Verification of a Cable Element for Cable Parametric Vibration of One-Cable-Beam System Subject to Harmonic Excitation and Random Excitation

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Verification of a Cable Element for Cable Parametric Vibration of One-Cable-Beam System Subject to Harmonic Excitation and Random Excitation

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Abstract: Nonlinear vibration of cables in cable-stayed bridges is usually studied by either a divide-and-conquer approach in which interaction between local motion of the cables and global motion of the bridge is not considered, or a coupled cable-beam system which can only represent a simple structure with few degrees of freedom. A cable finite element is presented in this paper which can account for in-plane and out-of-plane motions of the cable. A cable-beam system with frequency ratio of 1:1:2 is employed to verify feasibility of the cable element in analysing nonlinear vibration of the stay cable. Vibrations of the coupled cable-beam system under harmonic and random loadings are calculated and compared with the available analytical solutions that have been verified experimentally. The results demonstrate that the cable finite element can capture nonlinear vibration, in particular auto-parametric vibration of stay cables under both loading cases. Consequently it can be applied to practical structures with multiple cables.

Key words: stay cable, auto-parametric vibration, harmonic loading, random loading, cable beam interaction, cable element.

1. INTRODUCTION

Basically the vibration of stay cables can be classified into the following two categories. The first one is the vibration due to the direct loading on the cables. Rain-wind induced vibration (Hikami and Shiraishi 1988) and vortex-induced vibration are typical ones. The other type is the support-induced motion due to girder or pylon vibration. This type of vibration can be further classified into two: one is due to the linear coupling between the cable and the girder/pylon and the other is the parametrically excited vibration. In the recent years, the cable vibration caused by parametric excitation due to deck or tower motion has been studied, for example, Takahashi and Konishi (1987), Lilien and Pinto da Costa (1994), Yamaguchi and Fujino (1998), Wu *et al*. (2006), Gattulli and Lepidi (2003), and so on.

However, these analytical approaches either investigate the cable vibration given support motions of the girder/pylon, or study a cable-beam coupled system. The former approach does not consider interaction between the local motion (cable) and the global motion (cable/girder/pylon) (Abdel-Ghaffar and Khalifa 1991). The latter one simplifies the system into few degrees of freedom, usually no more than three (Fujino *et al*. 1993), otherwise the analytical solution is difficult to obtain. Consequently it cannot deal with real cable-stayed bridges usually with multiple cables and multiple modes. In this regard, finite element methods have to be employed.

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In literature, there are three types of cable elements available. In the first type, each cable is represented by a single truss element or single spring element with an equivalent modulus (Ernst 1965). This approach has been commonly used for analysis of cable-stayed bridges (Karoumi 1999). In the second approach, each cable is divided into several straight truss elements (Abdel-Ghaffar and Khalifa 1991). However, using truss elements cannot consider the transverse vibration and out-of-plane vibration of the cables, and thus parametric vibration of the cables cannot be obtained (Wu *et al.* 2006). The third approach developed by Broughton and Ndumbaro (1994) can account for the in-plane (longitudinal and transverse) and out-of-plane responses of cables. Wu *et al.* (2006) applied it to a cable-stayed bridge and obtained the coupled cable-deck vibration and the parametric vibration of the cables. However, the feasibility of the method hasn't been thoroughly verified through available analytical solutions, particularly for the case of random excitation.

To extend feasibility of the cable finite element in real cable-stayed bridges with multiple cables, in this paper the cable element developed by Broughton and Ndumbaro (1994) is applied to a simplified cable-beam system whose analytical solutions of nonlinear vibration to harmonic and random loadings have been obtained. Moreover, the analytical results of the system under harmonic loading have been verified through experiments (Fujino *et al.* 1993). Comparison shows that the cable element can deal with nonlinear vibration particularly auto-parametric oscillation of cables under harmonic and random loadings.

2. MODELLING OF A CABLE-BEAM SYSTEM

To compare the analytical approach and finite element approach, a simple cable-beam system as shown in Figure 1 is employed here, which has been studied by Fujino *et al.* (1993) for harmonic loading and Xia and Fujino (2006) for random loading.

2.1. Analytical Approach to the Cable-Beam System

With analytical approach, horizontal vibration of the beam is denoted as $u_b(x, t)$ and vertical vibration as $v_b(x, t)$, and local horizontal motion of the cable is represented by $u_c(s, t)$. As the analytical approach cannot deal with the system with degrees of freedom more than three, local vertical vibration of the cable is neglected. This simplification is proved acceptable by numerical solution, as shown in later sections. Based on assumption of small response, vibration of the system is described by the global horizontal motion (ϕ_h), global vertical motion (ϕ_g), local horizontal motion of the cable (ϕ_y) and the corresponding generalized coordinates, h, g and y, as illustrated in Figure 2.

With Lagrange's approach, equations of motion of the system are given as (Fujino *et al.* 1993)

$$y'' + 2\xi_y y' + y + \zeta_h h'' + 2\eta_g yg + \alpha y^3 = 0 \tag{1}$$

$$h'' + 2\xi_h \frac{f_h}{f_y} h' + \frac{f_h^2}{f_y^2} h + \zeta_y y'' = P_h(t) \tag{2}$$

$$g'' + 2\xi_g \frac{f}{f_{yg}} g'' + \frac{f_g^2}{f_y^2} g + \eta_y y^2 = P_g(t) \tag{3}$$

where ξ_y, ξ_h and ξ_g are viscous damping ratio; f_y, f_h and f_g are natural frequencies of the cable, beam in horizontal and vertical directions, respectively; $\zeta_h = \frac{2}{\pi}\phi_h(x_c)$, $\eta_g = \frac{1}{2}\phi_g(x_c)\sin\theta$, $\alpha = \frac{\pi^2 u_0}{4L_c}$, $\zeta_y = \frac{2}{\pi}\frac{M_y}{M_h}$ $\phi_h(x_c)$, $\eta_y = \frac{M_y}{2M_g}\phi_g(x_c)\sin\theta$, u_0 is the initial elongation of the cable; μ_c is the uniform mass per unit length of the cable; M_y, M_h and M_g are modal mass of the cable and beam; L_c is the chord length, θ the cable inclination angle, x_c is the distance between the cable stay point and the support of the beam; $P_h(t)$ and $P_g(t)$ are horizontal and vertical external forces applied to the cable-supported end of the beam, respectively; and prime denotes the derivative with respect to time. Detail derivation of the equations can be referred to the study by Fujino *et al.* (1993). From the above equations of

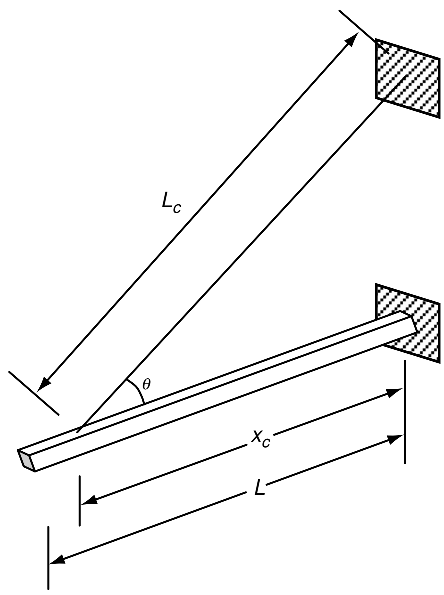
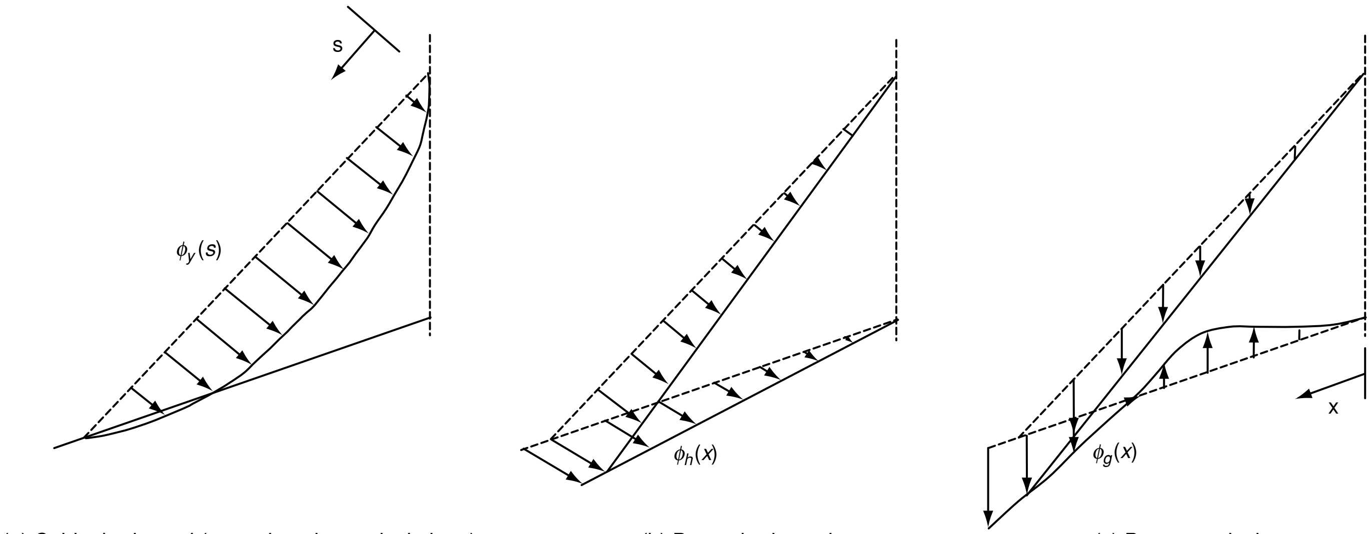


Figure 1. Cable-stayed beam as a model of cable-stayed bridges



(a) Cable: horizontal (normal to the vertical plane) (b) Beam: horizontal (c) Beam: vertical

Figure 2. Local and global vibration shapes

motion, it is observed that cubic nonlinear terms, linear and quadratic coupling terms are included.

Under the harmonic loading, the multiple scales method (Nayfeh and Mook 1979) can be employed to solve the above equations and obtain the responses of the system under different excitation amplitudes and excitation frequencies. For the case of random excitation, a general equivalent linearization with non-stationary approach (Roberts and Spanos 1990) can be employed to calculate the root-mean-square (RMS) of the system responses to different excitation levels.

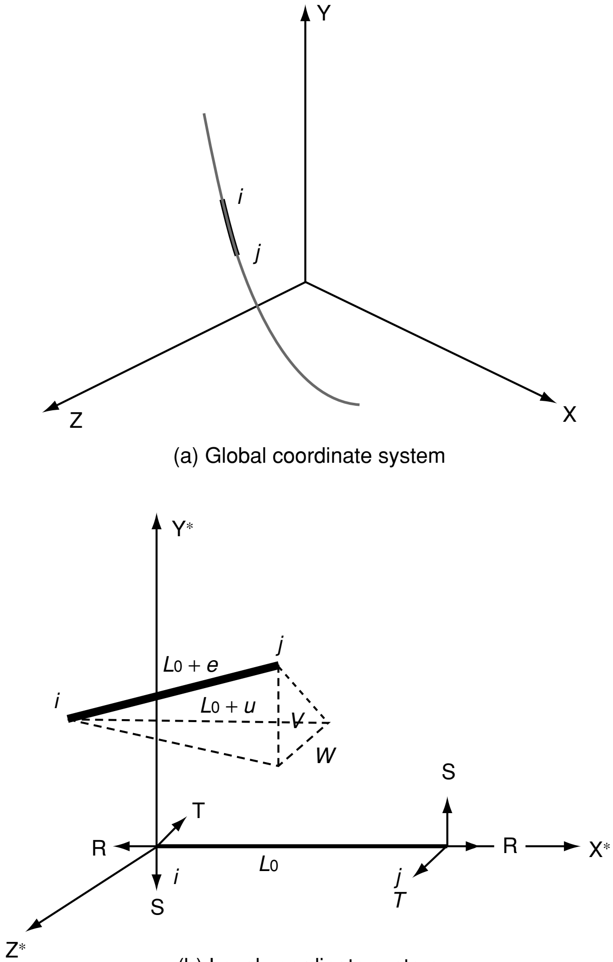
2.2. Finite Element Approach to the Cable-Beam System

With finite element approach, the cable is modelled by three-dimensional cable elements and the beam is modelled by three-dimensional Euler Bernoulli beam elements.

In the local coordinate system of the cable element as shown in Figure 3, the original length of the element is L_0, the initial basic force is P_0, and the displacements in three directions (x*, y*, z*) are (u_i, v_i, w_i) in node-*i* and (u_j, v_j, w_j) in node-*j*. The equilibrium equation of one cable element is given below.

$$P\left\{ -\frac{L_0+u}{L_0+e},\quad -\frac{v}{L_0+e},\quad -\frac{w}{L_0+e},\quad \frac{L_0+u}{L_0+e},\quad \frac{v}{L_0+e},\quad \frac{w}{L_0+e} \right\}^T = \{F\}^e \tag{4}$$

where $\{F\}^e = \{-R, -S, -T, R, S, T\}^T$ is the load vector applied to the ends of the element, $P = P_0 + E_C A_C / L_0 \times e$ is the updated element basic force, $e = \sqrt{(L_0+u)^2 + v^2 + w^2} - L_0$ is the element extension where $u = u_j - u_i$, $v = v_j - v_i$, and $w = w_j - w_i$, and E_c and A_c are Young's modulus and cross-sectional area of the cable element.



(a) Global coordinate system

(b) Local coordinate system

Figure 3. Cable element in global and local coordinate system

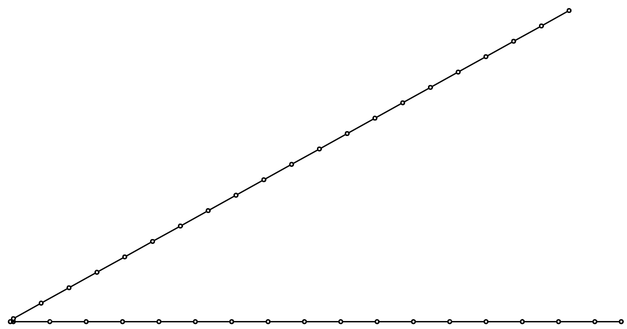


Figure 4. Finite element model of the cable-beam system

The above equation can be used to generate element incremental stiffness matrix, which includes transverse and out-of-plane actions of the cables. These are the major differences between the present cable element and other elements such as truss element and chord element that cannot evaluate the parametric vibrations of cables (Wu *et al*. 2006).

The restoring force of the cable element is a function of nodal displacements and element forces. Therefore, as the structure deforms, it needs to be reformulated using Newton-Raphson method. The stiffness matrix is updated during the iterative procedure. Direct integration is performed to calculate the time history dynamic response of the cable.

The lumped mass matrix and Rayleigh damping are used for the cable-beam system. For the cable-beam model shown in Figure 1, the cable is modelled by 20 cable elements and the beam is modelled by 18 Euler Bernoulli beam elements, as shown in Figure 4.

3. NONLINEAR VIBRATION UNDER HARMONIC LOADING

The cable-beam model is analysed with the two different approaches described above. Parameters of the system are listed in Table 1. It is noted that the frequency ratio of the system (local horizontal: global horizontal: global vertical) is approximately 1:1:2. Here the global vertical mode is the second in-plane mode of the beam.

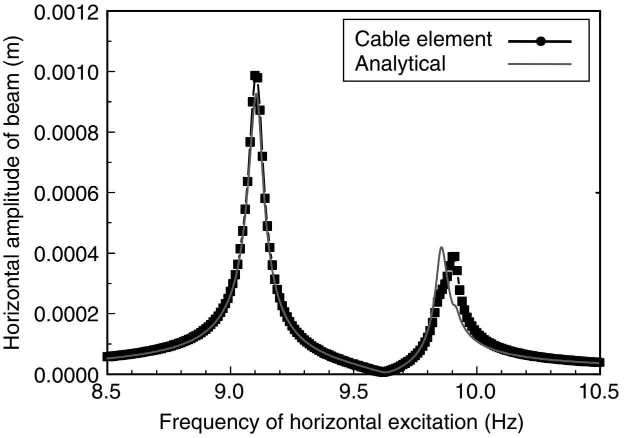
Under the horizontal excitation $P_h = F_h\cos(\Omega_h t)$ only, where F_h and Ω_h are the amplitude and frequency of the dynamic force, the vibration amplitudes of the cable and the beam obtained by the two methods are compared in Figure 5, in which $F_h = 0.15$ N. It is observed that the two methods give very similar results. There are two peaks in the frequency response diagram that correspond to the resonances of the system.

Similarly under the vertical excitation only with amplitude $F_h = 0.16$ N, the frequency responses of the system obtained by the two methods are compared in Figure 6. From the figure, one can conclude:

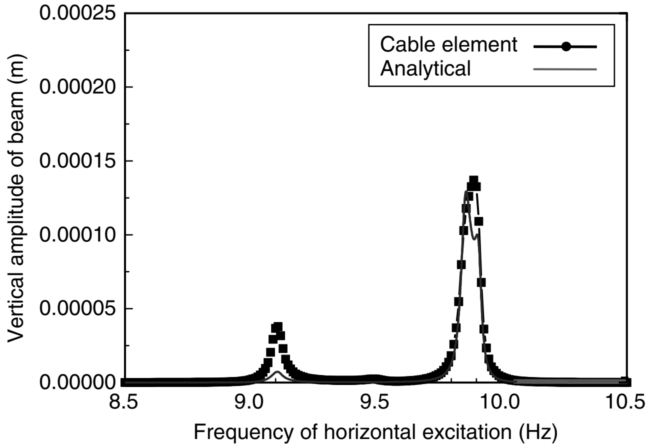
1) The results obtained by the two methods match very well, which verifies the accuracy of the cable finite element method.
2) When the excitation frequency is less than 19.73 Hz or larger than 19.84 Hz, horizontal motion of the cable and the beam is trivial (zero response), and only vertical vibration is excited, which is called as linear motion.
3) When the excitation frequency falls between 19.73 Hz and 19.84 Hz (1:2 superharmonic tuning), large horizontal vibration of the cable is excited at half excitation frequency. This is referred to as auto-parametric oscillation (Fujino *et al*. 1993), or internal resonance (Nayfeh and Mook 1979). At the same time, the vibration amplitude of the beam decreases. In the situation, the linear motion is unstable.
4) With the finite element approach, local vertical vibration of the cable at different excitation frequencies is plotted in Figure 6(d), which shows a similar manner as the beam's vertical motion. This, however, cannot be obtained through the analytical approach as only three degrees of freedom can be solved in the case. It is also found that the auto-parametric oscillation of the cable in the vertical direction is less than 10% of that in the horizontal direction. Therefore neglecting the vertical motion of the cable is acceptable in the analytical solution.

Table 1. Parameters of the cable-beam model

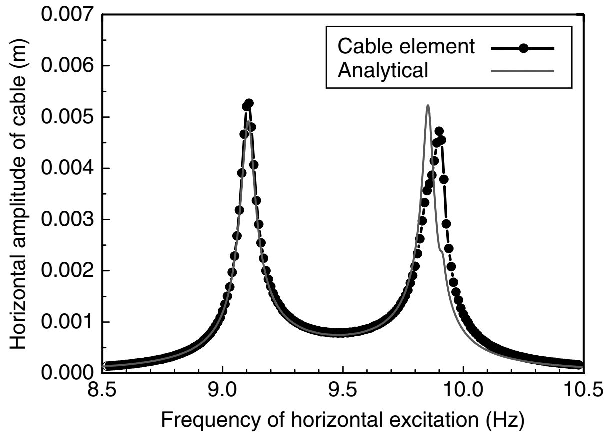
Parameter	Value	Parameter	Value	Parameter	Value
L_c	2.08 m	f_y	9.63 Hz	M_h	5.401 kg
L	2.00 m	f_h	9.38 Hz	M_g	6.200 kg
x_c	1.99 m	f_g	19.82 Hz	E_cA_c	35077.6 N
θ	29.0 deg	ξ_y	0.20%	$\phi_h(x_c)$	0.993
μ_c	0.07 kg/m	ξ_h	0.35%	$\phi_g(x_c)$	0.971
u_0	0.0067 m	ξ_g	0.14%		



(a) Horizontal amplitude of the beam

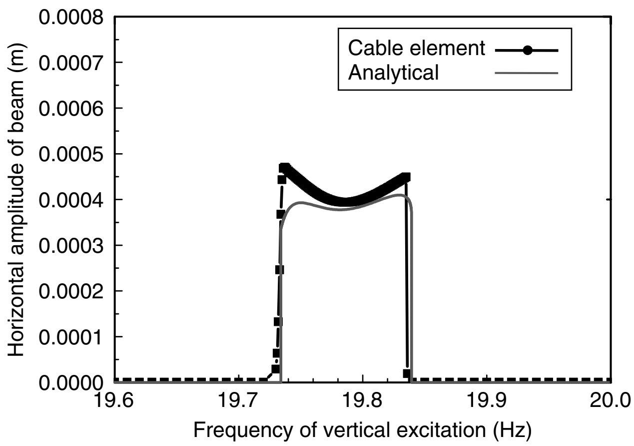


(b) Vertical amplitude of the beam

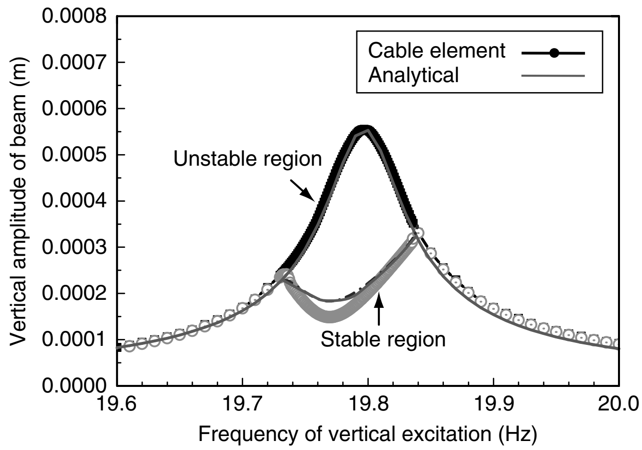


(c) Horizontal amplitude of the cable

Figure 5. Vibration of the system under horizontal excitation ($F_h = 0.15\,N$)

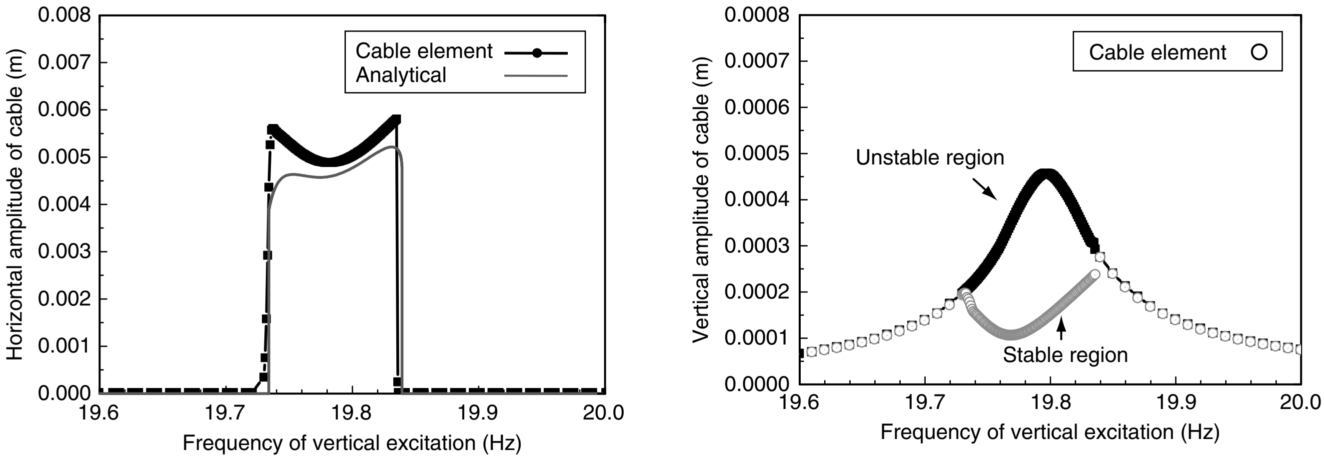


(a) Horizontal amplitude of the beam



(b) Vertical amplitude of the beam

Figure 6. (*Continued*)



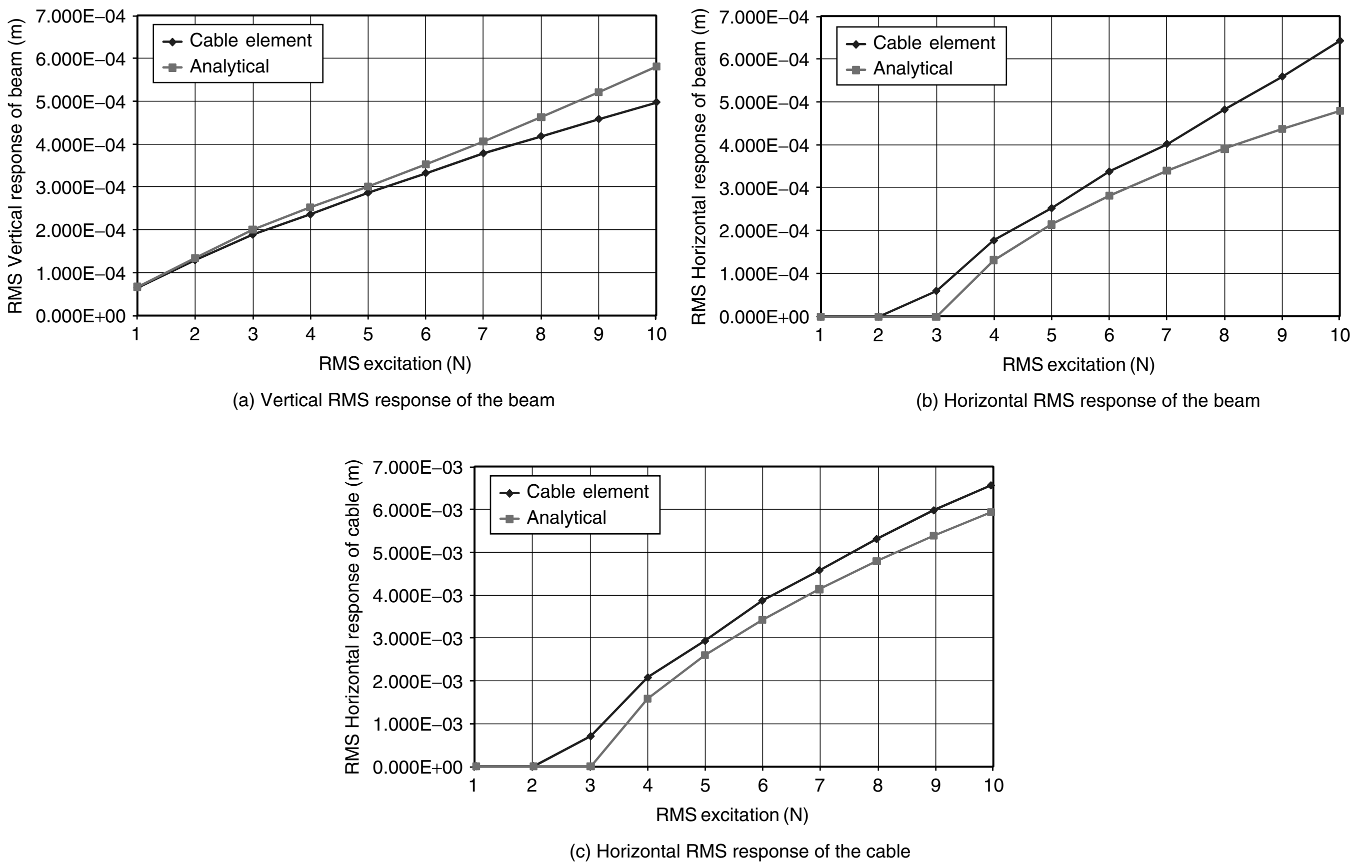
(c) Horizontal amplitude of the cable
(d) Vertical amplitude of the cable

Figure 6. Vibration amplitude under vertical excitation (F_g = 0.16 N)

4. NONLINEAR VIBRATION UNDER RANDOM LOADING

The real loading on the bridge is generally not purely harmonic but often random. Hence nonlinear responses of cable-stayed bridges due to random excitation are very important from practical point of view. Although the local and global random responses of the bridges can be estimated separately, the coupled cable-beam system under random excitation has been rarely studied so far.

Here only vertical random excitation is studied to examine the capability of the cable element in dealing with parametric vibration. For different excitation levels in terms of RMS, the responses are also obtained in sense of RMS, as shown in Figure 7. In the numerical



(a) Vertical RMS response of the beam
(b) Horizontal RMS response of the beam
(c) Horizontal RMS response of the cable

Figure 7. RMS responses under vertical random excitation

approach, 16 times simulation are applied and the mean values are presented here.

From the figure, one can find that:

1) In general the responses with the two methods agree very well.
2) When the excitation is less than a certain level, horizontal motions of the cable and the beam cannot be excited, and only vertical vibration is observed, which is similar to the case of harmonic excitation.
3) When the excitation is larger than a certain level, large horizontal vibration of the cable is excited. At the same time, vibration amplitude of the beam decreases. In the situation, the original linear motion is unstable.
4) Horizontal vibration of the beam and the cable via the finite element approach is larger than the counterpart with the analytical approach. Spectrum analysis shows that, in the analytical approach, the response of the beam is a banded process with three peaks corresponding to the three natural frequencies of the system. However, in the finite element approach, the system has much more degrees of freedom and higher modes are excited as well under the random excitation. The high frequency components contribute the vibration as well, which is not included in the analytical solution. This may result in different results.

5. CONCLUSIONS

Nonlinear vibration of the cable-beam system under harmonic loading and random loading is obtained with an analytical approach and finite element approach. Results show that the two methods give very close results even in the event of parametric vibration. Feasibility of the finite element approach to nonlinear vibration of cables is thus verified. The cable element cannot only deal with the structure with multiple cables, which is not possible for analytical methods, but also includes the cable-bridge interaction and out-of-plane vibration of cables. Therefore, it is deemed to be an ideal tool in analyzing vibration of real cable-stayed bridges. Application of the cable finite element approach to bridges with multiple stay cables will be investigated in the near future.

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