f_r (f_n - f_r)((f_n - 2f_r)(f_n - f_r)g_n + f_n^2(g_n - g_r))}{2(f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n)^2}$$
$$\lim_{q\to q_1^-} d_{n2}^G(q) = \lim_{q\to q_1^+} d_{n2}^G(q) = \frac{f_n f_r (f_n - f_r)^2((f_n - f_r)g_n + f_n(g_n - g_r))}{2(f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n)^2}$$
$$\lim_{q\to q_2^-} d_{r2}^G(q) = \lim_{q\to q_2^+} d_{r2}^G(q) = \frac{f_n(f_r g_n - f_n g_r)}{2(f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n)}$$

Given the continuity of $p_{ij}^G(q)$ and $d_{ij}^G(q)$, we discuss the impact of q for each segment. First, for $q < q_1$, we have

$$\frac{\partial p_{n2}^G(q)}{\partial q} = \frac{\partial p_{n1}^G(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2} > \frac{\partial p_{r2}^G(q)}{\partial q} = \frac{(f_r + 2g_r q)}{2} > 0;$$
$$\frac{\partial d_{n2}^G(q)}{\partial q} = -\frac{(g_n - g_r)}{2(f_n - f_r)} < -\frac{g_n}{2f_n} = \frac{\partial d_{n1}^G(q)}{\partial q} < 0; \quad \frac{\partial d_{r2}^G(q)}{\partial q} = \frac{(f_r g_n - f_n g_r)}{2f_n(f_n - f_r)} > 0.$$

Secondly, for $q_1 \le q < q_2$, we have

$$\frac{\partial p_{n2}^G(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2}$$
$$> \frac{\partial p_{n1}^G(q)}{\partial q} = \frac{f_n[f_n(f_n + 2g_n q) + 2(f_n - f_r)f_r - 2(f_r g_n - f_n g_r)q]}{2(f_n^2 + f_r(f_n - f_r))} > 0;$$
$$\frac{\partial p_{n2}^G(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2}$$
$$> \frac{\partial p_{r2}^G(q)}{\partial q} = \frac{f_r[(f_n - f_r)(f_r + 2(g_n + g_r)q) + f_n(f_r + 2g_n q)]}{2(f_n^2 + f_r(f_n - f_r))} > 0;$$
$$\frac{\partial d_{n1}^G(q)}{\partial q} = \frac{\partial d_{r2}^G(q)}{\partial q} = -\frac{((f_n - f_r)g_n + f_n g_r)}{2(f_n^2 + f_r(f_n - f_r))} < 0;$$
$$\frac{\partial d_{n2}^G(q)}{\partial q} = -\frac{(f_n g_n - f_r g_r)}{2(f_n^2 + f_r(f_n - f_r))} < 0.$$

Finally, for $q_2 \le q < q_3$, we have

$$\frac{\partial p_{n2}^G(q)}{\partial q} = \frac{f_r(f_n - f_r) + 2(f_n^2 + f_r(g_n + g_r)q)}{2(f_n + f_r)} >$$
$$\frac{\partial p_{n1}^G(q)}{\partial q} = \frac{f_n}{f_r}\frac{\partial p_{r2}^G(q)}{\partial q} = \frac{f_n((f_n + f_r) + 2(g_n + g_r)q)}{2(f_n + f_r)} > 0$$
$$\frac{\partial d_{n1}^G(q)}{\partial q} = \frac{\partial d_{r2}^G(q)}{\partial q} = -\frac{(g_n + g_r)}{2(f_n + f_r)} < 0 = \frac{\partial d_{n2}^G(q)}{\partial q}$$

□

Proof of Proposition 3.3: Given Lemma B.3 and its proof, $q^{G*} = q_S^G \equiv \frac{q_3}{3}$ if and only if $q_S^G \geq q_2$, which is equivalent to $g_r \leq g_{rS}$. Similarly, $q^{G*} = q_C^G \equiv \frac{2b_1 - \sqrt{4b_1^2 - 3b_0 b_2}}{3b_2}\frac{f_n}{g_n}$ if and only if $q_1 \leq q_C^G < q_2$, which is equivalent to $g_{rS} < g_r \leq g_{rC}$. Finally, $q^{G*} = q_P^G \equiv \frac{4 - \sqrt{16 - 6a_2}}{3a_2}\frac{f_n}{g_n}$ if and only if $q_P^G < q_1$, which is equivalent to $g_{rC} < g_r < \frac{f_r g_n}{f_n}$. □

Proof of Proposition 3.4: Assume that $g_r/f_r < g_n/f_n < f_n$. Then, two possibilities exist in an optimal solution: either $d_{r2}^* = 0$ or $d_{r2}^* > 0$. If $d_{r2}^* = 0$, then $k^* = 1$ by Proposition 3.1. If $d_{r2}^* > 0$, then $k^* = 1$ by definition. Thus, $k^* = 1$ is always true. Given this, Lemma B.1 implies $d_{r2}^* > 0$. □

Proof of Proposition 3.5: As per the proof of Proposition 3.4, $g_n/f_n < f_n$ implies $k^* = 1$, and $g_r/f_r < g_n/f_n$ implies $d_{r2}^* > 0$ by Lemma B.1. Given this, Proposition 3.3 applies, which indicates that $g_r \leq g_{rC}$ implies $d_{r2}^* = d_{n1}^* > 0$. □

Proof of Proposition 3.6: As per the proof of Proposition 3.4, $g_n/f_n < f_n$ implies $k^* = 1$. Moreover, by Lemma B.1, $g_r/f_r \geq g_n/f_n$ implies $d_{r2}^* = 0$. Thus $g_n/f_n < \min[f_n, g_r/f_r]$ implies that $\{k^* = 1, d_{r2}^* = 0\}$ is true. Next, to establish that $g_n/f_n \geq \min[f_n, g_r/f_r]$ implies that $\{k^* = 1, d_{r2}^* = 0\}$ is not true, consider two cases: If $g_n/f_n > g_r/f_r$, then either $g_n/f_n < f_n$, in which case $d_{r2}^* \neq 0$ by Proposition 3.4, or $g_n/f_n \geq f_n$, in which case $k^* \neq 1$ by Proposition 3.1. Thus, $g_n/f_n > g_r/f_r$ implies that $\{k^* = 1, d_{r2}^* = 0\}$ is not true. Similarly, if $g_r/f_r \geq g_n/f_n \geq f_n$, then either $d_{r2}^* > 0$ in which case $d_{r2}^* \neq 0$, or $d_{r2}^* = 0$, in which case $k^* = 0$ by Proposition 3.1. Thus, again, $\{k^* = 1, d_{r2}^* = 0\}$ is not true. □

Proof of Proposition 3.7: Assume that $g_n/f_n < \min[1, g_r/f_r]$. Then $g_n/f_n < \min[f_n, g_r/f_r]$, which implies $k^* = 1$ and $d_{r2}^* = 0$ by Proposition 3.6. Correspondingly, from Proposition 3.1, $f_n/g_n > 1$ implies that $\Delta(q^*) = \Delta(q^M) = \frac{2(e_1+e_2)f_n}{9g_n} > \frac{2(e_1+e_2)}{9} = \Delta(q^T)$. Thus, $g_n/f_n < \min[1, g_r/f_r]$ implies $\Delta(q^*) = \Delta(q^M) > \Delta(q^T)$. Next, to establish that $g_n/f_n \geq \min[1, g_r/f_r]$ implies that $\Delta(q^*) = \Delta(q^M) > \Delta(q^T)$ is not true, consider two cases: If $g_n/f_n \geq \min[f_n, g_r/f_r]$, then $\Delta(q^*) \neq \Delta(q^M)$ by Proposition 3.6. If $\min[1, g_r/f_r] \leq g_n/f_n < \min[f_n, g_r/f_r]$, then $1 \leq g_n/f_n < g_r/f_r$, which implies $\Delta(q^*) =$

$\Delta(q^M) = \frac{2(e_1+e_2)f_n}{9g_n} \leq \frac{2(e_1+e_2)}{9} = \Delta(q^T)$. Thus, $g_n/f_n \geq \min[1, g_r/f_r]$ implies that $\Delta(q^*) = \Delta(q^M) > \Delta(q^T)$ is not true. □

Proof of Proposition 3.8: If $g_r \leq g_{rC} < g_n f_r/f_n$ and $g_n/f_n < 1 - \frac{f_r^2(3f_n-f_r)}{(2f_n-f_r)(f_n+f_r)^2} < f_n$, then $d^*_{r2} = d^*_{n1}$ by Proposition 3.5. Thus, to complete the proof, it suffices to show that (1) $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})$ decreases in g_r, in which case $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1}) \geq \Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}}$; and (2) $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}} > \Delta(q^T)$. To prove (1), we establish that $\frac{d\Delta(q^*)}{dg_r} = \frac{\partial\Delta(q^*)}{\partial g_r} + \frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$ by demonstrating that $\frac{\partial\Delta(q^*)}{\partial g_r} < 0$ and $\frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$. First, to prove that $\frac{\partial\Delta(q^*)}{\partial g_r} < 0$, note that $g_{rS} < g_r \leq g_{rC} \Rightarrow \frac{\partial\Delta(q^*)}{\partial g_r} = -\frac{(e_1+e_2)(f_n-f_r)q^*}{2(f_n^2+f_r(f_n-f_r))} < 0$ and $g_r \leq g_{rS} \Rightarrow \frac{\partial\Delta(q^*)}{\partial g_r} = -\frac{(e_1+e_2)q^*}{2(f_n+f_r)} < 0$. Next, we prove that $\frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$ by establishing that $\frac{\partial\Delta(q^*)}{\partial q^*} > 0$ and $\frac{dq^*}{dg_r} < 0$. If $g_r \leq g_{rS}$, then $\frac{\partial\Delta(q^*)}{\partial q^*} = (e_1+e_2)/6 > 0$ and $\frac{\partial q^*}{\partial g_r} = -\frac{f_n+f_r}{3(g_n+g_r)^2} < 0$. If $g_{rS} < g_r \leq g_{rC}$, then recall from the proof of Lemma B.3 that $q^* = q_C^G = \frac{f_n}{g_n}\frac{b_0}{2b_1+H}$, where $H = \sqrt{4b_1^2 - 3b_0b_2}$. Thus,

$$\frac{\partial H}{\partial g_r} = \frac{(b_0-1)(b_1+3(\gamma-\beta))}{g_n H} > 0$$
$$\Rightarrow \text{sign}\left\{\frac{\partial q^*}{\partial g_r}\right\} = \text{sign}\left\{-\left(2\frac{db_1}{dg_r} + \frac{\partial H}{\partial g_r}\right)\right\} < 0$$

Moroever,

$$\frac{\partial\Delta(q^*)}{\partial q^*} = (e_1+e_2)\frac{f_n(f_n - 2g_nq^*) + (f_n-f_r)(f_n+f_r-2(g_n+g_r)q^*)}{2(f_n^2+f_r(f_n-f_r))}$$
$$= \frac{(e_1+e_2)}{2(1+\gamma-\gamma^2)}\frac{(2-\gamma^2)H - 2\gamma(\gamma-\beta)}{2b_1+H} \geq 0$$

where the inequality follows because $g_r > g_{rS}$ implies that $\beta \geq \left(\frac{3\gamma-2}{3-2\gamma}\right)^+$, which in turn implies that $(2-\gamma^2)H - 2\gamma(\gamma-\beta) \geq \frac{2((3\gamma-2)+\gamma^2(1-\gamma)(4-\gamma))}{(3-2\gamma)(1+\gamma-\gamma^2)} \geq 0$ if $\gamma \geq 2/3$, and $(2-\gamma^2)H - 2\gamma(\gamma-\beta) \geq \frac{2\gamma^2(1-\gamma)}{(1+\gamma-\gamma^2)} \geq 0$ if $\gamma < 2/3$. Thus, we conclude that $\frac{d\Delta(q^*)}{dg_r} < 0$ for $g_r \leq g_{rC}$, which implies $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1}) \geq \Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}}$.

Finally, to complete the proof, we establish that $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}} > \Delta(q^T)$ as follows. At $g_r = g_{rC}$, $g_n/f_n < 1 - \frac{f_r^2(3f_n-f_r)}{(2f_n-f_r)(f_n+f_r)^2}$ implies that $\Delta(q^*)|_{g_r=g_{rC}} = \frac{2(e_1+e_2)f_n^2(2f_n^2+3f_nf_r-3f_r^2)}{9(2f_n-f_r)(f_n+f_r)^2g_n} > \frac{2(e_1+e_2)}{9} = \Delta(q^T)$. □

Appendix C

Appendix for Chapter 4

C.1 Proofs of Propositions

Proof of Proposition 4.1: To prove that $\Delta^*(q) = (e_1+e_2)q(d_{n1}(q)+d_{n2}(q)) > (e_1+e_2)q(1-q) = \Delta^T(q)$, it suffices to show that $d_{n1}(q)+d_{n2}(q) > 1-q$. If $g_n/f_n < 1$, then, from (3.4), $\Pi^{NG}(q, k=1) = \frac{q(f_n-g_nq)^2}{2f_n} > \frac{q(1-q)^2}{2} = \Pi^{NG}(q, k=0)$. Moreover, if, on the one hand, $g_r \geq g_nf_r/f_n > g_nf_r/f_n - 2(f_n-f_r)(f_n-g_n)/f_n$, then $\Pi^{NG}(q, k=1) \geq \Pi^G(q)$ by Lemma B.1. Thus, in this case, the optimal remanufacturing strategy for given q is $k(q) = 1$ and $d_{r2}(q) = 0$, and the corresponding optimal pricing strategy implies, from (3.3), that $d_{n1}(q)+d_{n2}(q) = 1 - g_nq/f_n > 1-q$ which in turn implies that $\Delta^*(q) > \Delta^T(q)$. If, on the other hand, $g_nf_r/f_n > g_r > g_nf_r/f_n - 2(f_n-f_r)(f_n-g_n)/f_n$, then $\Pi^G(q) > \Pi^{NG}(q, k=1)$ by Lemma B.1. Thus in this case, the optimal remanufacturing strategy for a given q is $k(q) = 1$ and $d_{r2}(q) > 0$, and the corresponding optimal pricing strategy implies, from Lemma B.2, that $d_{n1}(q)+d_{n2}(q) \geq 1-q$ which in turn implies that $\Delta^*(q) > \Delta^T(q)$. □

Similar to the Appendix B, we first establish three lemmas as building blocks for proving Propositions 4.2–4.4. The proofs of Lemmas C.1 and C.2 are combined and follow the statement of Lemma C.2. The proof of Lemma C.3 directly follows its statement. For Lemmas C.1–C.3, let $p_i^G(q), d_i^G(q)$ denote the optimal Green strategy price and corresponding demand as a function of q for product $i = n, r$, respectively. Moreover, let $q_\gamma = \frac{\gamma f_r(f_n-f_r)}{(f_rg_n-f_ng_r)+\gamma f_r(g_n-g_r)}$ denote the quality threshold above which $d_r^G(q) = \gamma d_n^G(q)$, and $\bar{q} = \frac{f_n+\gamma f_r}{g_n+\gamma g_r}$ denote the quality threshold above which $d_r^G(q) = d_n^G(q) = 0$.

LEMMA C.1. *Given that $k=1$, then for given $q < \bar{q}$, $d_{r2} > 0$ if and only if $g_r/f_r < g_n/f_n$.*

LEMMA C.2. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $q^{G*} < \bar{q}$. Then, for given collection rate γ, a remanufacturing firm should*

i) If $0 < q < q_\gamma$, remanufacture a proportion of recycled products and set the demand, price as

$$d_n(q) = \tilde{d}_n - \frac{(f_r g_n - f_n g_r)q}{2f_n(f_n - f_r)}; \quad d_r(q) = \frac{(f_r g_n - f_n g_r)q}{2f_r(f_n - f_r)};$$
$$p_n(q) = \frac{(f_n + g_n q)q}{2}; \quad p_r(q) = \frac{(f_r + g_r q)q}{2}; \tag{C.1}$$

and enjoys a profit of $\Pi(q) = \frac{q(f_n - g_n q)^2}{4f_n} + \frac{(f_r g_n - f_n g_r)^2 q^3}{4 f_n f_r (f_n - f_r)}$;

ii) If $q_\gamma \leq q < \bar{q}$, remanufacture all recycled products and set the demand, price as,

$$d_n(q) = \frac{f_n - g_n q + \gamma(f_r - g_r q)}{2(f_n + \gamma f_r(2+\gamma))}; \quad d_r(q) = \gamma d_n^F$$
$$p_n(q) = \frac{(f_n + \gamma f_r)(f_n + g_n q + \gamma(f_r + g_r q)) + 2\gamma^2 f_r(f_n - f_r)}{2(f_n + \gamma f_r(2+\gamma))};$$
$$p_r(q) = \frac{((1+\gamma)(f_n + g_n q + \gamma(f_r + g_r q)) - 2\gamma(f_n - f_r))f_r q}{2(f_n + \gamma f_r(2+\gamma))}; \tag{C.2}$$

and enjoys a profit of $\Pi(q) = \frac{q(f_n - g_n q + \gamma(f_r - g_r q))^2}{4(f_n + \gamma f_r(2+\gamma))}$.

iii) If $q > \min(1, \bar{q})$, produce nothing.

Proofs for Lemmas C.1–C.2. We prove our results using lagrangian method. The corresponding lagrangean of profit (4.4) for given q is

$$\max_{p_n, p_r} \mathcal{L}(q) = \left(1 - \frac{p_n - p_r}{(f_n - f_r)q}\right)(p_n - g_n q^2 + \gamma\lambda) + \left(\frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q}\right)(p_r - g_r q^2 - \lambda + \mu)$$

where λ and μ are the lagrange multipliers for conditions $d_r \leq \gamma d_n$ and

$d_r \geq 0$, respectively. Correspondingly the KKT conditions are

$$\frac{\partial \Pi}{\partial p_n} = 1 - \frac{p_n - p_r}{(f_n - f_r)q} - \frac{(p_n - g_n q^2 + \gamma\lambda)}{(f_n - f_r)q} + \frac{(p_r - g_r q^2 - \lambda + \mu)}{(f_n - f_r)q} = 0$$
$$\frac{\partial \Pi}{\partial p_r} = \frac{(p_n - g_n q^2 + \gamma\lambda)}{(f_n - f_r)q} + \frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q} - \frac{f_n(p_r - g_r q^2 - \lambda + \mu)}{f_r(f_n - f_r)q} = 0$$
$$\lambda\left(\gamma\left(1 - \frac{p_n - p_{r2}}{(f_n - f_r)q}\right) - \left(\frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q}\right)\right) \geq 0$$
$$\mu\left(\frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q}\right) \geq 0$$

The solution to this lagrange system is

$$p_n = \frac{f_n q + g_n q^2 - \gamma\lambda}{2}; p_r = \frac{f_r q + g_r q^2 + \lambda - \mu}{2}$$
$$d_n = \frac{(f_n - f_r)q - g_n q^2 + g_r q^2 + \lambda(1+\gamma) - \mu}{2(f_n - f_r)q};$$
$$d_r = \frac{(f_r g_n - f_n g_r)q^2 - \lambda(f_n + \gamma f_r) + \mu f_n}{2 f_r (f_n - f_r) q}$$

To prove Lemma C.1, we first prove sufficient and then necessary conditions. To that end, we first assume $d_r > 0$ which implies $\mu = 0$.

$$0 < d_r = \frac{(f_r g_n - f_n g_r)q^2 - \lambda(f_n + \gamma f_r) + \mu f_n}{2 f_r (f_n - f_r) q} \Rightarrow f_r g_n - f_n g_r > \frac{\lambda(f_n + \gamma f_r)}{q^2} \geq 0$$

which completes the sufficient condition.

Next, we assume $f_r g_n > f_n g_r$, which implies. We consider four cases based on the multipliers λ and μ: (1) if $\lambda = \mu = 0$, then $d_r(q) = \frac{(f_r g_n - f_n g_r)q^2}{2 f_r (f_n - f_r) q} > 0$; (2) if $\lambda = 0$ and $\mu > 0$, $d_r > 0$ again holds; (3) If $\lambda > 0$ and $\mu = 0$, then $\lambda = \frac{(f_r g_n - f_n g_r)q^2 - f_r\gamma((f_n - f_r) - (g_n - g_r)q)q}{(f_n + f_r\gamma(2+\gamma))}$. $d_r = \gamma d_n = \frac{\gamma(f_n - g_n q + \gamma(f_r - g_r q))}{(f_n + f_r\gamma(2+\gamma))} > 0$ as $q < \bar{q}$; (4) if $\lambda => 0$ and $\mu > 0$, then $d_r = \gamma d_n = \frac{\gamma(f_n - g_n q + \gamma(f_r - g_r q))}{(f_n + f_r\gamma(2+\gamma))} = 0$, which contradicts $q < \bar{q}$. Thus, we complete our proof for Lemma C.1.

Second, we prove Lemma C.2. Note that $d_r(q) > 0$ (by definition); thus $\mu = 0$ and $f_r g_n > f_n g_r$ (by Lemma C.1). Accordingly,

$$\gamma d_n(q) - d_r(q) = \frac{\gamma(q_\gamma - q)}{2 q_\gamma} + \frac{\lambda(f_n + f_r\gamma(2+\gamma))}{2 f_r (f_n - f_r) q}$$

which decreases in q. Here we consider three possibilities.

Case 1. If $0 < q < q_\gamma$, then $\gamma d_n(q) - d_r(q) > \frac{\lambda(f_n + f_r\gamma(2+\gamma))}{2 f_r (f_n - f_r) q} \geq 0$ which implies that $\lambda = 0$. Thus, KKT conditions above directly imply (C.1).

Case 2. If $q_\gamma \leq q < \bar{q}$, then $0 \leq \gamma d_n(q) - d_r(q) = \frac{\gamma(q_\gamma - q)}{2q_\gamma} + \frac{\lambda(f_n + f_r\gamma(2+\gamma))}{2f_r(f_n - f_r)q}$. Thus $\lambda \geq 0$. If $\lambda = 0$, then $\gamma d_n(q) - d_r(q) = \frac{\gamma(q_\gamma - q)}{2q_\gamma} < 0$ which is impossible. Therefore, $\lambda = \frac{2f_r(f_n-f_r)q}{(f_n+f_r\gamma(2+\gamma))}\frac{\gamma(q-q_\gamma)}{2q_\gamma}$, and $\gamma d_n(q) - d_r(q) = 0$. Correspondingly, the KKT conditions above imply (C.2).

Case 3. If $q \geq \bar{q}$, then, similar to Case II, we again have $\lambda > 0$. Thus, $\gamma d_n(q) = d_r(q) = \frac{\gamma(f_n - g_n q + \gamma(f_r - g_r q))}{(f_n + f_r\gamma(2+\gamma))} < 0$ which is impossible. Thus, $q^{G*} < \bar{q}$.

□

Lemma C.3. *If $g_n/f_n > g_r/f_r$, then the profit function for remanufacturing strategy $\Pi^G(q)$ is unimodal in q over $q \in [0, f_n/g_n]$.*

Proof for Lemma C.3. From Lemma C.2, $\Pi^G(q)$ reduces to

$$\Pi^G(q) = \begin{cases} \Pi_P(q) = \dfrac{q(f_n - g_n q)^2}{4f_n} + \dfrac{(f_r g_n - f_n g_r)^2 q^3}{4f_n f_r(f_n - f_r)}, & q < q_\gamma; \\ \Pi_C(q) = \dfrac{q(f_n - g_n q + \gamma(f_r - g_r q))^2}{4(f_n + \gamma f_r(2+\gamma))}, & q_\gamma \leq q < \bar{q} \end{cases}$$

To prove Lemma C.3, we first establish that $\Pi^G(q)$ is continuous and differentiable over $[0, \bar{q}]$. Then $\Pi^G(q)$ is unimodal over $q \in [0, \bar{q}]$ such that $\Pi_P(q)$ is unimodal over $(0, q_\gamma)$, and $\Pi_C(q)$ is unimodal over $[q_\gamma, \bar{q})$.

Accordingly, we first prove the continuity and differentiability at quality level q_γ.

$$\lim_{q \to q_\gamma^-} \Pi^G(q) = \lim_{q \to q_\gamma^+} \Pi^G(q)$$
$$= \frac{\gamma(f_n - f_r)f_r(f_r g_n - f_n g_r)^2(f_n + \gamma f_r(2+\gamma))}{4((f_r g_n - f_n g_r) + \gamma f_r(g_n - g_r))^3}$$
$$\lim_{q \to q_\gamma^-} \frac{\partial \Pi^G(q)}{\partial q} = \lim_{q \to q_\gamma^+} \frac{\partial \Pi^G(q)}{\partial q}$$
$$= \frac{(f_r g_n - f_n g_r)((f_r g_n - f_n g_r)(f_n + \gamma f_r(2+\gamma)) - 2\gamma f_r(f_n - f_r)(g_r + \gamma g_r))}{4(f_r g_n - f_n g_r + \gamma f_r(g_n - g_r))^2}$$

Second, we establish that $\Pi_P(q)$ is unimodal over $(0, q_\gamma)$. To that end, we define the following notations:

$$f = \frac{f_r}{f_n}; g = \frac{g_r}{g_n}; a_1 = \frac{(f-g)}{f(1-f)}\frac{1+\gamma}{\gamma} + \frac{g}{f}; a_2 = \frac{(f-g)^2}{f(1-f)} + 1.$$

Accordingly,

$$a_1' = -\frac{1+\gamma f}{\gamma f(1-f)} < 0; a_2' = -\frac{2(f-g)}{f(1-f} < 0.$$

We have $q_\gamma = \frac{f_n}{g_n}\frac{\gamma f(1-f)}{(f-g)+\gamma f(1-g)}$, which implies $\frac{f_n}{g_n} = a_1 q_\gamma$.

$$\Pi^G(q) \propto \frac{f_n^2}{g_n^2}q - 2\frac{f_n}{g_n}q^2 + a_2 q^3$$
$$\frac{\partial \Pi^G(q)}{\partial q} \propto \frac{f_n^2}{g_n^2} - 4\frac{f_n}{g_n}q + 3a_2q^2 = 3a_2(q-\hat{q}^-)(q-\hat{q}^+)$$

where $\hat{q}^\pm = \frac{a_1 q_\gamma(2\pm\sqrt{4-3a_2})}{3a_2}$. Note if $3a_2 \geq 4$, then $\frac{\partial \Pi^G(q)}{\partial q} > 0$ over $[0, q_\gamma]$, in which case $\Pi^G(q)$ is unimodal. Thus, consider if $3a_2 < 4$. Let $H = \sqrt{4-3a_2}$, and $\hat{q}^- = \frac{a_1 q_\gamma(2-H)}{3a_2} < \hat{q}^+ = \frac{a_1 q_\gamma(2+H)}{3a_2}$. Then, to prove unimodality, it suffices to show that $q_\gamma \leq \hat{q}^+$. To that end, if $q_\gamma \leq \hat{q}^-$, then $q_\gamma \leq \hat{q}^- < \hat{q}^+$ is true. Otherwise, if $q_\gamma > \hat{q}^-$, then $H > \frac{2a_1-3a_2}{a_1}$, and $\frac{f((1-f)(1-2\gamma)+f(1+\gamma)^2)}{3\gamma^2 f(1-f)+(1+\gamma f)^2} < g < f$. In this case, to prove $q_\gamma \leq \hat{q}^+$ we need to show $K(g) = \frac{a_1(2+H)}{3a_2} \geq 1$. We consider two cases. First, if $2a_1 \geq 3a_2$, then $K(g) \geq \frac{2a_1}{3a_2} \geq 1$. Second, if $2a_1 < 3a_2$, then it suffices to show that (1) $K(g)$ is unimodal in g, and (2) $\min[K(g_{\max}), K(g_{\min})] \geq 1$. To that end, we establish (2) first: $K(g_{\max}) = K(f) = 1$; if $f \geq \frac{2\gamma-1}{\gamma(4+\gamma)}$, then $K(g_{\min}) = K(\frac{f((1-f)(1-2\gamma)+f(1+\gamma)^2)}{3\gamma^2 f(1-f)+(1+\gamma f)^2}) = 1$; and if $f < \frac{2\gamma-1}{\gamma(4+\gamma)}$, then

$$K(g_{\min}) = K(0) \geq 1 \iff \lim_{g\to 0} H^2 \geq \lim_{g\to 0}(\frac{2a_1-3a_2}{a_1})^2$$
$$\iff \lim_{g\to 0} a_1^2 - 4a_1 + 3a_2 \leq 0 \iff \frac{(1-f)(1-2\gamma)+f(1+\gamma)^2}{\gamma^2(1-f)^2} \leq 0$$

Finally, we establish that $K(g)$ is unimodal in g. Let $Z = K\frac{a_2}{a_1}$. Hence, $Z' = Z(\frac{K'}{K} + X)$ and $Z'' = \frac{(Z')^2}{Z} + Z(\frac{K''}{K} - (\frac{K'}{K})^2 + X')$, where $X = \frac{a_2'}{a_2} - \frac{a_1'}{a_1}$.

Correspondingly, we have

$$\begin{aligned}
&3Z-2=H=\sqrt{4-3a_2}>0 \Rightarrow Z'(3Z-2)=-a_2'/2>0\\
\Rightarrow &Z''(3Z-2)+3(Z')^2=-a_2''/2\\
\Rightarrow &\left.\frac{K''}{K}\right|_{K'=0}=-\frac{a_2''/2+Z(3Z-2)X'+2(3Z-2)\frac{(Z')^2}{Z}}{Z(3Z-2)}<0
\end{aligned}$$

where the inequality follows because $a_2''=\frac{2}{f(1-f)}>0$ and $X'=\frac{a_2''}{a_2}-(\frac{a_2'}{a_2}+\frac{a_1'}{a_1})X>0$ due to $-(\frac{a_2'}{a_2}+\frac{a_1'}{a_1})>0$ and $X|_{K'=0}>0$. This implies that $K(\beta)$ is unimodal, thereby completing the proof that $\Pi_P(q)$ is unimodal in q over $(0,q_\gamma)$.

Next, we follow a similar analysis to establish that $\Pi_C(q)$ is unimodal in q over $[q_\gamma,\bar{q}]$. Accordingly, we have $\bar{q}=\frac{f_n}{g_n}\frac{(1+\gamma f)}{(1+\gamma g)}$, which implies $\frac{f_n}{g_n}=\frac{(1+\gamma g)}{(1+\gamma f)}\bar{q}$.

$$\begin{aligned}
\Pi_C^G(q)&\propto\frac{(1+\gamma f)^2}{(1+\gamma f)+\gamma f(1+\gamma)}\left(\frac{f_n^2}{g_n^2}q-2\frac{(1+\gamma g)}{(1+\gamma f)}\frac{f_n}{g_n}q^2+\frac{(1+\gamma g)^2}{(1+\gamma f)^2}q^3\right)\\
\frac{\partial\Pi_C^G(q)}{\partial q}&\propto\frac{(1+\gamma f)^2}{(1+\gamma f)+\gamma f(1+\gamma)}\left(\frac{f_n^2}{g_n^2}-4\frac{(1+\gamma g)}{(1+\gamma f)}\frac{f_n}{g_n}q+\frac{(1+\gamma g)^2}{(1+\gamma f)^2}q^2\right)\\
&\propto 3\frac{(1+\gamma g)^2}{(1+\gamma f)^2}(q-\bar{q})(q-\frac{\bar{q}}{3})
\end{aligned}$$

Thus, $\frac{\partial\Pi_C^G(q)}{\partial q}>0$ for $q<\bar{q}/3$ and $\frac{\partial\Pi_C^G(q)}{\partial q}<0$ for $\bar{q}/3<q<\bar{q}$, which implies that $\Pi_C^G(q)$ is unimodal in q over $[q_\gamma,\bar{q}]$. We thereby complete the proof for Lemma C.3. □

Proof for Proposition 4.2. Given Lemma C.3 and its proof, $q^{G*}=q_C^G\equiv\frac{\bar{q}}{3}$ if and only if $q_C^G\geq q_\gamma$, which is equivalent to $g_r\leq g_{rC}$. Similarly, $q^{G*}=q_P^G\equiv\frac{2-\sqrt{4-3a_2}}{3a_2}\frac{f_n}{g_n}$ if and only if $q_C^G<q_\gamma$ and $\frac{g_r}{f_r}<\frac{g_n}{f_n}$, which is equivalent to $g_{rC}<g_r\leq\frac{f_r g_n}{f_n}$. □

Proof for Proposition 4.3. To prove the first part, assume that $g_r/f_r<g_n/f_n<f_n$. Then, two possibilities exist in an optimal solution: either $d_{r2}^*=0$ or $d_{r2}^*>0$. If $d_{r2}^*=0$, then $k^*=1$ by Lemma C.2. If $d_{r2}^*>0$, then $k^*=1$ by definition. Thus, $k^*=1$ is always true. Given this, Lemma C.2 implies $d_{r2}^*>0$. In this case, we know from Proposition 4.2 that $0<d_r^*<\gamma d_n^*$ requires $g_r>g_{rC}$ which translates to $\gamma>\hat{\gamma}$.

To prove the second part, assume that $g_r/f_r<\frac{(1+\gamma)(f_n+\gamma f_r)-3\gamma(f_n-f_r)}{(f_n+\gamma f_r)^2+3f_r\gamma^2(f_n-f_r)}g_n/f_n$. As per the proof of Proposition 4.2, $g_r/f_r<f_n$ means $k^*=1$, and $g_r<g_{rC}$

means $d_r^* = \gamma d_n^* > 0$. This requires $g_r \leq g_{rC}$ which translates to $\gamma \leq \hat{\gamma}$. And $\gamma \leq 1$ by definition. □

Proof for Proposition 4.4. From Proposition 4.2, we know that $k^* = 1$ when $g_n/f_n < f_n$. If $g_r > g_{rC}$, then $0 \leq d_r^* < \gamma d_n^*$ is true, which implies that the collection rate has no effect on the optimal solution. If $g_r \leq g_{rC}$, we have $q^* = q_C^G = \frac{f_n+\gamma f_r}{3(g_n+\gamma g_r)}$ and, correspondingly, demands are $\gamma d_n^* = d_r^* = \frac{\gamma(f_n+\gamma f_r)}{3(f_n+\gamma f_r(2+\gamma))}$. Hence,

$$\frac{\partial q^*}{\partial \gamma} = \frac{f_r g_n - f_n g_r}{3(g_n + \gamma g_r)^2} > 0$$
$$\frac{\partial d_n^*}{\partial \gamma} = -\frac{f_r(f_n(1+2\gamma) + f_r\gamma^2)}{3(f_n + \gamma f_r(2+\gamma))^2} < 0$$
$$\frac{\partial d_r^*}{\partial \gamma} = \frac{(f_n + \gamma f_r)^2 - (f_n - f_r)f_r\gamma^2)}{3(f_n + \gamma f_r(2+\gamma))^2} > 0$$

□

Proof for Proposition 4.5. From Proposition 4.2, we know that $k^* = 1$ when $g_n/f_n < f_n$. If $g_r > g_{rC}$, then $0 \leq d_r^* < \gamma d_n^*$ is true, which implies that the collection rate has no effect on the optimal solution. If $g_r \leq g_{rC}$, we have $q^* = q_C^G = \frac{f_n+\gamma f_r}{3(g_n+\gamma g_r)}$ and, correspondingly, demands are $\gamma d_n^* = d_r^* = \frac{\gamma(f_n+\gamma f_r)}{3(f_n+\gamma f_r(2+\gamma))}$. Hence, the associated environmental damage $\Delta^* = e d_n^* q^* = \frac{e(f_n+\gamma f_r)^2}{9(g_n+\gamma g_r)(f_n+\gamma f_r(2+\gamma))}$, and

$$\frac{\partial \Delta^*}{\partial \gamma} = -\frac{e(f_n + \gamma f_r)((f_n + \gamma f_r)(f_n + \gamma^2 f_r) + 2\gamma f_r)}{9(g_n + \gamma g_r)(f_n + \gamma f_r(2+\gamma))} \cdot \frac{((f_n - f_r)g_n + f_n(g_n + \gamma g_r))}{9(g_n + \gamma g_r)(f_n + \gamma f_r(2+\gamma))} < 0.$$

□

To prove Propositions 4.6–4.7, we again first introduce and prove Lemma C.4 for environmental planner. Let $p_i^{G'}(q), d_i^{G'}(q)$ denote the optimal Green strategy price and corresponding demand as a function of q for product $i = n, r$, respectively. Moreover, let $q'_\gamma = \frac{f_r(\gamma(f_n-f_r)-e(1+\gamma))}{(f_r g_n - f_n g_r)+\gamma f_r(g_n-g_r)}$ denote the quality threshold above which $d_r^{G'}(q) = \gamma d_n^{G'}(q)$, and $\bar{q}' = \frac{f_n+\gamma f_r - e}{g_n+\gamma g_r}$ denote the quality threshold above which $d_r^{G'}(q) = d_n^{G'}(q) = 0$.

LEMMA C.4. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $q^{G'} < \bar{q}'$. Then, for given collection rate γ, an environmental planner should*

i) If $0 < q < q'_\gamma$, remanufacture a proportion of recycled products and set the demand, price as

$$\begin{aligned} p_n^{G'}(q) &= p_n^G(q) + \frac{eq}{2}; \quad p_n^{G'}(q) = p_r^G(q) \\ d_n^{G'}(q) &= d_n^G(q) - \frac{e}{2(f_n - f_r)}; \quad d_r^{G'}(q) = d_r^G(q) + \frac{e}{2(f_n - f_r)} \end{aligned} \tag{C.3}$$

and enjoys a profit of $\Phi(q) = \Pi_P(q) - \frac{eq(2((f_n - f_r) - (g_n - g_r)q) - e)}{4(f_n - f_r)}$;

ii) If $q'_\gamma \leq q < \bar{q}'$, remanufacture all recycled products and set the demand, price as,

$$\begin{aligned} p_n^{G'}(q) &= p_n^G(q) + \frac{eq(f_n + \gamma f_r)}{2(f_n + \gamma f_r(2 + \gamma)}; \\ p_r^{G'}(q) &= p_r^G(q) + \frac{eq(1 + \gamma)}{2(f_n + \gamma f_r(2 + \gamma)}; \\ d_r^{G'}(q) &= \gamma d_n^{G'}(q) = d_r^G(q) - \frac{e\gamma}{2(f_n + \gamma f_r(2 + \gamma))} \end{aligned} \tag{C.4}$$

and enjoys a profit of $\Phi(q) = \frac{q(f_n - g_n q - e + \gamma(f_r - g_r q))^2}{4(f_n + \gamma f_r(2 + \gamma))}$.

iii) If $q > \min(1, \bar{q}')$, produce nothing.

Proofs for Lemmas C.4. The proof is similar to that for Lemma C.2. We prove our results using lagrangian method. The corresponding lagrange function for profit (4.8) is

$$\begin{aligned} \max_{p_n, p_r} \Phi(q) = &\left(1 - \frac{p_n - p_r}{(f_n - f_r)q}\right)(p_n - g_n q^2 - eq + \gamma\lambda) \\ &+ \left(\frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q}\right)(p_r - g_r q^2 - \lambda + \mu) \end{aligned}$$

where λ and μ are the lagrange multipliers corresponding to $d_r \leq \gamma d_n$ and $d_r \geq 0$, respectively. And the KKT conditions are

$$\begin{aligned} \frac{\partial\Phi}{\partial p_n} &= 1 - \frac{p_n - p_r}{(f_n - f_r)q} - \frac{(p_n - g_n q^2 - eq + \gamma\lambda)}{(f_n - f_r)q} + \frac{(p_r - g_r q^2 - \lambda + \mu)}{(f_n - f_r)q} = 0 \\ \frac{\partial\Phi}{\partial p_r} &= \frac{(p_n - g_n q^2 - eq + \gamma\lambda)}{(f_n - f_r)q} + \frac{p_n - p_r}{(f_n - f_r)q} - \frac{p_r}{f_r q} - \frac{f_n(p_r - g_r q^2 - \lambda + \mu)}{f_r(f_n - f_r)q} = 0 \end{aligned}$$

and the corresponding orthogonal conditions. The solution to this lagrange system is

$$p'_n(q) = \frac{f_n q + g_n q^2 + eq - \gamma\lambda}{2}; p'_r(q) = \frac{f_r q + g_r q^2 + \lambda - \mu}{2}$$
$$d'_n(q) = \frac{(f_n - f_r - e)q - (g_n - g_r)q^2 + \lambda(1+\gamma) - \mu}{2(f_n - f_r)q};$$
$$d'_r(q) = \frac{(f_r g_n - f_n g_r)q^2 + f_r eq - (f_n + \gamma f_r)\lambda + f_n\mu}{2f_r(f_n - f_r)q}$$

We first prove that $g_r/f_r < g_n/f_n\frac{(f_n+2e)}{(f_n-e)}$ is sufficient for $d'_r(q) > 0$ by contradiction. Assume that $d'_r(q) = 0$. Then the orthogonal conditions dictate that $\lambda = 0$ and $\mu \geq 0$. From the lagrange solution, we know that $\mu = -q\frac{(f_r g_n - f_n g_r)q + f_r e}{f_n}$. If $g_r/f_r \leq g_n/f_n$, then $\mu < 0$ which implies a contradiction. Thus, $g_r/f_r < g_n/f_n$ is a sufficient condition for $d'_r(q) > 0$. Given this, we have

$$\gamma d'_n(q) - d'_r(q) = \frac{\gamma(q'_\gamma - q)}{2q_\gamma} + \frac{\lambda(f_n + f_r\gamma(2+\gamma))}{2f_r(f_n - f_r)q}$$

which again decreases in q. Here, we consider three cases.

Case 1. If $0 < q < q'_\gamma$, then $\gamma d_n(q) - d_r(q) > \frac{\lambda(f_n+f_r\gamma(2+\gamma))}{2f_r(f_n-f_r)q} \geq 0$ which implies that $\lambda = 0$. Thus, KKT conditions above directly imply (C.3).

Case 2. If $q'_\gamma \leq q < \bar{q}'$, then $0 \leq \gamma d_n(q) - d_r(q) = \frac{\gamma(q'_\gamma - q)}{2q_\gamma} + \frac{\lambda(f_n+f_r\gamma(2+\gamma))}{2f_r(f_n-f_r)q}$. Thus $\lambda \geq 0$. If $\lambda = 0$, then $\gamma d_n(q) - d_r(q) = \frac{\gamma(q_\gamma - q)}{2q_\gamma} < 0$ which is impossible. Therefore, $\lambda = \frac{2f_r(f_n-f_r)q}{(f_n+f_r\gamma(2+\gamma))}\frac{\gamma(q-q'_\gamma)}{2q_\gamma} > 0$, and $\gamma d_n(q) - d_r(q) = 0$. Correspondingly, the KKT conditions above imply (C.4).

Case 3. If $q \geq \bar{q}'$, then, similar to Case 2, we again have $\lambda > 0$. Thus, $\gamma d_n(q) = d_r(q) = \frac{\gamma(f_n - g_n q + \gamma(f_r - g_r q))}{(f_n+f_r\gamma(2+\gamma))} < 0$ which is impossible. Thus, $q^{G'} < \bar{q}$.

□

Proof for Proposition 4.6. The proof is similar to that in Proposition 4.2. First, we prove $g_n \leq \frac{(f_n-e)^2}{(1-e)^2}$ implies $k' = 1$ by considering two scenarios. If $d'_r = 0$, then $g_n \leq \frac{(f_n-e)^2}{(1-e)^2}$ implies that $\Phi^M \equiv \frac{(f_n-e)^2}{27g_n} > \Phi^T \equiv \frac{(1-e)^2}{27}$. If $d'_r = 0$, then $k = 1$ by definition.

Second, given that $g_n \leq \frac{(f_n-e)^2}{(1-e)^2}$, the optimal solution is one of the three cases: (1) $d_r' = 0$; (2) $0 < d_r' < \gamma d_n'$; and (3) $0 < d_r' = \gamma d_n'$.

(1) If $d_r' = 0$, then $q' = q^{M'}$. Accordingly, $\mu = q' \frac{-(f_r g_n - f_n g_r)q' - f_r e}{f_n} \geq 0$ which implies $g_r \geq \frac{f_r g_n}{f_n} \frac{(f_n+2e)}{(f_n-e)}$. Also $g_r \leq 1$ by definition.

(2) If $0 < d_r' < \gamma d_n'$, then $q' = q_P^{G'}$, $\mu = 0$ and $\lambda = 0$. Accordingly, $d_r' = \frac{(f_r g_n - f_n g_r)q' + f_r e}{2 f_r (f_n - f_r)} > 0$ implies that $g_r < \frac{f_r g_n}{f_n} \frac{(f_n+2e)}{(f_n-e)}$. Also, $q' < q_\gamma'$ translates to $\gamma > \hat{\gamma}'$.

(3) If $0 < d_r' = \gamma d_n'$, then $q' = q_C^{G'}$. In this case, $q' \geq q_\gamma'$ translates to $\gamma \leq \hat{\gamma}'$.

Where,

$$\hat{\gamma}' = \frac{-f_r(2 f_r g_n - f_n(g_n + g_r) + e(g_n + 2g_r))}{f_r(f_r(g_n + 2g_r) - 3(f_n - e)g_r)} \left(1 + \sqrt{f_r - \frac{(f_r g_n (f_n + 2e) - f_n g_r (f_n - e))(f_r(g_n + 2g_r) - 3(f_n - e)g_r)}{(f_r(2 f_r g_n - f_n(g_n + g_r) + e(g_n + 2g_r))^2}}\right)$$

$$q_P^{G'} = \frac{2((f_n - f_r)g_n - e(g_n - g_r))\sqrt{f_r}}{3(f_r g_n (g_n - g_r) - (f_r g_n - f_n g_r) g_r)} \left(\sqrt{f_r} - \sqrt{1 - \frac{3(f_r g_n (g_n - g_r) - (f_r g_n - f_n g_r) g_r)(f_r(f_n - 2e) - (f_n - e)^2)}{4((f_n - f_r)g_n - e(g_n - g_r))^2}}\right)$$

□

Proof for Proposition 4.7. We prove our results in two parts: $d_r'(q) = 0$ and $d_r'(q) > 0$. First when $d_r'(q) = 0$, the global planner's objective function is $\Theta(p, q, k) = (p - g_n q^2 - eq)d_n + \frac{f_n q - p}{2} d_n$. Following the same solution procedure as in §4.3.3, we have $d_n''(q) = 2d_n'(q)$ and $\Theta(q, k) = \frac{q(f_n - e - g_n q)^2}{2 f_n} = 2\Phi(q, k)$. Second when $d_r'(q) > 0$, the global planner's objective function is (4.9). Following the same solution procedure as in §4.3.3, we again have $d_n''(q) = 2d_n'(q)$ and $d_r''(q) = 2d_r'(q)$, and

$$\Theta(q) = 2\Phi(q) = \begin{cases} \Pi_P(q) - \dfrac{eq(2((f_n - f_r) - (g_n - g_r)q) - e)}{4(f_n - f_r)}, & q < q_\gamma'; \\ \dfrac{q(f_n - g_n q - e + \gamma(f_r - g_r q))^2}{4(f_n + \gamma f_r (2 + \gamma))}, & q_\gamma' \leq q < \bar{q}' \end{cases}$$

Consequently, $\Theta(q) = 2\Phi(q)$ implies the same optimal quality, or $q' = q''$, and $d_i(q'') = 2d_i(q'') = 2d_i(q')$. Thus, we conclude our proof. □

Proof for Proposition 4.8. First, the relationship between a firm's profit Π and an environmental planner's objective Φ implies that taxation can transform a firm to an environmental planner. Next, we focus on the second part of proposition. For the firm to behave like a global planner, taxation is required to reduce the product quality. Moreover, a global planner charges lower prices, which means lower profit for the firm. Thus, subsidies must compensate up to an amount of $\Pi' - \Pi''$, which represents the difference in the firm's profit under 2BL and under 3BL. Thus, the subsidy per unit product remanufactured $s = (\Pi' - \Pi'')/d''_r$. □

C.2 Derivation

Remanufacturability Model

We discuss the implications of continuous $k \in [0, 1]$ for the Non-Green and Green strategies, respectively. First, if the Non-Green strategy is implemented, then the profit function (3.5) becomes $\Pi^{NG}(k) = \frac{2(1+\theta k)^2}{27(1+c_1 k)}$, which implies that $\frac{\partial^2 \Pi^{NG}(k)}{\partial k^2} = \frac{4(c_1-\theta)^2}{27(1+c_1 k)^3} \geq 0$. Therefore $\Pi^{NG}(k)$ is convex in k, which implies that $\arg\max \Pi_{NG}(k) = 0$ or 1. Thus, $k^{NG} \in \{0, 1\}$. Next, if the Green strategy is implemented, then the profit function (3.6) becomes

$$\begin{aligned}\Pi^G(p_{n1}, p_{n2}, p_{r2}, q, k) = {} & d_{n1}(p_{n1} - g_n(k)q^2) \\ & + d_{n2}(p_{n2} - g_n(k)q^2) + d_{r2}(p_{r2} - g_r(k)q^2) \qquad \text{(C.5)}\end{aligned}$$

Following the same procedure as in §3.3, we find that Lemmas B.1-B.3 change only in two regards: (1) $k = 1$ is replaced by $k > 0$, and (2) f_i and g_i are replaced by $f_i(k)$ and $g_i(k)$, respectively, for $i = n, r$. Accordingly, for given k, we solve for $q^G(k)$ and $p^G_{n1}(k), p^G_{n2}(k), p^G_{r2}(k)$, the optimal design and pricing decisions given that $d_{r2} > 0$, as per Proposition 3.3. Substituting these decisions into (C.5) thus reduces the Green strategy profit to $\Pi^G(k)$, a function of the single variable k. Given $\Pi^G(k)$, we then conducted a line search to determine $k^* = \arg\max\{\Pi^G(k), \Pi^{NG}(k = 1), \Pi^{NG}(k = 0)\}$. We implemented this line search for an extensive array of input parameters such that $(\theta, \alpha, c_1, c_2) \in \{0, 0.1, 0.2, \ldots, 1.0\}$, and in every instance we found that either $k^* = 1$ or $k^* = 0$.

Infinite-Horizon Model

We solve the infinite-horizon model by following the same solution procedure as in §§3.3–3.4. If the Non-Green strategy is implemented (4.3), then the solution is the same as in the two-stage model. If the Green strategy is implemented (4.4), then for given q, let $p_i^{IG}(q)$ and $d_i^{IG}(q)$ denote the price and demand for product $i = n, r$ in the infinite-horizon problem. Accordingly, for $q < q_1^I \equiv \frac{f_r(f_n - f_r)}{(f_r g_n - f_n g_r) + f_r(g_n - g_r)}$,

$$p_n^{IG}(q) = \frac{(f_n + g_n q)q}{2}; \quad p_r^{IG}(q) = \frac{(f_r + g_r q)q}{2};$$
$$d_n^{IG}(q) = \frac{(f_n - f_r) - (g_n - g_r)q}{2(f_n - f_r)}; \quad \text{and} \quad d_r^{IG}(q) = \frac{(f_r g_n - f_n g_r)q}{2f_r(f_n - f_r)};$$

And for $q \geq q_1^I$,

$$p_n^{IG}(q) = \frac{((f_n + f_r)(f_n + g_n q + (f_r + g_r q)) + 2f_r(f_n - f_r))q}{2(f_n + 3f_r)};$$
$$p_r^{IG}(q) = \frac{(2f_r + (g_n + g_r)q)f_r q}{(f_n + 3f_r)};$$
$$d_n^{IG}(q) = d_r^{IG}(q) = \frac{(f_n + f_r - (g_n + g_r)q)}{2(f_n + 3f_r)}$$

Given this, Lemma B.1 and Proposition 3.1 in Chapter 3 still hold. Therefore, similar to the two-stage model, we establish that $\Pi^{IG}(q)$ is unimodal in q over $(0, q_3)$ if the Green strategy is implemented. Correspondingly, we determine the optimal q^{I*} by analyzing first-order conditions to establish $q^{IG*} = \arg\max \Pi^{IG}(q)$ and then comparing $\Pi^{IG}(q^{IG*})$ to $\Pi^{ING}(q^{ING*})$.

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