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Original article

Investigating the Impact of Random and Systematic Errors on GPS Precise Point Positioning Ambiguity Resolution

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Abstract

Precise Point Positioning (PPP) is an increasingly recognized precisely the GPS/GNSS positioning technique. In order to improve the accuracy of PPP, the error sources in PPP measurements should be reduced as much as possible and the ambiguities should be correctly resolved. The correct ambiguity resolution requires a careful control of residual errors that are normally categorized into random and systematic errors. To understand effects from two categorized errors on the PPP ambiguity resolution, those two GPS datasets are simulated by generating in locations in South Korea (denoted as SUWN) and Hong Kong (PolyU). Both simulation cases are studied for each dataset; the first case is that all the satellites are affected by systematic and random errors, and the second case is that only a few satellites are affected. In the first case with random errors only, when the magnitude of random errors is increased, L1 ambiguities have a much higher chance to be incorrectly fixed. However, the size of ambiguity error is not exactly proportional to the magnitude of random error. Satellite geometry has more impacts on the L1 ambiguity resolution than the magnitude of random errors. In the first case when all the satellites have both random and systematic errors, the accuracy of fixed ambiguities is considerably affected by the systematic error. A pseudorange systematic error of 5 cm is the much more detrimental to ambiguity resolutions than carrier phase systematic error of 2 mm. In the 2^{nd} case when only a portion of satellites have systematic and random errors, the L1 ambiguity resolution in PPP can be still corrected. The number of allowable satellites varies from stations to stations, depending on the geometry of satellites.

Through extensive simulation tests under different schemes, this paper sheds light on how the PPP ambiguity resolution (more precisely L1 ambiguity resolution) is affected by the characteristics of the residual errors in PPP observations. The numerical examples recall the PPP data analysts that how accurate the error correction models must achieve in order to get all the ambiguities resolved correctly.

Keywords : Global Positioning System (GPS), Precise Point Positioning (PPP), Ambiguity Resolution (AR), Random and Systematic Errors

1. Introduction

The Global Positioning System (GPS) has been widely used for various precise positioning applications. Over the past decades, a new positioning technique, called the Precise Point Positioning (PPP) has gotten significant attentions (Zumberge *et al*., 1997; Kouba and Héroux, 2001; Gao and Shen, 2002; Le and Tiberius, 2007; Bisnath and Gao, 2009; Huber *et al*., 2010). The PPP technique determines a receiver's position using the un-differenced code and carrier-phase observations from one dual or multiple frequency receivers, with the use of precise orbit and satellite clock data. The precise data are usually obtained from International GNSS Service (IGS) (Dow *et al*., 2009). Compared with traditional relative positioning, the major difficulty in the PPP technique happens in fixing ambiguities to integer values due to the fractional-

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cycle biases (FCBs) in un-differenced observations (Collins, 2008; Ge *et al*., 2008). In recent years, as various techniques for ambiguity resolution have been developed (e.g. Bertiger *et al*., 2010; Collins *et al*., 2010; Ge *et al*., 2008; Geng *et al*., 2009), the PPP is able to provide millimeter positioning accuracies in static mode, and centimeter accuracies in kinematic mode. Due to its unique efficiency and reasonable accuracy, the PPP technique has been used in various applications, such as airborne mapping, atmospheric science, and precise positioning for mobile objects.

The correct ambiguity resolution is the most crucial procedure in applying the PPP technique (Ge *et al*., 2008; Laurichesse *et al*., 2009; Geng *et al*., 2011). All error sources must be mitigated as much as possible to resolve the PPP ambiguities correctly. To correct every error source in the PPP, compared to relative positioning, more attentions need to be paid. This is because many errors, which are mostly able to cancelled or mitigated in relative positioning, cannot be cancelled in the PPP. Thus, a major task in PPP application is to correct, model or estimate all the errors as much as possible (Abdel-salam, 2005). Nevertheless, residual errors always are remain due to the imperfection of models or data. Since the direct consequence of residual errors has impacts on PPP ambiguity resolution, those incorrect ambiguities may be resulted in the worst case.

In many of past years, efforts have been invested to correct the errors in PPP to achieve correct PPP ambiguity resolution (Kouba and Héroux, 2001). The ionospheric delay generally could be cancelled through ionosphere-free combination observations. The receiver clock offset and wet tropospheric delay are usually estimated as unknown parameters. The other errors are supposed to have been eliminated through modeling processes. For instance, errors in satellite clock and orbit could be corrected by IGS precise products (Kouba and Héroux, 2001), while effects from site displacements and satellite (such as satellite antenna offsets and phase wind-up) could be also corrected by mathematics models (Kouba, 2003). However, these errors are unable to be eliminated completely due to limitations in modeling accuracy. Taking the precise IGS products as an example, the final products of precise IGS orbit and clock currently have an accuracy of ~2.5 cm and ~2.2 cm, respectively (Dow *et al*., 2009). The residual errors resulting from imperfect models will aggregately affect the PPP ambiguity resolution.

Only limited attention has been paid to study, however, on how the residual errors aggregately affect the PPP ambiguity resolution. This paper aims to investigate the impact of residual errors, categorized as random errors and systematic errors, on PPP ambiguity resolution. Since only the aggregate effect is studied, the source of each individual contributing residual error is not important in this study, so which will not be identified. The impact of aggregate residual errors is investigated through data and error simulations. Such a study will help understand the relationship between residual errors and ambiguity resolution errors. It will allow PPP researchers to understand what sizes and types of residual errors are tolerable to exist in the carrier phase and pseudorange measurements, while still getting the PPP ambiguities resolved correctly.

Section 2 of this paper introduces the methods of generating simulation data, random errors as well as systematic errors. In Section 3, the PPP mathematics model and the method of ambiguity fixing are presented. In Section 4, the effect of both errors in random and systematic on PPP ambiguity resolution is analyzed. The conclusion is given in Section 5.

2. GPS Data Simulation and Random and Systematic Error Generation

In order to analyze the impact of residual errors on PPP ambiguity resolution, we first generate GPS simulation data and random and systematic errors. Since we know the true values of the simulated errors, the influence of the random and systematic errors on PPP can be analyzed.

2.1. GPS data simulation

In this study, to compare the effect of satellite geometry, the static data for the "SUWN" reference station are simulated, where located in Korea GPS network, and the "PolyU" station located in Hong Kong. The simulation data from 1 January, 2011 are generated at an interval of 30 seconds for 30 minutes period. The known coordinates of both "SUWN" and "PolyU" stations are in Table 1, used as the receiver's positions in data simulation.

Table 1. Coordinates of two test datasets

Station	Latitude	Longitude	Ellipsoidal Height
SUWN	37°16' 31.8529"	127°03' 15.2638"	83.816 m
PolyU	22°18' 13.5558"	114°10' 43.4467"	45.479 m

The satellite positions are calculated from IGS precise ephemerides corresponding to the time period of the simulation data. With the known positions of both receiver and GPS satellites, the geometric distances between them can be calculated. To simulate ionospheric delay by the single-layer ionospheric model, we assumed that heights of ionosphere and VTEC are 400 km and 40TECU respectively. The tropospheric delay is simulated by the Saastamoinen model and the Niell mapping function. The zenith of total delay can be computed by Saastamoinen model, as the meteorological parameters (such as pressure, temperature and partial water vapor pressure) are derived from the standard atmosphere model, based on the height of station. The simulated ionospheric and tropospheric errors are added to the geometric distances. The receiver clock error is simulated by the clock model, which applies the 2nd order process with the correlated white noise (Brown and Hwang, 1997). In the clock model, the typical Allan variance parameters for ovenized crystal oscillators (OCXO) values are used. A satellite clock error, orbit error or other errors, such as multipath, are not simulated at here since these errors are not estimated anyway in the PPP data processing. The generation of carrier-phase measurements is similar to that of pseudorange data, except an opposite sign in ionosphere delay and an introduction of carrier-phased integer ambiguities. The values of integer ambiguities are randomly generated in the range from -10^8 to 10^8 cycles with an assumption in no loss of lock with a cut-off angle of 15°. In this way, the simulated carrier-phase and pseudorange measurements for GPS L1 and L2 frequencies are generated. Mathematically, the carrier-phase and pseudorange data are produced using the following formulas:

$$L_{i,m}^k = \rho_i^k + T_i^k + cdt_i - \frac{I_i^k}{f_m^2} + \lambda_m N_{i,m}^k + \varepsilon_{i,m}^k$$
$$P_{i,m}^k = \rho_i^k + T_i^k + cdt_i + \frac{I_i^k}{f_m^2} + e_{i,m}^k \tag{1}$$

where $L_{i,m}^k$ and $P_{i,m}^k$ are un-differenced carrier-phase and pseudorange measurement in frequency band m, respectively; ρ_i^k is geometric distance between satellite k and receiver i; T_i^k is tropospheric delay; c is the speed of light in vacuum; dt_i is receiver clock offset; I_i^k / f_m^2 is the ionospheric group delay at frequency band m; λ_m is the wavelength of band m; $N_{i,m}^k$ is the integer ambiguity associated with band m; $\varepsilon_{i,m}^k$ and $e_{i,m}^k$ are random errors with potentially existing systematic errors, contributing to carrier-phase and pseudorange measurements, respectively. Neither multipath effect nor the initial fractional phase at transmitter and receiver is included in the simulation. Their effects on PPP ambiguity resolution, together with other unmodeled residual errors, such as residual errors in satellite precise orbit and satellite clock, will be investigated by treating all the aggregate residual errors as random plus systematic errors.

2.2. Random and systematic errors

As discussed previously, multiple types of errors could not be removed completely due to the limitation of modeling accuracy. The residual errors could generally be categorized into random and systematic errors. Accordingly, we can assume that the characteristics of residual errors are either random error, systematic error, or both. The systematic error may result from different error sources in GPS measurements. In this study, we define the systematic error as a constant. Two cases are considered in the data simulation; the first case is that all satellites are considered to have random errors and systematic errors with generating three types of simulation data, as shown in Table 2. The first type, denoted as Type 1.1, includes only random error, while the second and third types, denoted as Types 1.2 and 1.3, respectively, include both random and systematic errors. The type 1.3 has systematic errors twice larger than the type 1.2. For each type in Table 2, the standard deviation of random errors in the pseudorange measurements varies from 5 to 50 cm with an interval of 5 cm. Similarly, the standard deviation for carrier-phase data varies from 0.5 to 5 mm with an interval of 0.5 mm. There are 100 pairs of pseudorange and carrier-phase random errors, when all the combinations are considered.

For the second case, some satellites have systematic and random errors, while other satellites have random errors only. In the 2nd case, each standard deviation of pseudorange and carrier-phase random errors is simulated by 5 cm and 0.5 mm, orderly, when the systematic errors in pseudorange and carrier-phase data are fixed as 5 cm and 1 mm, respectively. The simulation data types of the 2nd case for both SUWN and PolyU stations are summarized following in Table 3 and 4.

Table 2. The range of random and systematic errors in first case

	Systematic Error		Random Error			
	pseudo-range (cm)	carrier-phase (cm)	pseudo-range (cm)	interval (cm)	carrier-phase (cm)	interval (cm)
Type1.1	0	0	5 ~ 50	5	0.05 ~ 0.5	0.05
Type1.2	5	0.1	5 ~ 50	5	0.05 ~ 0.5	0.05
Type1.3	10	0.2	5 ~ 50	5	0.05 ~ 0.5	0.05

Table 3. Data types of systematic errors in second case at SUWN station; O (with systematic error); X (without systematic error)

	PRN #								
	7	8	11	17	19	20	24	28	32
Type 2.1	O	X	X	X	X	X	X	X	X
Type 2.2	O	O	X	X	X	X	X	X	X
Type 2.3	O	O	O	X	X	X	X	X	X
Type 2.4	O	O	O	O	X	X	X	X	X
Type 2.5	O	O	O	O	O	X	X	X	X
Type 2.6	O	O	O	O	O	O	X	X	X
Type 2.7	O	O	O	O	O	O	O	X	X
Type 2.8	O	O	O	O	O	O	O	O	X
Type 2.9	O	O	O	O	O	O	O	O	O

Table 4. Data types of systematic errors in second case at PolyU station; O (with systematic error); X (without systematic error)

	PRN #							
	7	8	11	17	20	24	28	32
Type 3.1	O	X	X	X	X	X	X	X
Type 3.2	O	O	X	X	X	X	X	X
Type 3.3	O	O	O	X	X	X	X	X
Type 3.4	O	O	O	O	X	X	X	X
Type 3.5	O	O	O	O	O	X	X	X
Type 3.6	O	O	O	O	O	O	X	X
Type 3.7	O	O	O	O	O	O	O	X
Type 3.8	O	O	O	O	O	O	O	O

3. PPP Model and Ambiguity Fixing

In this section, the PPP mathematic model and adjustment model are presented, which are followed by PPP ambiguity resolution method.

3.1. PPP mathematic model and adjustment model

PPP model using dual frequency measurements usually is based on ionosphere-free (IF) combinations in order to eliminate the first-order ionospheric effect. The IF combinations for carrier-phase and pseudorange measurements are given by:

$$L_{i,IF}^{k} = \frac{f_1^2 L_{i,1}^k - f_2^2 L_{i,2}^k}{f_1^2 - f_2^2} = \rho_i^k + cdt_i + T_i^k + B_{i,IF}^k + \varepsilon_{i,IF}^k$$
$$P_{i,IF}^{k} = \frac{f_1^2 P_{i,1}^k - f_2^2 P_{i,2}^k}{f_1^2 - f_2^2} = \rho_i^k + cdt_i + T_i^k + e_{i,IF}^k \quad (2)$$

where $L_{i,IF}^k$ and $P_{i,IF}^k$ are carrier-phase and pseudorange IF combinations, respectively; f_1 and f_2 are carrier frequencies of GPS L1 and L2 signals, respectively; $B_{i,IF}^k$ is IF combination ambiguity term; $\varepsilon_{i,IF}^k$ and $e_{i,IF}^k$ are random errors of IF combination contributing to carrier-phase and pseudorange measurements, respectively. All the other terms are same as those in Eq. (1). The other errors such as multipath, satellite clock offset and satellite orbit error are assumed to be completely mitigated or corrected.

In this study, the block-wise least squares adjustment (Xu, 2007) for IF combinations was applied. This adjustment model is able to separate the unknowns into two groups: ambiguities and other parameters such as receiver's position, clock offset, and zenith tropospheric delay. The block-wise observations of IF combination at a particular epoch are presented by:

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{11} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \; P = \begin{bmatrix} w_c P_0 & 0 \\ 0 & w_p P_0 \end{bmatrix} \quad (3)$$

where l_1 and l_2 denote the observed minus computed measurements for carrier-phase and pseudorange, respectively; vector X_1 contains the receiver position vector, receiver clock offset and zenith tropospheric delay and they are epoch-dependent parameters; vector X_2 contains the non-integer IF ambiguities that are epoch-independent parameters; A_{11} and A_{12} are design matrices for the X_1 and X_2 vectors, respectively; P_0 is weight matrix; w_c and w_p are weight factors for IF combination carrier-phase and pseudorange measurements, respectively.

3.2. Ambiguity fixing

The ambiguities in carrier-phases measurements should be fixed to integers in order for ensuring the high accuracy in the PPP solutions, if the fractional-cycle biases (FCBs) are not considered at this moment. It is assumed in this study that the FCBs have been correctly compensated. This will allow the exploitation of the integer property of the ambiguities. The impact of residual FCBs, if not fully compensated, can be assessed in the systematic error analysis, since they can be treated as a type of systematic errors. In this study, the procedure of ambiguity fixing includes five steps; the first step is those float solutions for widelane and IF ambiguities are estimated by the least squared adjustment, together with their variance-covariance matrices; at second, the estimated float solution of widelane ambiguities is adjusted and fixed to integer values; thirdly, the float solution for L1 ambiguity is calculated by float solution of IF ambiguities and integer solution of widelane ambiguities; at fourth, the estimated real-value solution of L1 ambiguities is fixed to integer values. Integer L2 ambiguities are obtained by subtracting the L1 ambiguities from the widelane ones; lastly, the IF ambiguities are recalculated with integer solutions of widelane and L1 ambiguities.

The IF ambiguity can be decomposed as the following equation of the widelane and L1 ambiguities:

$$B_{i,IF}^{k} = c\frac{f_2}{f_1^2 - f_2^2} N_{i,w}^{k} + \frac{c}{f_1 + f_2} N_{i,1}^{k} \tag{4}$$

where $N_{i,w}^{k}$ and $N_{i,1}^{k}$ are integer ambiguities for widelane combination and L1 carrier-phase, respectively. Other terms in Eq. (4) have been defined in Eq. (2). The Melboune-Wübbena combination is used to estimate widelane ambiguity. At the un-differencing level, this combination uses both carrier-phase and pseudorange measurements to yield widelane ambiguity as:

$$L_{i,6}^{k} = \frac{1}{f_1 - f_2}\left(f_1 L_{i,1}^{k} - f_2 L_{i,2}^{k}\right) - \frac{1}{f_1 + f_2}\left(f_1 P_{i,1}^{k} + f_2 P_{i,2}^{k}\right) = \lambda_w N_{i,w}^{k} \tag{5}$$

where $\lambda_w = c/(f_1 - f_2)$ is the wavelength of widelane observation. The estimated float solutions for widelane ambiguities should be fixed to integer values.

A number of studies have been developed for the resolution of ambiguities in PPP (e.g. Ge *et al.*, 2008; Laurichesse *et al.*, 2009). The LAMBDA (Least-squares Ambiguity Decorrelation Adjustment) is applied in this study because of its high success rate in the resolution of ambiguities. In the ambiguity validation, the ratio test of the best and second-best solutions (Leick, 2004) is used, e.g. with the following formula:

$$\frac{v^T P v_{\text{2nd smallest}}}{v^T P v_{\text{smallest}}} > 3 \tag{6}$$

After the widelane ambiguities are successfully fixed, the L1 ambiguities can be derived according to Eq. (4) as

$$\hat{N}_{i,1}^{k} = \frac{f_1 + f_2}{c}\hat{B}_{i,IF}^{k} - \frac{f_2}{f_1 - f_2}\hat{N}_{i,w}^{k} \tag{7}$$

where $\hat{N}_{i,w}^{k}$ is the fixed integer widelane ambiguity and $\hat{B}_{i,IF}^{k}$ is the estimated IF ambiguity.

The ambiguity resolution and fixing decision for L1 ambiguities are same as the case of widelane ambiguities. After both the widelane and L1 ambiguities are fixed into their integer values, the IF ambiguities can be recalculated with Eq. (4)

4. Numerical Result and Analysis

The data analysis is performed, according to the random and systematic error simulation cases, described in Section 2. Each data set has a length of 30 minutes at an interval of 30 seconds, equivalent to the total of 60 epochs at SUWN station. Nine satellites are observed at the beginning, but the satellite PRN 7 drops at epoch 47. At the PolyU station, eight satellites are observed at the beginning, as a new satellite PRN 4 is observed after the 52nd epoch. Fig. 1 shows the observed satellites at both stations. In the PPP data processing, the Saastamoinen tropospheric model and Niell mapping functions are used to correct the errors in the same way that is used to generate simulation data. Therefore, all the errors, except the systematic and random errors, were completely corrected. We use the LAMBDA method for ambiguity resolution, while tolerance values for the ratio test is chosen as 3.

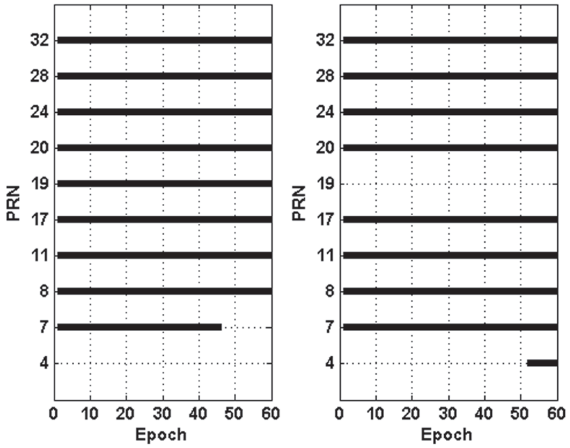

Fig. 1. The observed satellites at SUWN (left plot) and PolyU (right plot) stations

4.1 PPP ambiguity resolution with random error only (Type 1.1)

In this section, we analyze the time-to-ambiguity-fix (TTAF) and the error of ambiguity resolution, when all the observations for all the satellites have random errors, specified in Type 1.1. The standard deviation of the pseudorange random errors varies from 5 cm to 50 cm, with an interval of 5 cm. The standard deviation of the carrier-phase random errors varies from 0.05 cm to 0.5 cm, with an interval of 0.05 cm. Our ambiguity resolution result shows that both widelane and L1 ambiguities were fixed in integer ambiguities (not necessarily correct ones). Fig. 2 contains the TTAF, computed for Type 1.1. As seen in Fig. 2, the TTAF at SUWN station is generally increased along with magnitude of random errors, especially, the TTAF at SUWN station is affected more by the magnitude of pseudorange random error than by that of carrier-phase random error. However, as seen in the case of PolyU station, the TTAF is not clearly associated with the magnitude of random errors. Also, the TTAF at PolyU was smaller than that at SUWN. This result might be caused by complex effect of satellite geometry and the number of unknown ambiguities appearing in the resolution of ambiguities. As seen in Fig. 3, the GDOP at SUWN is actually smaller than that at PolyU, but the TTAF at SUWN is generally longer than at PolyU, as displayed in Fig. 2. On the other hand, the number of unknown ambiguities at SUWN is 9 but 8 at PolyU. Therefore, the TTAF is combined function of magnitudes of random errors, satellite geometry, and the number of ambiguities.

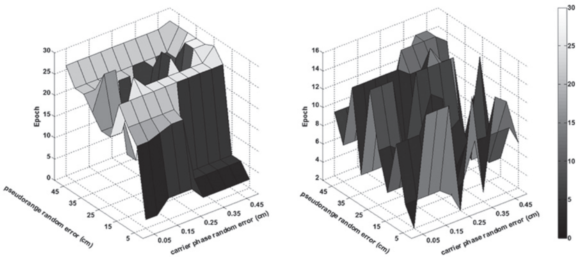

Fig. 2. The time-to-ambiguity-fix with random errors at both SUWN (left plot) and PolyU (right plot) stations using simulated GPS data. SUWN and PolyU stations have 9 and 8 ambiguities, respectively

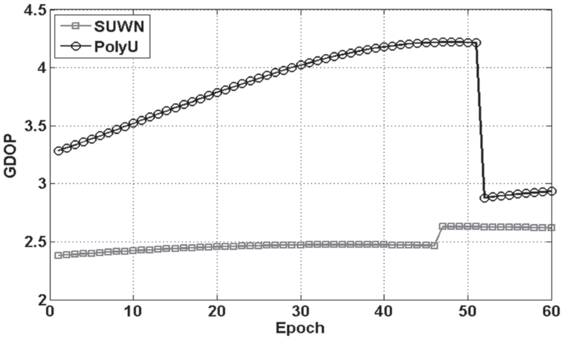

Fig. 3. GDOPs at SUWN and PolyU stations

All the widelane ambiguities at both SUWN and PolyU stations are resolved correctly, when compared to the known widelane ambiguities derived from simulated L1 and L2 ambiguities. Since the widelane ambiguities are fixed correctly, the focus of our analysis is placed on the errors in L1 ambiguities. Fig. 4 and 5 show the errors of L1 fixed ambiguities at SUWN and PolyU stations, respectively. In the case of PolyU station, ambiguities of 8 satellites are simultaneously fixed first and the ambiguity of PRN 4 is fixed after the 52nd epoch. At the SUWN station, all the L1 ambiguities are fixed correctly, when pseudorange random error varies within 5~20 cm and carrier-phase random error varies within 0.05~0.25 cm. If the pseudorange random error is larger than 25 cm, L1 ambiguities at SUWN are almost definitely incorrectly fixed, regardless of sizes of random error in carrier-phase measurements

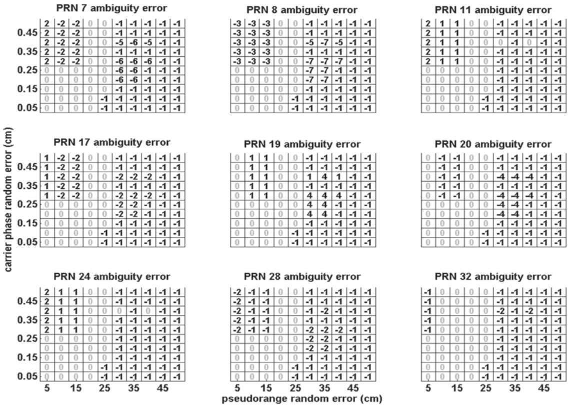

Fig. 4. The error of fixing L1 ambiguities at SUWN station in Type 1.1 (in unit of cycle); Overall, 321 out of 900 (35.67%) L1 ambiguities at SUWN are fixed correctly

At the PolyU station, when pseudorange random error is at 5 cm, and carrier-phase random error varies within 0.05~0.20 cm, all the L1 ambiguities, except the PRN 4 (PRN4 is observed after the 52nd epoch), are fixed exactly. Same as other satellites, the ambiguity of PRN 4 is fixed correctly without errors when pseudorange random error is at 5 cm, and carrier-phase random error varies within 0.05~0.20 cm. Even if the pseudorange random error increases up to 30 cm, the PRN 4 ambiguity can be correctly fixed, however, if the carrier phase random error decreases proportionally. This might be a benefit from facts; one that ambiguities for other satellites have been resolved, while the other one ambiguity from the PRN 4 needs to be resolved.

It can be seen from Fig. 4 and 5 that the size of ambiguity error is not necessarily proportional to the magnitude of random errors. For example, the largest ambiguity errors occur when the pseudorange errors are in the range of 30~35 cm and carrier-phase error in the range of 0.20~0.30 cm in the case of SUWN station. At both stations, the magnitude of the ambiguity fixing error varies for different satellites. This might be explained that different satellites have different positions, so as their contributions to the geometry matrixes in the Least Squares are different. As seen in Fig. 3, the GDOP at SUWN is better than that at PolyU. Comparing the Fig. 4 with the Fig. 5 the percentage of correctly fixed L1 ambiguities at SUWN (35.67%) is higher than PolyU (11.44%). This implies that the ambiguity resolution is considerably affected by satellite geometry, in addition to the magnitude of random errors.

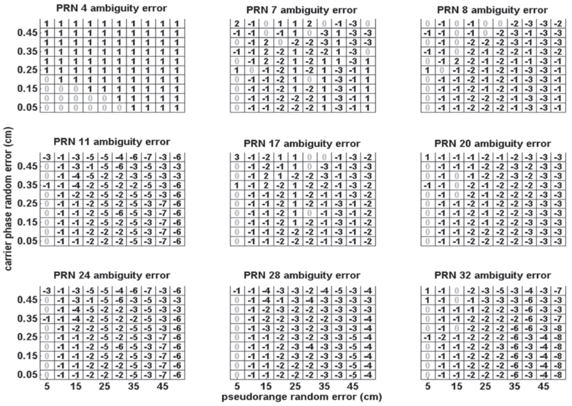

Fig. 5. The error of fixing L1 ambiguities at PolyU station in Type 1.1 (in unit of cycle); 88 out of 800 (11%) L1 ambiguities for 8 satellite (not including PRN 4) are fixed correctly; 15 out of 100 (15%) L1 ambiguities of PRN 4 are fixed correctly

In order to verify the effect of geometry on the resolution of ambiguities, we generate an additional simulation dataset. The starting time of this dataset is from the 11th epoch of dataset type 1.1 at SUWN station, but other conditions (such as the number of satellites, the magnitude and shape of random error) are exactly identical. Therefore, the difference between the additional dataset and Type 1.1 at SUWN is only the geometry of satellites. As shown in Fig. 6, the GDOP within the first 25 epochs is worse than that in Type 1.1 at SUWN.

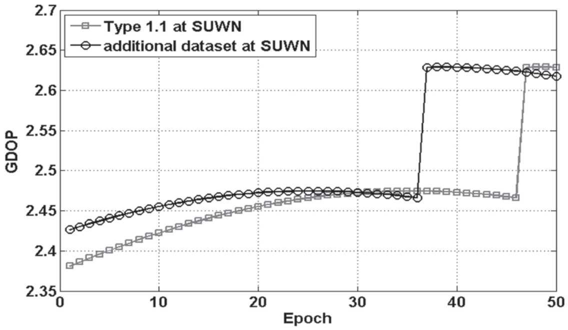

Fig. 6. GDOP at SUWN and additional dataset

All widelane ambiguities of this additional dataset are fixed correctly. Fig. 7 shows the error of L1 fixed ambiguities in the dataset. Comparing Fig. 7 with Fig. 4, the overall percentage of correct ambiguities decreases from 35.67% to 21.44% in the additional dataset, a degradation of 14.23%. This clearly shows that the resolution of ambiguities is dependent on geometry of satellites more than the magnitude of random errors.

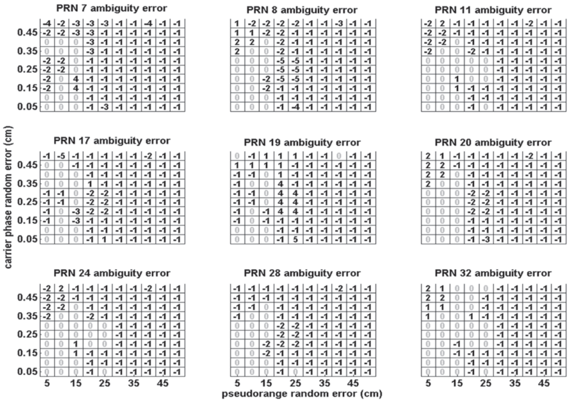

Fig. 7. The error of fixing L1 ambiguities in the additional dataset (in unit of cycle); Overall, 193 out of 900 (21.44%) L1 ambiguities are fixed correctly

4.2 PPP ambiguity resolution with both random and systematic errors on all the satellites (Type 1.2 and 1.3)

In this section, the scenario of having both systematic and random errors in all GPS satellites is studied. The systematic errors simulated in pseudorange and carrier-phase observations are 5 cm and 1 mm, respectively in Type 1.2. In Type 1.3, the systematic errors are increased to 10 cm and 2 mm for pseudorange and carrier-phase data, respectively. The random errors simulated in Types 1.2 and 1.3 are same as Type 1.1.

Our data analysis shows that both the widelane and L1 ambiguities at both SUWN and PolyU stations are successfully fixed to integer values (not necessarily correct ones) in both Types 1.2 and 1.3. The time-to-ambiguity-fix of both Types 1.2 and 1.3 are shown in Fig. 8. Fig. 8(a) and 8(b) shows the TTAF values for Types 1.2 and 1.3 at SUWN station. The TTAF values for the PolyU station are shown in Fig. 8(c) and 8(d). It can be seen that larger systematic errors result in longer time to fix the ambiguities.

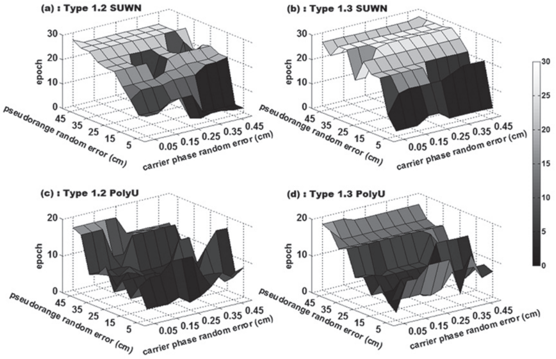

Fig. 8. The TTAF in both Types 1.2 and 1.3 at SUWN (top plots) and PolyU (bottom plots). (a) and (c): random error plus systematic error (pseudorange: 5 cm, carrier-phase: 1 mm); (b) and (d): random error plus systematic error (pseudorange: 10 cm, carrier-phase: 2 mm)

From results analysis of the study, in both Types 1.2 and 1.3, all the widelane ambiguities at SUWN stations are fixed correctly, while the majority of the L1 ambiguities are incorrectly fixed, as shown in Fig. 9 and Fig.10. Comparing three of Fig. 4, Fig. 9 and 10 clearly indicate that the L1 ambiguities have a much higher probability to be incorrectly fixed, when systematic error is included. In the case of Type 1.2 shown in Fig. 9, only 27 out of 900 (3%) L1 ambiguities are correctly fixed, while in the Type 1.3 shown in Fig. 10, only 20 out 900 (2.22%) L1 ambiguities are correctly resolved. By examining Figs. 9 and 10, the L1 ambiguity resolution is incorrect even if the random errors of both pseudorange and carrier phase measurements at their smallest level. The results in Fig. 9 and 10 clearly suggest that systematic errors can drastically degrade L1 ambiguity resolution, even if the systematic errors in carrier phase and pseudorange data are, respectively, 1~2 mm and 5-10 cm, only.

To understand which type of systematic error (pseudorange or carrier phase) results in bigger impacts on the incorrect L1 ambiguity resolution, one additional test with the SUWN dataset is conducted. Various combinations of different sizes of pseudorange and carrier phase systematic errors are tested. The Table 5 of L1 ambiguity resolution results corresponding to the combinations clearly demonstrates that pseudorange systematic errors (of 5-10 cm) have a remarkably larger impact on the L1 ambiguity resolution than do the carrier-phase random errors (of 1-2 mm). resolution than do the carrier-phase random errors (of 1-2 mm).

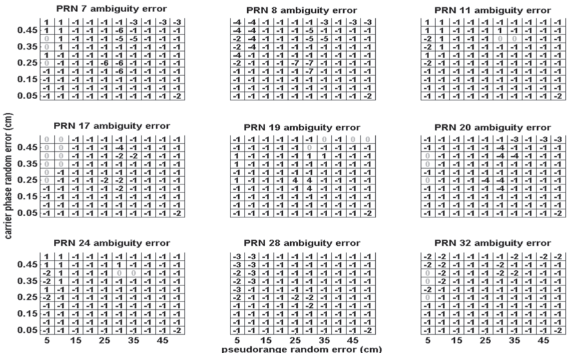

Fig. 9. The error of fixing L1 ambiguities in Type 1.2 at SUWN station (in unit of cycle); only 27 out of 900 (3%) L1 ambiguities are fixed correctly

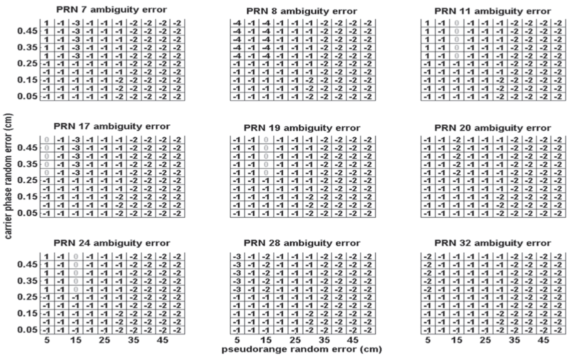

Fig. 10. The error of fixing L1 ambiguities in Type 1.3 at SUWN stations (in unit of cycle); only 20 out of 900 (2.22%) L1 ambiguities are fixed correctly

Table 5. The success rate of ambiguity resolution at SUWN station for different combinations of systematic error and random error

Systematic error		Correctly fixing rate (%)
Pseudorange (cm)	Carrier-phase (mm)	
0	1	35.67 %
0	2	35.44 %
5	0	2.33 %
10	0	3.00 %
5	1	3.00 %
10	2	2.22 %

In the case of PolyU station, all widelane ambiguities are fixed correctly too. Fig. 11 and 12 show the error of L1 ambiguity resolution for Types 1.2 and 1.3, respectively. When compared to the results in Type 1.1 and the first 8 satellites are considered (PRN 4 is observed at the 52nd epoch), the percentages of correct L1 ambiguities decreases by 6.12% and 6.37% in Types 1.2 and 1.3, respectively. This has consistent results at SUWN station because of the introduction of systematic errors in Types 1.2 and 1.3. The L1 ambiguity resolution of the first 8 satellites (not including the PRN 4) is wrong, according to Figs. 11 and 12, even if the random errors of both pseudorange and carrier phase measurements at their smallest level. This again clearly indicates that systematic errors have a dramatic effect on L1 ambiguity resolution. For the PRN 4, however, the number of correct L1 ambiguities increases dramatically in both Types 1.2 and 1.3, which is believed to be just a coincident. It can be explained that the L1 ambiguity error of the PRN 4 and the introduced systematic errors cancel each other. Thus, the L1 ambiguity resolution of the PRN 4 increases instead.

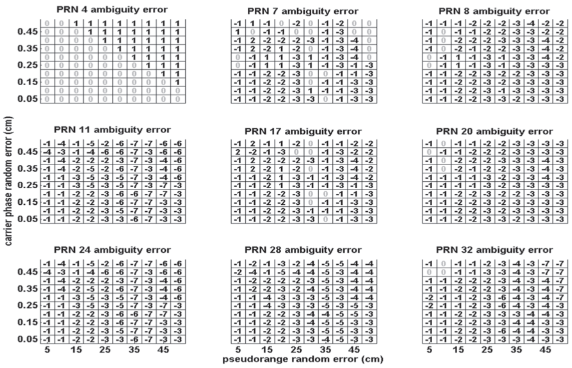

Fig. 11. The error of fixing L1 ambiguities in Type 1.2 at PolyU station (in unit of cycle); 39 out of 800 (4.88%) L1 ambiguities (not including PRN 4) are fixed correctly; 64 out of 100 (64%) L1 ambiguities of PRN 4 are fixed correctly

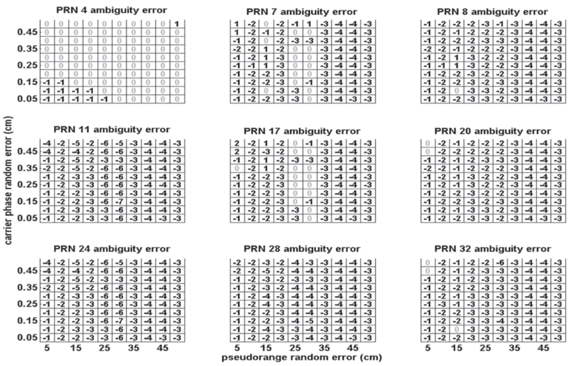

Fig. 12. The error of fixing L1 ambiguities in Type 1.3 at PolyU station (in unit of cycle); 37 out of 800 (4.63%) L1 ambiguities (not including PRN 4) are fixed correctly; 88 out of 100 (88%) L1 ambiguities of PRN 4 are fixed correctly

Similarly, different sizes of systematic errors in pseudorange and carrier phase data are combined and their impact on L1 ambiguity resolution is investigated. As seen in Table 6, the pseudorange systematic errors (of 5-10 cm) affect the resolution of ambiguity resolution considerably more than the carrier-phase systematic error (of 1-2 mm). This is consistent with the results obtained at SUWN station.

Table 6. The success rate of ambiguity resolution at PolyU station excluding PRN 4 for different combinations of systematic error and random error

Systematic error		Correctly fixing rate (%)
Pseudorange(cm)	Carrier-phase (mm)	
0	1	12.00 %
0	2	12.38 %
5	0	5.38 %
10	0	4.13 %
5	1	4.88 %
10	2	4.63 %

4.3 PPP ambiguity resolution with both random and systematic errors on some satellites (Type 2.2 to 2.9)

Different from the above section where all the satellites are simulated with systematic errors, this section studies the case of only a portion of satellites having systematic errors. As mentioned before, the systematic errors in pseudorange and carrier-phase data are fixed as 5 cm and 1 mm, respectively. The pseudorange and carrier-phase random errors are chosen as 5 cm and 0.5 mm, respectively. This is because the L1 ambiguities for all the satellites are fixed correctly at SUWN and PolyU stations at this level of random errors, as shown in Figs. 4 and 5.

The analysis results show that all the widelane ambiguities at both stations can be correctly resolved but it is not true for L1 ambiguity. As seen in Table 7 for station SUWN, when 6 or less satellites have systematic errors (5 cm for pseudorange and 1 mm for carrier-phase data), the L1 ambiguities can be resolved correctly. And when 7 satellites have systematic error, the L1 ambiguities fail to fix to integer values. Incorrect L1 ambiguities are obtained if 8 or more satellites are simulated with systematic errors. The failure to fix ambiguity is denoted as N/A in Table 7.

Table 7. Error of L1 ambiguities at SUWN station in case of systematic error in some satellites (in unit of cycle)

	PRN #								
	7	8	11	17	19	20	24	28	32
Type 2.1	0	0	0	0	0	0	0	0	0
Type 2.2	0	0	0	0	0	0	0	0	0
Type 2.3	0	0	0	0	0	0	0	0	0
Type 2.4	0	0	0	0	0	0	0	0	0
Type 2.5	0	0	0	0	0	0	0	0	0
Type 2.6	0	0	0	0	0	0	0	0	0
Type 2.7	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Type 2.8	-1	-1	-1	-1	-1	-1	-1	-1	-1
Type 2.9	-1	-1	-1	-1	-1	-1	-1	-1	-1

Same as the SUWN station, the widelane ambiguities for all satellites at PolyU station can be fixed correctly, even if systematic errors are present. In fact, the resolution of L1 ambiguities depends on the number of satellites containing systematic error. Table 8 indicates when if 2 or fewer of satellites have systematic errors (5 cm for pseudorange and 1 mm for carrier-phase data), the L1 ambiguities for all satellites are fixed correctly. When 3 or more of satellites contain systematic errors, the L1 ambiguities for all the satellites will be fixed incorrectly. Compared to Table 7 in the case of SUWN, the resolution of L1 ambiguities at PolyU is more influenced by the number of satellites having systematic errors. This result can probably be explained by the different geometry contributions from different satellites, which is same as the scenario of Type 1.1.

Table 8. Error of L1 ambiguities at PolyU station in case of systematic error in some satellites (in unit of cycle)

	PRN #							
	7	8	11	17	20	24	28	32
Type 3.1	0	0	0	0	0	0	0	0
Type 3.2	0	0	0	0	0	0	0	0
Type 3.3	-1	-1	-1	-1	-1	-1	-1	-1
Type 3.4	-1	-1	-1	-1	-1	-1	-1	-1
Type 3.5	-1	-1	-1	-1	-1	-1	-1	-1
Type 3.6	-1	-1	-1	-1	-1	-1	-1	-1
Type 3.7	-1	-1	-1	-1	-1	-1	-1	-1
Type 3.8	-1	-1	-1	-1	-1	-1	-1	-1

Tables 7 and 8 suggest that in the PPP ambiguity resolution, small magnitude of systematic errors (e.g. 5 cm for pseudorange and 1 mm for carrier-phase data in this study) is allowed to exist in a given number of satellites. The appearance of systematic errors in some satellites can still produce correct ambiguity resolutions for all the satellites. However the number of allowable satellites varies from station to station, largely depending on the satellite geometry.

5. Conclusions

In GPS/GNSS PPP processing, after correcting various types of errors in pseudorange and carrier phase measurements, residual errors still remain in the form of systematic and random errors. The impact of systematic and random errors on PPP ambiguity resolution is studied using simulation data for two GPS stations of different geometries: SUWN site located in South Korea and PolyU site located in Hong Kong. The results can be summarized as follows:

- The time-to-ambiguity-fix (TTAF) is affected by the magnitude of pseudorange random error than that of carrier phase random error.
- When the magnitude of random error is increased, PPP L1 ambiguities have a much higher chance to be incorrectly fixed. However, the size of ambiguity error is not exactly proportional to the magnitude of random error.
- The satellite geometry has more impacts on the PPP L1 ambiguity resolution than the magnitude of systematic and random errors.

This study can help understand the impacts from random and systematic errors on PPP ambiguity resolution, which of more precisely on L1 ambiguity resolution, since the widelane ambiguity resolution is relative easy. It is useful for the PPP researchers to control and budget the random and systematic errors in PPP data analysis. It can be concluded, as if the aggregate errors in PPP measurements are over certain limits, it will be difficult or even impossible to resolve correct integer ambiguities, even if the fractional-cycle biases have been compensated.

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