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A complex network based model for detecting isolated communities in water distribution networks

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Water distribution network (WDN) is a typical real-world complex network of major infrastructure that plays an important role in human's daily life. In this paper, we explore the formation of isolated communities in WDN based on complex network theory. A graph-algebraic model is proposed to effectively detect the potential communities due to pipeline failures. This model can properly illustrate the connectivity and evolution of WDN during different stages of contingency events, and identify the emerging isolated communities through spectral analysis on Laplacian matrix. A case study on a practical urban WDN in China is conducted, and the consistency between the simulation results and the historical data are reported to showcase the feasibility and effectiveness of the proposed model. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4823803]

Nowadays, the quality of human's life is significantly dependent on a reliable water supply system, which is one of the most critical infrastructures in modern society. With fast development of metropolitans, the structural and operational complexities of modern cities are increasingly dependent on the efficacy of Water distribution network (WDN). Quite often, any partial system failure or malfunction in WDN are greatly amplified through the whole network and thereby causing unexpected cascading contingencies, which potentially could split the WDN into isolated communities. The serious subsequent consequences due to system split can involve immediate suspensions of water supply in multiple areas. To some extent, traditional technologies are insufficient to study complicated behaviors of WDN, especially under contingency conditions. The model being proposed based on the view point of network structure will serve to provide a new solution of isolated community detection in the WDN.

I. INTRODUCTION

Critical infrastructures (e.g., water supply system, electrical power system, and transportation system, etc.) constitute the indispensable "lifeline systems" in a modern society.[1–3] They provide essential service to the society by normally and continuously operating their functionalities. Disruptions of major infrastructural services could cause serious consequences, such as health and safety problems, security issues, and economic loss.[1–4] With the high pace of economic development, critical infrastructures are increasingly constructed with highly sophisticated functionalities. Along with rapid development, operation of many infrastructures is often associated with various degrees of uncertainties and risks, and all these could potentially jeopardize the reliability of the infrastructural systems. Small internal or external disturbances might negatively impact a local system; and in some cases, some apparently unremarkable malfunctions or failures may be amplified significantly through the whole infrastructure system and cascaded to other parts, which would eventually cause the whole system to lose most of its functions or even collapse.[1,5]

Water supply system, as one of the most critical infrastructures today, plays an essential role for both social consumption and industrial production. Undoubtedly, local stable and reliable water supply is an essential commodity for residential and industrial consumers. It is also the primary need of public institutions (e.g., school, hospital, asylum, etc.). Nowadays, water supply systems are expanding rapidly in order to meet the increasing demands from all aspects in the society. Consequently, more complexities and uncertainties are getting involved in this large-scale network, which presents new challenges for network security monitoring and control.[6,7] As for WDNs, previous technologies of WDN operation analysis are mostly focusing on the social demands and network structural designs, whereas the structural properties, interdependent relations of each part in WDNs and their dynamic behaviors under different conditions are also neglected. On the context of such complex and large-scale WDNs, complex network researchers are presented with a good opportunity to create new concepts and research directions to enhance the operation reliability of WDNs.

A typical WDN consists of different types of fixed junctions (e.g., water source, reservoirs, demand points, pumping stations, and tanks) and flexible junctions (e.g., intersections of pipes, temporary tanks), which are connected by actual physical links (e.g., pipes with different sizes, lengths, and materials). WDNs are complex in the sense that all of their structural components constitute nontrivial configurations and interact in a fairly complex manner.[8,9] Obviously, the main function of WDNs is to transfer water from finite hydraulic sources to many possible demand points through water pipes. The time-varying water demand and uncertain perturbations

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(e.g., environmental disasters, damaged by animals, etc.) will naturally contribute more complexities and uncertainties to WDNs. Therefore, through such a complex network system, little changes in the WDNs layout (i.e., malfunction or outage of water pipes, pumping stations, or any other components) impose high risks to operation of the whole networks. The formation of isolated communities in WDNs is one of most serious drawbacks, as isolated sub-systems (i.e., communities) are no longer connected to the main part. In the absence of hydraulic source in the isolated communities, water supply will be suspended locally when failures occur. In cases that even the component outages do not bring down the whole system, there can still exists some weak areas, which could potentially evolve into isolated communities due to, for example, abnormal hydraulic pressures. In particular, as water pipes are installed underground, the difficulties of monitoring the operation conditions and status cannot be underestimated.

Based on the theory of complex network, we propose a comprehensive model in this paper to detect and identify the isolated communities in WDNs. This structure based modeling is designed to effectively capture network spectral properties and dynamic behaviors. According to statistical analysis and algebraic calculation, we can have an in-depth understanding on the network system layout (i.e., topology, patterns of connectivity), network revolution (i.e., operation status under different system conditions), and design (i.e., network sizing, water pipes layout). Furthermore, by addressing the physical features of the networks and complexity involved in the whole WDN system, we can evaluate the efficacy of a practical WDN including its topological design and resilience, i.e., the capacity of system in resisting disturbances.

II. RELATED STUDY

In recent years, the field of complex network has attracted increasing attentions by engineering researchers. The concept of complex network being incorporated into concrete engineering consideration is a new and promising research direction. Thus far, many applications of complex network based statistical and graphic analysis in artificial and natural networks (e.g., power system networks, transportation networks, logistic networks, etc.) are fairly successful and effective, thereby providing many novel and useful solutions on practical issues. Initially, physicists create complex network theory in researches that focus mainly on abstract networks.[10–16] Some excellent research outcomes (e.g., Erdos-Renyi random networks) were reported on analyzing the structure and linkage of networks mathematically.[11] However, significant gaps exist when applying such theories to practical networks. Motter *et al.* studied internet and power system networks and found out that there was a robust-but-fragile feature under cascade-based attacks.[17,18] Simultaneously, some complex network models were proposed to investigate the structural vulnerability of power grids, water distribution networks in city, logistic networks, etc.[9,19–25] Generally, previous researches were focusing on the use of complex network theories to analyze the physical topologies and structural connection of networks, but most prior works mostly neglect the specific structural properties, interaction of network components governed by practical operation conditions and concrete engineering features.

The function of a distribution network is significantly dependent on the structure, which is greatly affected by its organizational complexity and interplay among all structural components.[26] In Ref. 27, Barabasi-Albert discovers scale-free networks, which are characterized by heterogeneous structures and non-uniform degree distributions. In general, scale-free networks are robust against random failures while vulnerable to attacks on their hubs (i.e., nodes with high degree, high connectivity).[28] Once the network is altered (i.e., in case of component outages), the function may be significantly weakened. Recently, there are some reported research works that study the specific features and interplay of components in infrastructure networks,[29–35] which include power grids, rail networks, WDNs, gas pipeline networks, and logistic networks. Though theoretical basis has been established by previous complex network scientists, researches for specific structural and dynamical properties of practical networks are still in a primary stage.

To the best knowledge of the authors, there are few applications of complex network based approaches to WDNs. Basically, WDNs contain small-world features to some degree.[36,37] Different parts in a WDN are highly interdependent to each other and are interacting directly or indirectly. One particular aspect, which is worthy of considerations in WDN research, is the formation of isolated communities. In small-world network, one failure in a part may notably propagate to other parts to result in separation or even a collapse of the whole network.[37–41] The concept of "communities" in WDN is referring to those areas with water supply suspension and the losing of the connection to the main part of the WDN. Detecting those isolated communities and analyzing their nature is an important task, as it can produce hints for maintainers to take actions for risk reduction and control, so as to assist the entire WDN to quickly recover its full function.

Detecting communities in networks is closely related to the research of graph-partitioning. Many relevant algorithms and approaches have been developed thus far. At the beginning, Girvan and Newman proposed an iterative method based on the concept of *betweenness*,[42,43] which can well indentify the patterns of network separations by recursively removing edges with the largest betweenness. This approach yields very good partitioning results. However, it is computationally heavy and time consuming.[44,45] Therefore, some alternative methods stemming from different considerations were developed. In Ref. 46, Radicchi *et al.* create a divisive algorithm that can well handle triangular or higher order arcs (i.e., loops), which are exposed in networks. The Wu-Huberman algorithm is based on the idea of voltage drop,[47] which is a fast method while iterations are still necessary. The Reichard-Bornholdt method is reported in Ref. 48 and that is based on a q-state Potts Hamiltonian. This is a pioneering approach for identifying fuzzy communities. In Ref. 49, the Capocci-Servedio-Colaiori-Caldarelli method combines spectral properties of networks with correlation measurements to detect the closeness of communities. For further discussions of some methods mentioned above, the readers are referred to a comprehensive review as given in Ref. 44.

III. THE COMPLEX NETWORK BASED MODEL

A. Complex network based modeling of water distribution network

Based on the concept of complex network, WDNs can be abstracted as a graph with n vertices connected by m edges, which represent flexible or fixed junctions (e.g., water source, reservoirs, demand points, intersections of pipes, etc.) and physical links (e.g., pipes), respectively. Typically, most links in WDNs are bilateral except a small number of inlet branches (connected by water source) and outflow tubes (connected by demand points). In order to simplify the networks, inlet branches and water source points are combined as source-vertices. Likewise, the outflow tubes and demand points are combined as sink-vertices. Therefore, WDNs are generally modeled as non-directed graphs $G(N, L)$, where $N = \{1, 2, ..., n\}$ denotes the set of all vertices and $L = \{l^1, l^2, l^3, ...l^m\}$ denotes the set of all edges. Each edge is associated with a vertex pair (u, v), which is denoted as $l_{u,v}$.

Adjacency matrix $\boldsymbol{A}$ is frequently used to illustrate the connection of a graph. In a non-directed graph, $\boldsymbol{A}$ is a symmetric matrix, which is defined as

$$A = A(G) = [a_{u,v}]_{u\in N(G), v\in N(G)},$$

where $a_{u,v}$ is equal to 1 if there is an edge that connects vertices u and v, otherwise $a_{u,v}$ is equal to 0.

Let $\boldsymbol{D} = \text{diag}(d_1, d_2, \ldots, d_n)$ denotes the degree matrix of graph G, which is a diagonal matrix with $d_v = \sum_{i\in N(G)} a_{v,i}$, $v \in N(G)$.

In a practical WDN, the length, width, and material of each water pipe differ significantly due to different design considerations. We assign a specific value $\omega_{u,v}$ as the *weight* of each edge $(u, v \in N(G))$. All $\omega_{u,v}$ form an n-by-n weighting matrix $\boldsymbol{W}$. In this paper, $\omega_{u,v}$ is defined as

$$\omega_{u,v} = \frac{\phi_{u,v}}{l_{u,v}} \cdot \beta_{u,v}, \tag{1}$$

where $\omega_{u,v}$ is a non-negative value. $\phi_{u,v}$ and $l_{u,v}$ are the inside diameter and length of water pipe (u,v), respectively. $\beta_{u,v}$ represents the coefficient of pipe material to reflect its compressive strength.

In a weighted graph abstracted from WDN, the degree matrix can be redefined as: $\boldsymbol{D^*} = \text{diag}(d_1^*, d_2^*, \ldots, d_n^*)$, with diagonal elements

$$d_v^* = \sum_{i\in N(G)} \omega_{v,i},\ \ v \in N(G).$$

The topology of WDN can be expressed by a *Laplacian matrix* $\boldsymbol{L}$, which is defined as follows:

$$L = D^* - W.$$

In the matrix $\boldsymbol{L}$, the diagonal elements are given by the degree of each vertex in the WDN, and the off-diagonal element indicates the physical attribute of the associated edge, which is governed by its corresponding weight. Laplacian matrix is symmetric, singular, and positive semidefinite.[50] Unweighted graphs are considered as special cases of the weighted graph. Thus, $L = D - A$, which means all weights of the corresponding connected edges are equal to either 1 or 0.

B. Spectral properties of WDN

Under multiple lines outage or unexpected malfunction in WDN, the topology of graph is altered and the connection of each component is most likely loosened. Some isolated or weakly connected communities (named as *clusters* based on complex network theory) can therefore emerge. Such clustering is identified as a NP problem.[51] Some previous researches have studied the spectral properties of graph that can be useful for isolated community detection.[49,52] In the case of WDN, spectral analysis based on Laplacian matrix can be a new and effective approach for fast and accurate detection of isolated communities.

The spectrum of Laplacian matrix contains much topological information of WDNs. Multidimensional *eigenvector-space* is established for spectral analysis. Eigenvectors of Laplacian matrix $\boldsymbol{L}$ are denoted by $X_v, v \in N(G)$, which are associated with n eigenvalues $\lambda(L) = \{\lambda_1, \lambda_2 ... \lambda_n\}$. For the ith eigenvector, its jth element is represented by $x_j^i, i \in N(G), j \in N(G)$. We use the notation $\lambda_1 \leq \lambda_2 \leq \lambda_3 ... \leq \lambda_n$ to present an ascending order of Laplacian eigenvalues. By applying Perron-Frobenius theorem to $\boldsymbol{L}$, some useful results can be obtained as follows:

(i) $L(G)$ has only real eigenvalues;
(ii) $\lambda_1 = 0$ and $\lambda_2 > 0$ if and only if G is connected;
(iii) The smallest eigenvalue λ_1 is equal to 0 and its corresponding eigenvector is constant, i.e., $(1, 1, ...1)^T$. The City Number of zero eigenvalues is determined by the number of splitting components of the graph G.

Based on Courant's theorem,[50] the second smallest eigenvalue (non-zero) can be formulated as

$$\lambda_2 = \min X_v{}^T L X_v, v \in N(G). \tag{2}$$

We term the second smallest eigenvalue λ_2 the *algebraic connectivity* of the graph G.

In order to investigate the eigenvector profile, the following constrained optimization problem should be studied.

Here, the objective function is given by

$$f(x_1, x_2 ..., x_n) = \sum_{v\in N(G)} \sum_{u\in N(G)} (x_u - x_v)^2 \cdot \omega_{u,v},$$
$$s.t. \quad \sum_{u\in N(G)} \sum_{v\in N(G)} x_u \cdot x_v \cdot c_{u,v} = 1, \tag{3}$$

where $c_{u,v}$ is the element of a given symmetric matrix C. The optimization problem in (3) can be converted to its Lagrange dual function in (4), where γ is the Lagrange multiplier

$$\Lambda(x_1, x_2, ...x_n, \gamma) = \sum_{v\in N(G)} \sum_{u\in N(G)} (x_u - x_v)^2 \cdot \omega_{u,v} + \gamma\left(\sum_{u\in N(G)} \sum_{v\in N(G)} x_u \cdot x_v \cdot c_{u,v} - 1\right). \tag{4}$$

Applying differentiation, (4) becomes

$$\begin{cases} \left.\dfrac{\partial \Lambda}{\partial x_u}\right|_{u\in G} = 2\sum_{v\in G}(x_1 - x_v)\cdot \omega_{1,v} + \gamma\sum_{v\in G} x_v \cdot c_{u,v} = 0 \\ \dfrac{\partial \Lambda}{\partial \gamma} = \sum_{u\in N(G)}\sum_{v\in N(G)} x_u\cdot x_v \cdot c_{u,v} - 1 = 0. \end{cases}$$

Thus,

$$\begin{cases} \left(x_1\sum_{v\in N(G)}\omega_{1v} - x_1\omega_{11}\right) - x_2\omega_{12} - x_3\omega_{13} - \ldots - x_n\omega_{1n} = -\frac{1}{2}\gamma(x_1c_{11} + x_2c_{12}\ldots + x_nc_{1n}) \\ -x_1\omega_{21} + \left(x_2\sum_{v\in N(G)}\omega_{2v} - x_2\omega_{22}\right) - x_3\omega_{23} - \ldots - x_n\omega_{2n} = -\frac{1}{2}\gamma(x_1c_{21} + x_2c_{22}\ldots + x_nc_{2n}) \\ -x_1\omega_{31} - x_2\omega_{32} + \left(x_3\sum_{v\in N(G)}\omega_{3v} - x_3\omega_{33}\right) - \ldots - x_n\omega_{3n} = -\frac{1}{2}\gamma(x_1c_{31} + x_2c_{32}\ldots + x_nc_{3n}) \\ \vdots \\ -x_1\omega_{n1} - x_2\omega_{n2} - x_3\omega_{n3} - \ldots + \left(x_n\sum_{v\in N(G)}\omega_{nv} - x_n\omega_{nn}\right) = -\frac{1}{2}\gamma(x_1c_{n1} + x_2c_{n2}\ldots + x_nc_{nn}). \end{cases}$$

The solution of (4) can be expressed in a generalized matrix form as given below

$$(D - W)X = \eta CX \Longleftrightarrow C^{-1}(D - W)X = \eta X, \quad (5)$$

where $\eta = -\frac{1}{2}\gamma$. Thus, based on the analysis above, the constrained optimization problem (3) is translated into an eigenproblem with different constraints associated with C. When C is an identity matrix, optimization problem (3) results in an eigenvalue problem of the Laplacian matrix of a graph.

Therefore, in the case of a concrete graph G, the elements x_u of eigenvector corresponding to close-connected vertices are mathematically approaching to each other. In this paper, this numerical pattern is termed as *spectrum aggregation*. In a large WDN consisting of two or more connected areas, spectrum aggregation may not be obvious when strong inter area connections exist. However, such characteristic patterns are much clearer in d ($d \geq 2$) dimensional eigenvector space, which is formed by d eigenvectors (corresponding to d non-vanishing eigenvalues). The application of these spectral properties of Laplacian matrix will be discussed in Sec. III C.

C. Detection of isolated communities in WDNs

In order to detect the communities in WDNs, a d-dimensional eigenvector space (2-dimensional space is adopted in this paper) is first established, whose co-ordinates are determined by the projections of the first d non-trivial eigenvectors. Based on the analysis in Sec. II, the following procedure for spectral analysis of WDN is made

(i) Compute algebraic connectivity. $\lambda_2 > 0$ if and only if the whole graph is connected;
(ii) Identify the isolated communities if the graph is splitting;
(iii) Detect the weakly connected parts of the graph (potential communities). Angular distance (i.e., the angle between two vectors stemming from the origin of a 2-dimensional space) is introduced as a metric to quantitatively measure eye-inspected communities. Closeness is defined to distinguish each community, which is formulated as

$$\sigma_i \triangleq \frac{1}{n}\sum_{j\in S_i}||a_j^i - c_i||^2,$$

where S_i is the set which consists of all nodes in community i. a_j^i and c_i indicate the angular distance of node j and the angular centroid of community i, respectively.

IV. CASE STUDY

In this section, a practical case study, i.e., an urban WDN in *City N, China*, is carried out in details. The proposed model is applied to this network to detect isolated

FIG. 1. Sketch map of urban WDN in *City N*.

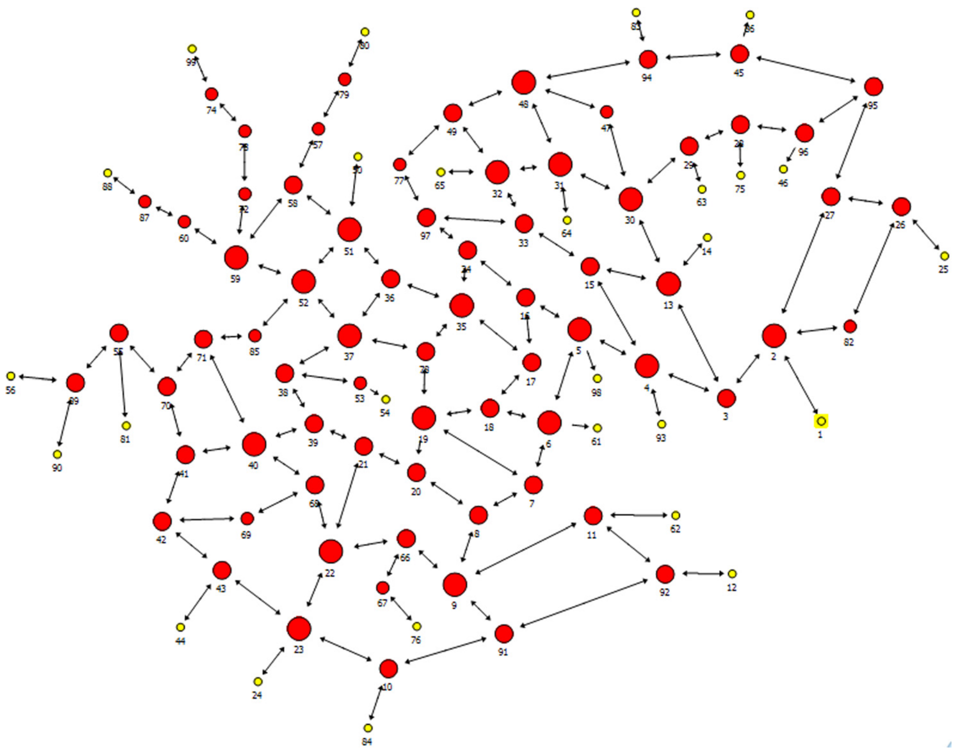

FIG. 2. Abstracted graph of urban WDN in *City N*.

communities under multiple operation conditions and contingencies. A topology based evaluation for this network will be given finally via the following graphic analysis and discussions.

A. Urban water distribution network in *City N*, China

Fig. 1 shows the sketch map of the urban WDN in City N, which consists of one water source, one reservoir, 98 demand points, and 124 physical links. There are rivers passing across this network. There are some water pipes installed on the riverbed. An abstracted graph with one source vertex, 98 sink vertices, and 124 connected edges is shown in Fig. 2.

The detection of communities in WDNs has to rely on limited information such as abnormal hydraulic pressure, leakage, burst, etc., since water pipes are mostly buried underground without direct exposures. Nevertheless, once community emergence is confirmed, timely decisions of countermeasures can then be made to mitigate the negative impacts.

B. Simulation and discussion

Simulation based on this real case is conducted in a given scenario. MATLAB is the simulation platform. Fig. 3 illustrates the flowchart of the proposed detection scheme for the isolated communities in WDNs.

Scenario description: initially, one of the core inlet branches (i.e., $l_{2,3}$, $l_{2,27}$ and $l_{2,82}$) bursts or malfunctions in an unexpected event. This would subsequently change the connection and interdependence of each area in the whole

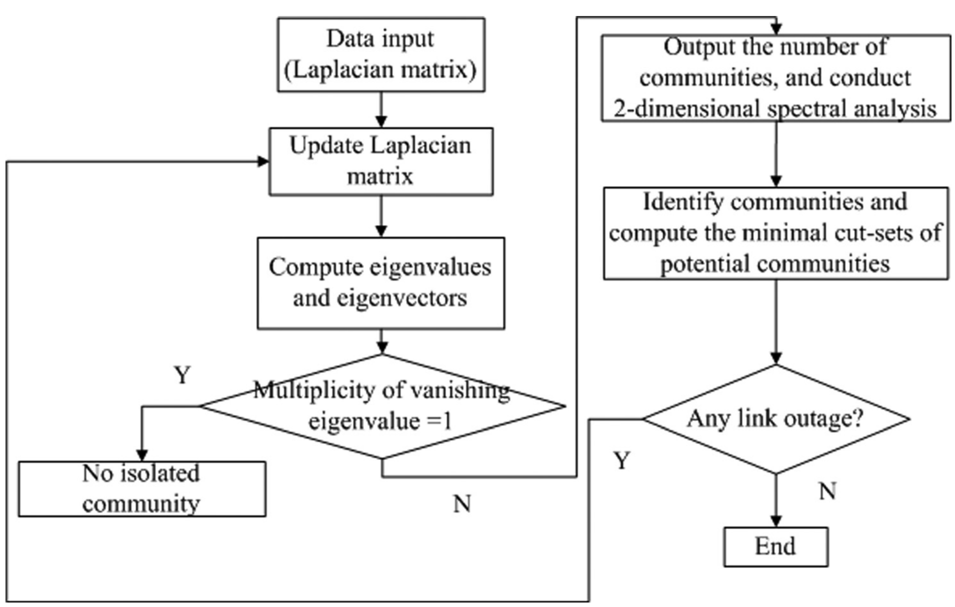

FIG. 3. Flowchart of the proposed detection scheme for the isolated communities.

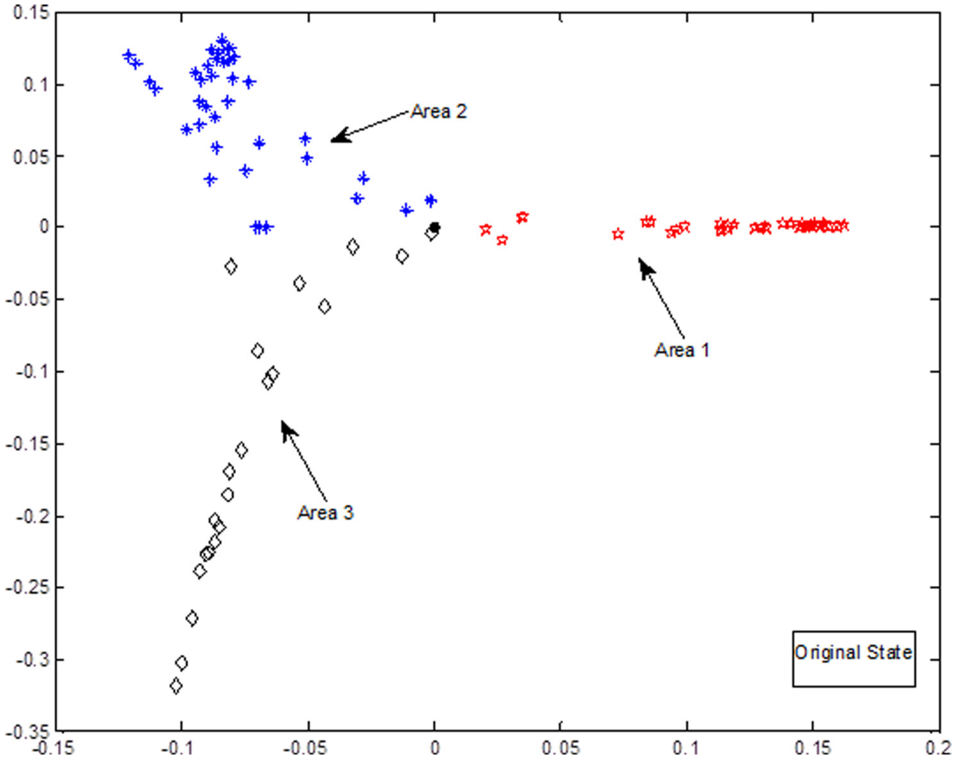

FIG. 4. Laplacian spectrum of City N WDN in the original state.

network. Thus, some tie lines—pipes connecting two areas—may be stressful and eventually burst. Such network component outages would cause severe cascading failures to result in the formation of isolated communities with no water supply.

Fig. 4 shows the original spectrum plot of the City N WDN. It is obvious that the whole network could be roughly divided into three areas, which are marked with different colors and shapes in Fig. 4. Figs. 5–8 show the network Laplacian spectrum of each state, which is subject to multiple pipe outages. The spectral information of each state is listed accordingly in Tables I to V.

In the original state, the spectrum pattern illustrates the connectivity of the whole graph. It can be visually partitioned into three areas according to the spectral distribution, which is consistent with the original network topology. The spectrum is distributed in a relatively narrow space with the smallest closeness, i.e., 0.02714 in area 1, which

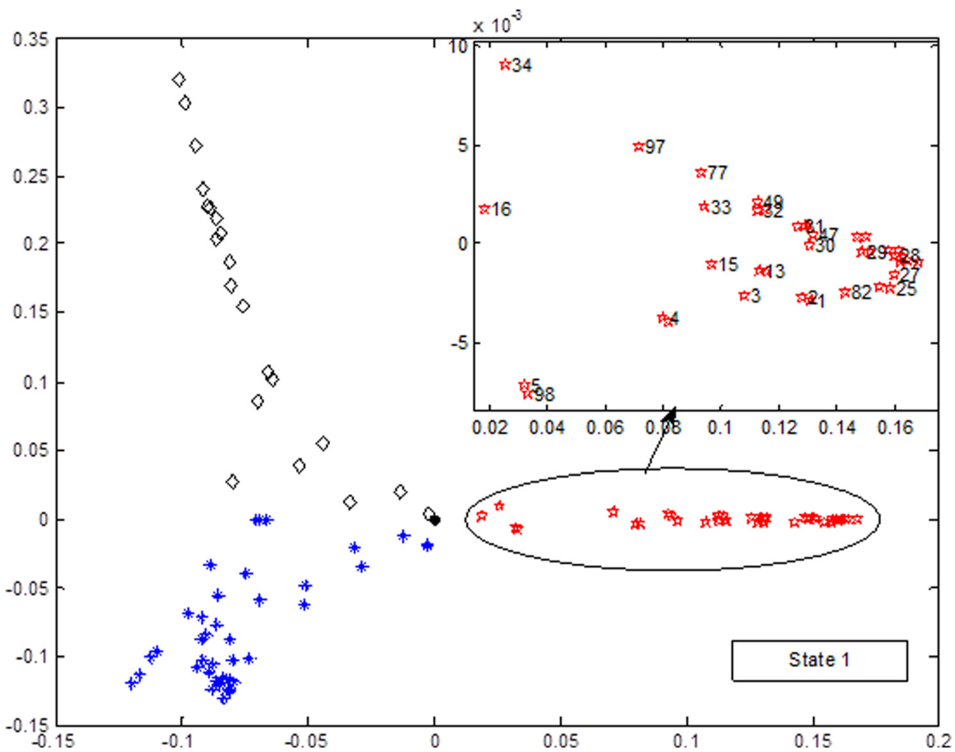

FIG. 5. Laplacian spectrum of WDN in state 1 ($l_{2,27}$ is out of service).

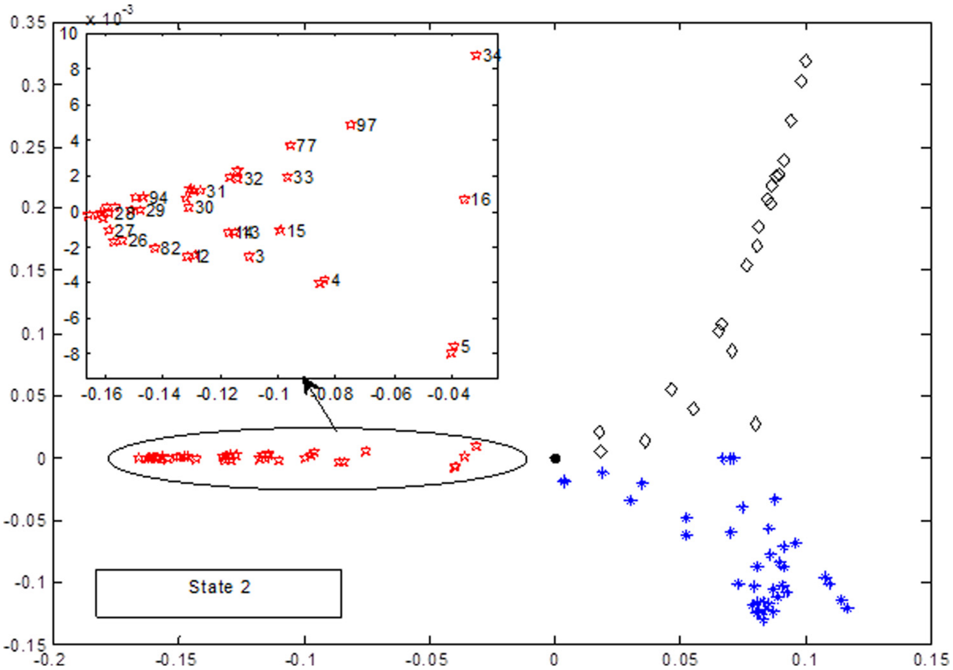

FIG. 6. Laplacian spectrum of WDN in state 2 ($l_{2,27}$ and $l_{16,17}$ are out of service).

demonstrates that area 1 is much more independent than the other two areas. All areas are interconnected by a few tie lines, which are used to transmit water from one area to another. The only water source is located in area 1 (i.e., No.1 node). This implies that areas 2 and 3 are facing a high risk of water supply interruption if area 1 is isolated. In state 1, $l_{2,27}$ is out of service and closeness of area 1 is slightly increased from 0.02714 to 0.02882. In contrast, closeness of areas 2 and 3 is decreased to 0.10789 and 0.09480 respectively. This illustrates that the interdependence of all areas are strengthened, which is consistent with the fact that outage of the core inlet branches (e.g., $l_{2,27}$) would make some tie lines (e.g., $l_{16,17}$) stressful. In state 2, $l_{16,17}$ bursts due to overpressure. Water can only be transmitted from area 1 to areas 2 and 3 via edges $l_{5,6}$ and $l_{34,35}$. Likewise, $l_{5,6}$ and $l_{34,35}$ would be very likely to be out of service due to increasing stress. In state 3, closeness of area 1 is sharply decreased to 0.02186 and spectrum pattern of isolated community emerges, which is shown in Fig. 7 (all spectra are approximately aligned in the same direction). In state 4, isolated community has been formed (shown in Fig. 8) and all tie lines connecting to the source area (i.e., area 1) are cut off. This is the most critical situation that most parts of the WDN collapse. These simulation results have been proven to be consistent with the experience and assessment of WDN operators in City N while such critical situations happened.

In practice, with the use of spectral analysis, one can carry out an in-depth analysis of a WDN, including its

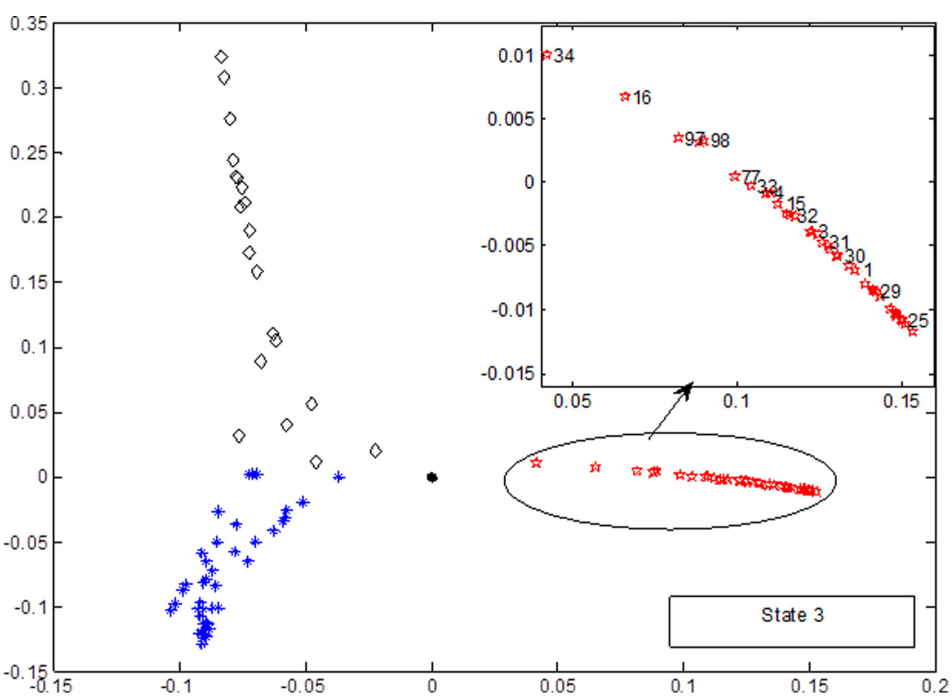

FIG. 7. Laplacian spectrum of WDN in state 3 ($l_{2,27}$, $l_{16,17}$, and $l_{5,6}$ are out of service).

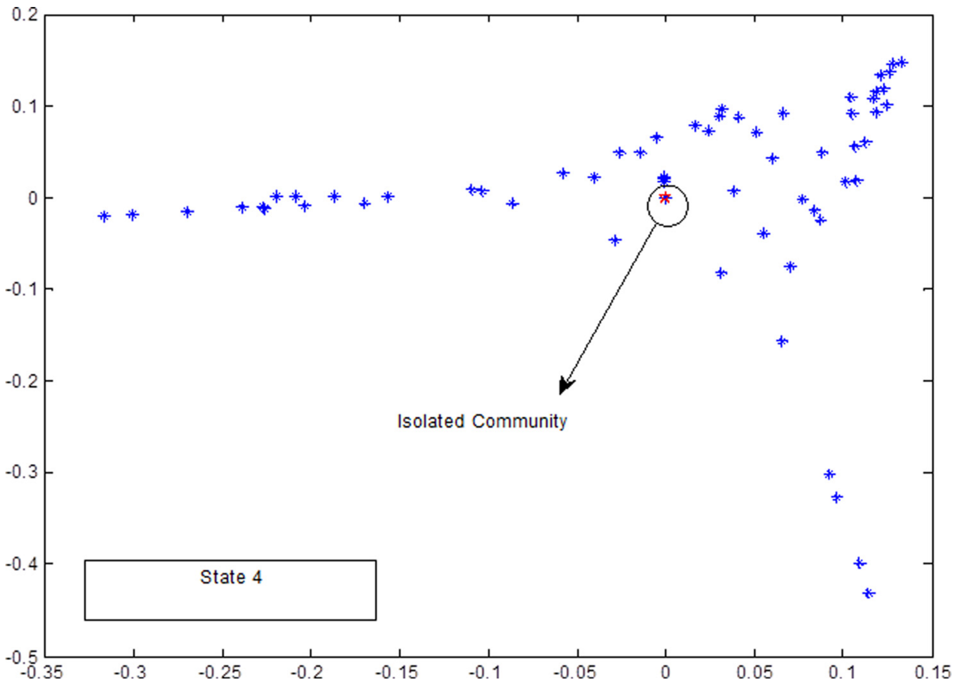

FIG. 8. Laplacian spectrum of WDN in state 4 ($l_{2,27}$, $l_{16,17}$, $l_{5,6}$, and $l_{34,35}$ are out of service).

TABLE I. Spectral information of City N WDN in the original state.

Graph partitioning	Components (nodes)	Closeness	Formation of isolated community
Area 1	1 2 3 4 5 13 14 15 16 25 26 27 28 29 30 31 32 33 34 45 46 47 48 49 63 64 65 75 77 82 83 86 93 94 95 96 97 98	0.02714	Nil
Area 2	6 7 8 9 10 11 12 18 19 20 21 22 23 24 38 39 40 41 42 43 44 53 54 55 56 61 62 66 67 68 69 70 71 76 81 84 89 90 91 92	0.11217	Nil
Area 3	17 35 36 37 50 51 52 57 58 59 60 72 73 74 78 79 80 85 87 88 99	0.09773	Nil

TABLE II. Spectral information of WDN in state 1.

Graph partitioning	Components (nodes)	Closeness	Formation of isolated community
Area 1	1 2 3 4 5 13 14 15 16 25 26 27 28 29 30 31 32 33 34 45 46 47 48 49 63 64 65 75 77 82 83 86 93 94 95 96 97 98	0.028821	Nil
Area 2	6 7 8 9 10 11 12 18 19 20 21 22 23 24 38 39 40 41 42 43 44 53 54 55 56 61 62 66 67 68 69 70 71 76 81 84 89 90 91 92	0.10789	Nil
Area 3	17 35 36 37 50 51 52 57 58 59 60 72 73 74 78 79 80 85 87 88 99	0.09480	Nil

TABLE III. Spectral information of WDN in state 2.

Graph partitioning	Components (nodes)	Closeness	Formation of isolated community
Area 1	1 2 3 4 5 13 14 15 16 25 26 27 28 29 30 31 32 33 34 45 46 47 48 49 63 64 65 75 77 82 83 86 93 94 95 96 97 98	0.02426	Nil
Area 2	6 7 8 9 10 11 12 18 19 20 21 22 23 24 38 39 40 41 42 43 44 53 54 55 56 61 62 66 67 68 69 70 71 76 81 84 89 90 91 92	0.10787	Nil
Area 3	17 35 36 37 50 51 52 57 58 59 60 72 73 74 78 79 80 85 87 88 99	0.11669	Nil

structural connectivity and drawbacks, in the early stage. Such analysis is useful in order to devise timely countermeasures when unexpected events (e.g., pipe bursts, malfunctions, etc.) happen. Furthermore, this proposed model can provide significant guidance for WDN planning and reconstruction.

V. CONCLUSIONS

Complex network based analysis in practical engineering is a new research direction and has attracted increasing attentions in recent years. This paper attempts to propose an algebraic-graph model based on complex network theory to

TABLE IV. Spectral information of WDN in state 3.

Graph partitioning	Components (nodes)	Closeness	Formation of isolated community
Area 1	1 2 3 4 5 13 14 15 16 25 26 27 28 29 30 31 32 33 34 45 46 47 48 49 63 64 65 75 77 82 83 86 93 94 95 96 97 98	0.02186	Nil
Area 2	6 7 8 9 10 11 12 17 18 19 20 21 22 23 24 38 39 40 41 42 43 44 53 54 55 56 61 62 66 67 68 69 70 71 76 81 84 89 90 91 92	0.10521	Nil
Area 3	35 36 37 50 51 52 57 58 59 60 72 73 74 78 79 80 85 87 88 99	0.11172	Nil

TABLE V. Spectral information of WDN in state 4.

Isolated communities	Components (nodes)	Closeness
Community 1	1 2 3 4 5 13 14 15 16 25 26 27 28 29 30 31 32 33 34 45 46 47 48 49 63 64 65 75 77 82 83 86 93 94 95 96 97 98	0

investigate the structural properties of WDNs, which are highly related to the emergences of isolated communities under contingency conditions. Based on the proposed model and spectral analysis, the detection of isolated communities in WDNs becomes a matrix eigen-problem in assessing the spectrum aggregation of different areas. By doing so, existing and emerging communities with possible service interruptions can be quickly and accurately detected. Simulation results from a real case study are reported to validate the feasibility and effectiveness of this model.

Admittedly, the dynamic behaviors of a WDN are fairly complex and unpredictable. It is challenging to comprehensively model a WDN by taking into account all factors. Instead, this paper focuses mainly on the formation of isolated communities in WDNs with respect to structural connectivity. Our future work is to further develop the proposed model by considering water flow on each pipe and time-varying water demands over the whole network.

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