

ON THE EXPERIMENTAL ASSESSMENT OF DE MARCHI'S DISCHARGE COEFFICIENT FOR INCLINED SIDE WEIRS: TRANSFER FUNCTIONS FOR THE APPLICATION OF ALTERNATIVE METHODS

MARIO DI BACCO⁽¹⁾, ANNA RITA SCORZINI⁽¹⁾ & MAURIZIO LEOPARDI⁽¹⁾

⁽¹⁾ University of L'Aquila – Department of Civil, Environmental and Architectural Engineering, L'Aquila, Italy
e-mail: annarita.scorzini@univaq.it

ABSTRACT

The use of De Marchi's approach, solving the 1D dynamic equation of spatially varied steady flow with non-uniform discharge, is commonly accepted in the design of side weirs. A key issue in applying De Marchi's formula is the assessment of the discharge coefficient C_M . However, no explicit equation for the experimental determination of C_M can be derived for inclined side weirs, due to the longitudinal change of crest's height. In this context, the present study analyzes the feasibility of using alternative methods for the estimation of the discharge coefficient (i.e. Dominguez's, Schimdt's or other approaches), which may be suitable to be used in De Marchi's equation also for inclined side weirs. However, this solution necessarily yields an additional error in the estimation of C_M , due to the different modelling assumptions underlying these other methods. Therefore, in this study, the magnitude of this error is first quantified using a 1D numerical model for different tested hydraulic conditions and geometric configurations of the side weir, including: Froude number (only subcritical flows), channel and friction slope, crest angle, water depth/weir height and weir length/channel width ratios. Results indicate that error factors (i.e. observed/predicted ratio) in the assessment of De Marchi's coefficient can range from 0.57 to 15.60 for inclined lateral weirs, depending on the selected modelling approach. As a second step, a Multilayer Perceptron Neural network is applied to derive transfer functions from the discharge coefficients calculated using the different methods to the corresponding values of C_M to be used in De Marchi's weir equation.

Keywords: discharge coefficient, De Marchi, side weirs, inclined, neural networks

1 INTRODUCTION

Side weirs are particular hydraulic structures used in many engineering applications, ranging from irrigation and drainage networks to flood hazard mitigation in river systems.

The investigation of flow features over side weirs has been the object of many analytical and experimental studies since the first decades of the past century (e.g., Velatta, 1934; Coleman and Smith, 1923; Engels, 1917). The fundamental phenomenon that needs to be taken into account when describing the outflow from a later weir is the non-constant, decreasing discharge along the structure. Under the accepted assumption of the validity of Poleni's formula (1717), which describes the outflow from frontal weirs (with the exception of neglecting the kinetic component of the total head in the case of side weirs), the flow rate derived per unit of length of the lateral weir q_d can be expressed as:

$$q_d = \frac{dQ_d}{dx} = -\frac{dQ}{dx} = \frac{2}{3} C \sqrt{2g} (y - p)^{3/2} \quad [1]$$

where y is water depth in the channel, p is weir height (i.e., $y-p$ is the pressure head over the weir) and C is the dimensionless discharge coefficient, which depends on crest shape and, in theory, on relative water height $((y-p)/y)$, although its influence is usually considered to be negligible (Hager, 1987). In Eq. [1], under Poleni's assumption, the discharge coefficient C assumes the same meaning which has for frontal weirs and gives information on the efficiency of the outflow process. The total diverted flow can be calculated by integrating Eq. [1] over the total length of the later weir, L :

$$Q_d = \frac{2}{3} C \sqrt{2g} \int_0^L (y(x) - p)^{3/2} dx \quad [2]$$

Eq. [2] indicates that the discharge coefficient C can be determined experimentally by measuring the diverted discharge Q_d and water depths along the side weir (i.e. the water surface profile, $y(x)$) with a sufficient spatial resolution in the flow direction.

De Marchi's (1934) approach is commonly accepted in the design of side weirs. The method is based on the solution of the 1D dynamic equation of spatially varied steady flow with non-uniform discharge, by assuming energy conservation in the main channel, irrespective of friction and channel slope. Under these hypotheses, the differential equation describing the water surface profile can be expressed as:

$$\frac{dy}{dx} = \frac{Qy \left(-\frac{dQ}{dx} \right)}{gB^2y^3 - Q^2} \quad [3]$$

where B is channel width.

The flow conveyed in the main channel can be calculated as a function of water depth y , as shown in Eq. [4], and then the diverted flow can be estimated as the difference between the inflow and outflow (Eq. [5]):

$$Q = B y \sqrt{2g(E - y)} \quad [4]$$

$$Q_d = B y_1 \sqrt{2g(E - y_1)} - B y_2 \sqrt{2g(E - y_2)} \quad [5]$$

where E is the energy head along the weir (assumed constant under De Marchi's approach) and $y_{1,2}$ are water depths at the upstream and downstream ends of the structure.

By combining Eq. [1], [3] and [4], we obtain:

$$\frac{dy}{dx} = \frac{4C \sqrt{(E - y)(y - p)^3}}{3B(3y - 2E)} \quad [6]$$

Therefore, the length of the side weir necessary to reduce the inflow Q_1 to an assigned value Q_2 is given by the integration of Eq. [6], that yields to:

$$L = \frac{3B}{2C_M} (\Phi(y_2) - \Phi(y_1)) \quad [7]$$

in which $\Phi_{1,2}$ are calculated immediately upstream and downstream of the side weir, as follows:

$$\Phi(y) = \frac{2E - 3p}{E - p} \sqrt{\frac{E - y}{y - p}} - 3 \arcsin \frac{E - y}{E - p} \quad [8]$$

The discharge coefficient C (denoted here as C_M to indicate that it refers to De Marchi's approach), obtained by inverting Eq. [7], can be then derived experimentally by means of water depth and flow measurements:

$$C_M = \frac{3B}{2L} (\Phi(y_2) - \Phi(y_1)) \quad [9]$$

The C_M coefficient takes into account the differences between the "real" measured water surface profile $y(x)$ and the one approximated by the De Marchi's model, while it can give reliable indications on the discharging efficiency only if the difference between the two is very small. In other words, the discharge coefficient C_M can be intended as the product of two factors ($C_M = C_m \cdot C_d$), represented by Poleni's coefficient C_d and another component C_m accounting for the differences in the shape of the water surface profiles, inherent to the simplifying hypotheses of De Marchi's approach.

To extend the weir equation to the case of inclined lateral spillways (Figure 1a and 1c), it is necessary to replace the weir height p with its expression as a linear function of x :

$$p(x) = p_0 + x \tan(\vartheta) \quad [10]$$

where p_0 is the minimum weir height and ϑ is crest angle. With this substitution, Eq. [6] becomes:

$$\frac{dy}{dx} = \frac{4C}{3B} \frac{\sqrt{(E-y)(y-(p_0+x \tan(\vartheta)))^3}}{3y-2E} \quad [11]$$

This equation cannot be integrated explicitly, but it can be solved numerically. Consequently, in this case it is not possible to obtain a direct formula for estimating the C_M coefficient to be applied in De Marchi's model and then for calculating the weir length L . Therefore, in order to use the extended model for the inclined weirs [11], it is necessary to find a proper expression for the C_M coefficient.

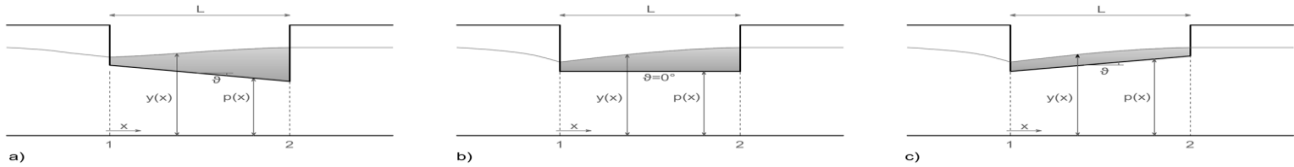


Figure 1. Schematization of side weir configurations: a) inclined downward; b) horizontal; c) inclined upward.

The idea we propose in this study is to verify the feasibility of using alternative methods for the estimation of the discharge coefficient, which may be suitable to be applied in the extended version of the De Marchi's equation for inclined side weirs.

For example, Dominguez (1935) introduced a simplified method for estimating the discharge diverted from a lateral weir, assuming a linear water surface profile along the structure; under this hypothesis, the pressure head can be expressed as (Bagheri et al., 2014a; Shariq, 2018):

$$h(x) = y - p = h_1 + (h_2 - h_1) \frac{x}{L} \quad [13]$$

and, consequently, the diverted flow can be determined by substituting Eq. [13] in [1] and integrating it:

$$Q_d = \frac{4}{15} C_D L \sqrt{2g} \frac{(h_2^{5/2} - h_1^{5/2})}{h_2 - h_1} \quad [14]$$

This equation can be inverted to compute the discharge coefficient (here denoted as C_D , as a reminder to Dominguez's approach) and, in theory, has no restriction to be applied also to inclined side weirs:

$$C_D = \frac{15}{4} \frac{Q_d}{L \sqrt{2g}} \frac{h_2 - h_1}{(h_2^{5/2} - h_1^{5/2})} \quad [15]$$

A similar approach was proposed by Schmidt (1954), who assumed energy conservation between the extreme upstream and downstream sections of the lateral weir. In this method, the outflow Q_d can be determined by means of Poleni's formula, under the assumption of a pressure head equal to the mean water depth h_a calculated between the upstream and downstream ends of the structure:

$$Q_d = \frac{2}{3} C_S L \sqrt{2g} h_a^{3/2} \quad [16]$$

Consequently, an explicit expression for the discharge coefficient under the Schmidt's approach, C_S , can be easily obtained:

$$C_S = \frac{3Q_d}{2L \sqrt{2g} h_a^{3/2}} \quad [17]$$

Eqs. [18-19] are other expressions used in many studies in the literature for calculating the diverted flow and the discharge coefficient in lateral weirs, under the erroneously definition of "De Marchi's formula" (e.g., Emiroglu et al., 2016 and 2014; Bagheri et al. 2014a and 2014b):

$$Q_d = \frac{2}{3} CL \sqrt{2g} (y-p)^{3/2} \quad [18]$$

$$C_1 = \frac{3Q_d}{2L \sqrt{2g} (y_1 - p)^{3/2}} \quad [19]$$

In this last approach, a key role is played by the method chosen to evaluate y . Still, this is not often explicitly stated in many studies: the most common solution, where indicated, is to consider it equal to the water depth in the upstream section of the weir, y_1 (Emiroglu et al., 2016; Bagheri et al., 2014a; Emiroglu et al., 2011).

However, in any case, the solution of adopting alternative expressions for the discharge coefficient to be used in the extended De Marchi's model for inclined side weirs necessarily yields an additional error, inherent to the different modelling assumptions underlying these other methods. Within this framework, the discharge coefficient C_M can be then regarded as the product of three factors:

$$C_M = f \cdot C_m \cdot C \quad [20]$$

where f can be intended as a transfer function between the considered coefficient (C_D , C_S or C_1) for the extended model and the corresponding (unknown) value of De Marchi's coefficient.

Therefore, in the following sections of this paper, the magnitude of the "modelling error" is first quantified using a 1D numerical model for different tested hydraulic conditions and geometric configurations of lateral weirs, in order to evaluate which approach is more suitable to be applied with the modified De Marchi's method for inclined weirs. Then, as a second step, a multilayer perceptron neural network is implemented to analyze the transfer functions f to convert the discharge coefficients calculated using the alternative methods to the corresponding values of C_M to be used in De Marchi's equation.

2 DATA AND METHODS

2.1 Numerical analysis of the error in the estimation of De Marchi's discharge coefficient with alternative methods

Based on the exposed considerations, four expressions for the discharge coefficient can be used (Eq. [9, 15, 17, 19]). In order to verify their adaptability to the De Marchi's model, we analyzed the error they introduce in it by means of the numerical solution of the 1D energy equation combined with Poleni's formula for assigned values of the discharge coefficient, C^* , and different sets of hydraulic boundary conditions and geometric configurations of the lateral weir. In detail, the system of mathematical equations [21, 22]

$$Q_{k+1} = Q_k - C^* dx \sqrt{2g} \left(\frac{y_k + y_{k+1}}{2} - \frac{p_k + p_{k+1}}{2} \right)^{3/2} \quad [21]$$

$$Q_{k+1} = B \cdot y_{k+1} \sqrt{2g (E - y_{k+1})} \quad [22]$$

was solved using a predictor-corrector method for the unknown parameters Q , y and E , for an horizontal test channel ($B=0.20$ m) under different combinations of the following input variables:

- 13 values of the side weir crest angle ϑ (i.e., 1° , 2° , 3° , 4° , 6° , 9°), for both upward and downward configurations, including the horizontal case, $\vartheta = 0^\circ$ (Figure 1);
- Discharge coefficient C^* , randomly selected in the range $0.3 \div 0.45$ or $0.45 \div 0.6$;
- Upstream Froude number Fr , randomly selected in the range $0.4 \div 0.6$ or $0.6 \div 0.8$;
- 9 values of the upstream y/p ratio, in the range $1.125 \div 3.25$.

Furthermore, in a second step, possible changes in the energy head E were also taken into account, by adding to the previous system (Eqs. [21], [22]) the following equation:

$$E_{k+1} = E_k + (i - j) dx \quad [23]$$

where i and j are channel and friction slope, respectively; j has been expressed by means of the Chezy formula, as commonly used in the literature (e.g., Singh et al., 1994; Ghodsian, 2003; Aghayari et al., 2009) and also suggested by De Marchi (1934). For these variable-energy runs, the following tested parameters were included in the analysis:

- 3 values of the channel slope, i : 0.0001, 0.001 and 0.01;
- 2 values of the Chezy constant χ , randomly selected in the range $20 \div 40$ m^{1/2}/s or $40 \div 60$ m^{1/2}/s.

For each combination of the selected variables, the equations were integrated until at least one of the following conditions was satisfied:

- $y_k = p_k$, i.e. the water surface profile reaches the crest of the weir (possible for the cases with $\vartheta > 0^\circ$);
- $p_k = 0$ (possible for the cases with $\vartheta < 0^\circ$);
- $Q_d/Q_1 > 0.9$.

In detail, the implemented procedure is described hereinafter and schematized in Figure 2: for each integration step ($dx=2.5$ mm), the first predictor y_p , estimated by Eq. [11], is used for determining the diverted flow dQ_p with Poleni's formula. For comparison, an additional value of the diverted flow dQ_c is computed as the difference between the one determined in the previous step and the one estimated with Eq. [22]; then, the corrector y_c is calculated as a function of Q_c by inverting Poleni's equation. The procedure is iterated until the error in the reconstruction of each single step satisfies a prescribed tolerance of $\pm 4\%$, equal to an error in the estimation of the water surface elevation of 10^{-2} mm over a reference integration step of 2.5 mm. y_c values for the following iterations are calculated in order to ensure the convergence of the algorithm, which is obtained using the following weighted average:

$$y_p = (1 - w) \cdot y_c + w \cdot y_p \quad [24]$$

where the weight w is calculated as a function of the integration step dx , i.e. $w = 1+dx/20$. The discharge coefficient was evaluated at each integration step of the water surface profile over the side weir (Figure 2), according to Eq. [9, 15, 17, 19], provided that the L/B ratio was between 0.1 and 5 (i.e., the length of the structure is physically acceptable), obtaining a total of 150000 samples.

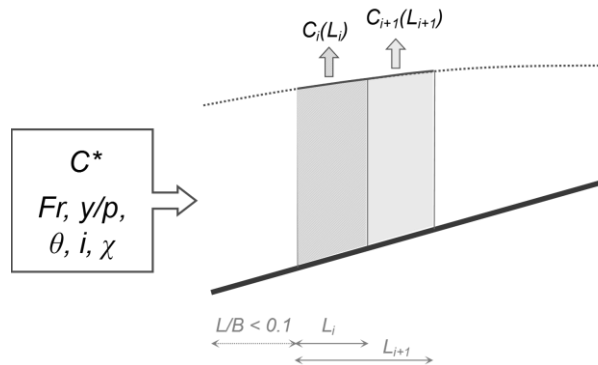


Figure 2. Schematization of the implemented procedure: input and output variables.

2.2 Analysis of the transfer functions

The transfer functions f compensating for the differences in the alternative modelling approaches for the determination of the discharge coefficients ($C_i = C_D, C_s$ and C_t) to the corresponding values of C_M to be used in the De Marchi's differential equation have been identified by studying regression functions of the type $C^* = f(C_i, Fr, \vartheta, i, \chi, y/p, L/B)$, using the synthetic datasets resulting from the numerical runs described in the previous section. To this aim, in this study we implemented a multilayer perceptron, or multilayer feedforward neural network, with two layers of neurons and a sigmoid as activation function, while the Levenberg-Marquardt algorithm (Marquardt, 1963) was used to train the network. The various datasets were divided between training, testing and validation in the proportion 70-15-15, with maximum six validation checks. The minimum dataset was constituted by 38693 elements, while the network size ranged from a minimum of two layers with two neurons each to a maximum of two layers with seven neurons on the first layer and five on the second; therefore, given the large sample size, the risk of overfitting can be excluded. In addition, due to the complexity of the problem, training was repeated several times for the different datasets in order to avoid that the identified solution could be characterized by a local minimum.

The transfer function from C^* to C_M (with C_M evaluated by means of the traditional formula in the non-constant energy runs), which accounts for the error introduced in the model by neglecting the contribution of channel and friction slope, was obviously analyzed by considering only the horizontal cases ($\vartheta = 0^\circ$).

3 RESULTS AND DISCUSSION

3.1 Numerical analysis of the error in the estimation of De Marchi's discharge coefficient with alternative methods

Results for the constant and variable energy simulations are presented respectively in Figure 3 and 4 in terms of the adimensional C_i/C^* ratio as a function of the L/B ratio for downward, horizontal and upward side weirs. The choice of visualizing the results in $(L/B, C_i/C^*)$ plots makes it possible to follow the trend of the

calculated discharge coefficients over the progress of each single run, for a given set of upstream boundary conditions.

Figure 3 (top row) clearly indicates that the adoption of C_1 formula is not a suitable choice, especially for downward configurations ($\vartheta < 0^\circ$), in which overestimation of C^* values (up to more than an order of magnitude) is always observed. This can be explained by considering that the C_1 formula is not able to capture the increase in the pressure head in the flow direction resulting from the concurrent reduction of crest height and increase of water surface elevation when moving from upstream to downstream. Also Wang et al. (2018), who analyzed the discharge coefficients for downward inclined side weirs with a physical and a numerical model, using the total head in the upstream section instead of the pressure head, observed an increase in C_1 values with crest angle, which was justified by them by only hydrodynamic considerations; still, we demonstrated here that the nature of this relationship is essentially geometric, given that the used expression of the discharge coefficient is not influenced by the characteristics of the water surface profile. In addition, also the relationship of C_1 with y/p reported in Wang et al. (2018) seems likely to be spurious, because, for a given Fr value, a greater y/p implies a higher C_1 , due to the larger wetted area at an elevation higher than that of the crest in the upstream section (i.e. p_1). Differently, for upward configurations ($\vartheta > 0^\circ$), the C_1 equation does not always lead to an overestimation of the discharge coefficient, given that, while water elevation necessarily increases in the flow direction, the same cannot be said for the pressure head on the weir crest. However, also in this case, the results are not very satisfactory, as the error between C_1 and C^* is found to be greater than 50% in some conditions.

On the contrary, C_D and C_S equations provide much better estimates, especially for downward configurations ($\vartheta < 0^\circ$), with generally positive errors up to 15%, as shown in Figure 3 (central and bottom rows). This more frequent overestimation is due to the concavity of the water surface profile, which is usually concave down. However, when considering the effect of friction and channel slope (Figure 4), it is possible to find C_D/C^* or C_S/C^* values much smaller than 1 (up to ~ 0.85), which can be explained by observing that, for subcritical flows and for a given value of the discharge, a reduction in the energy level is associated with a shallower water depth. This is also the reason that justifies the change in the concavity of some profiles for small L/B values, given that the large part of the energy losses generally occurs in the first part of the lateral weir; such effect is amplified by the increase in the diverted flow rate occurring along the structure, as a consequence of the reduction of crest height (more evident for those cases characterized by smaller y/p values in the extreme downstream section of the weir).

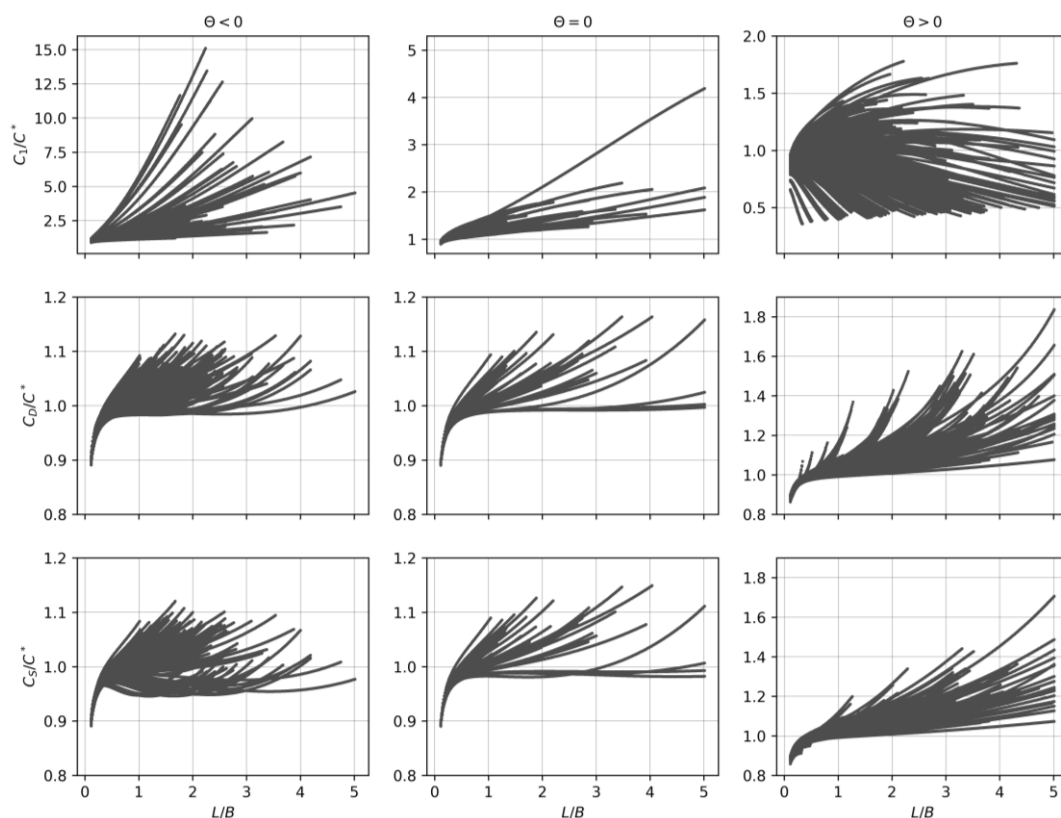


Figure 3. Analysis of the error in the estimation of De Marchi's discharge coefficient with alternative methods (constant energy simulations): downward (left), horizontal (central) and upward (right) configurations.

For upward inclinations ($\vartheta > 0^\circ$), C_D and C_S formulas almost always tend to overestimate reference C^* values, more markedly for longer weirs, as a consequence of the larger concavity of the profile, which becomes more pronounced in the final part of the structure, due to reduction of the derived flow rate.

Therefore, even for upward configurations, the use of C_D and C_S expressions provide more reliable estimates of the discharge coefficient, even though they introduce a spurious relationship with the parameter L/B . In addition, Figure 4 indicates that the inclusion in the model of friction and channel slope, which limit the increase of water depth in the initial part of the weir, has a slight mitigation effect of the estimation error.

For completeness, Figures 3 and 4 also report the results for the horizontal case ($\vartheta = 0^\circ$). It is interesting to note that C_1 formula, although usually applied in the literature for assessing the De Marchi's discharge coefficient in horizontal weirs (Bagheri et al., 2014a; Emiroglu et al., 2011), provides high overestimations of the reference C^* values, with an almost linear dependence on the L/B parameter caused by the increasing water surface elevation in the flow direction, which can partly explain the results reported by Emiroglu et al. (2011), who instead attributed this behavior to the enhancement of the weir's efficiency with its length. Similarly to inclined configurations, the C_S and C_D equations produce more accurate results, with overestimations around 15%. Once more, it can be noted that, for a given set of upstream conditions (i.e. following the evolution of the single run), the discharge coefficient increases with L/B , thus indicating again a spurious dependence on this parameter, which physically explains the results found in Emiroglu et al. (2016). The simulations for the horizontal case with constant energy (not shown) using Eq. [9], provide C_M values essentially equal to C^* , except for an essentially numerical error smaller than 0.1%: this is obvious as the solved system of equations [21, 22] is exactly the one at the base of De Marchi's model. On the contrary, as shown in Figure 5, large errors (up to a factor around 2) are found when analyzing the simulations that consider the effect of friction and channel slope; in particular, it can be observed that a reduction in the energy level leads to an underestimation of the reference C^* values, which can lead also (non-physical) to negative C_M values, if the energy reduction is very significant.

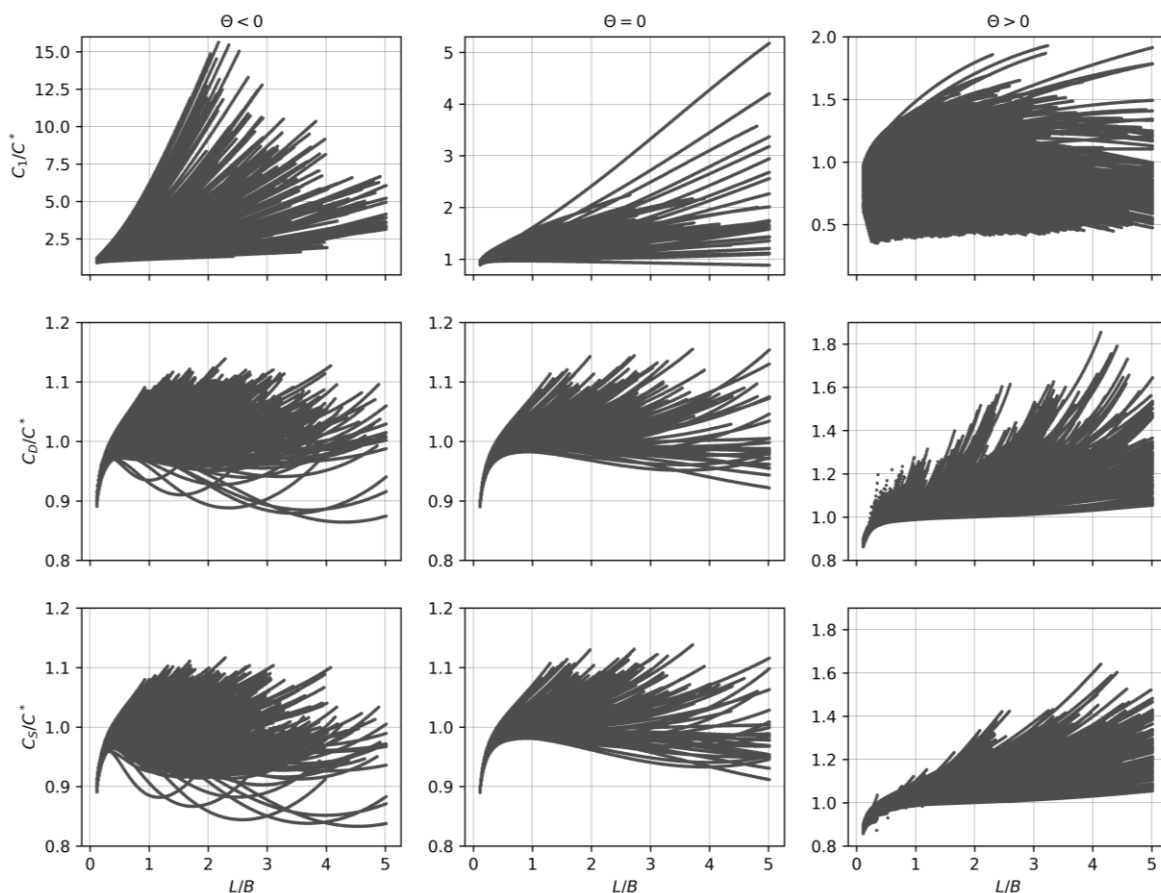


Figure 4. Analysis of the error in the estimation of De Marchi's discharge coefficient with alternative methods (non-constant energy simulations): downward (left), horizontal (center) and upward (right) configurations.

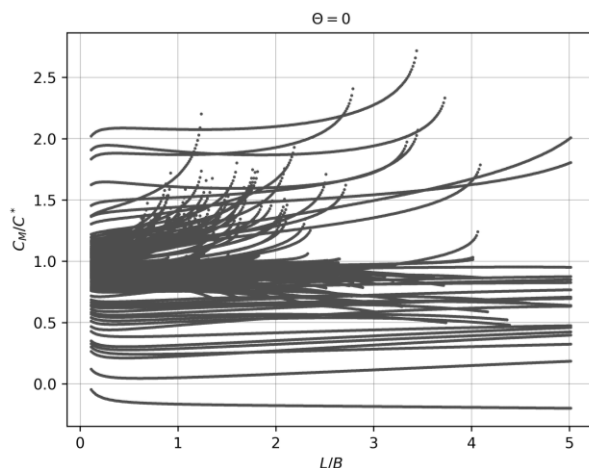


Figure 5. Effect of the non-constant energy hypothesis on the assessment of De Marchi's discharge coefficient in horizontal weirs.

Therefore, these results indicate that, for horizontal side weirs, C_M is more sensitive to energy variations along the structure than C_D and C_s , while it is an accurate estimator of the discharge coefficient when these changes are negligible, as also well known in the literature (De Marchi, 1934).

3.2 Analysis of the transfer functions

Table 1 summarizes the results of the different analyses carried out with the identified transfer functions to convert the C_i coefficients into the corresponding C_M values to be used in the De Marchi's equation for lateral weirs.

Table 1. Results of the analysis of the transfer functions.

Neurons [1 st layer]	Neurons [2 nd layer]	C_i	Parameters						R^2	$\sigma(R^2)$
			Fr	y/p	L/B	ϑ	i	χ		
2	2	C_D	x	x	x	x		x	0.973	0.003
			x	x	x	x			0.975	0.003
			x	x	x	x	x	x	0.976	0.003
			x	x	x	x	x		0.976	0.002
				x	x	x	x		0.972	0.005
				x	x	x	x	x	0.972	0.005
2	2	C_s		x	x	x		x	0.978	0.010
				x	x	x	x		0.975	0.009
			x	x	x	x	x	x	0.977	0.009
			x	x	x	x	x		0.978	0.010
				x	x	x		x	0.972	0.010
				x	x	x	x	x	0.972	0.009
7	5	C_1		x	x	x		x	0.931	0.006
				x	x	x	x		0.915	0.005
			x	x	x	x	x	x	0.962	0.007
			x	x	x	x	x		0.942	0.006
				x	x	x		x	0.856	0.003
				x	x	x	x	x	0.884	0.008
7	5	C_M		x	x	x		x	0.878	0.012
				x	x	x	x		0.731	0.116
			x	x	x	x	x	x	0.987	0.008
			x	x	x	x	x		0.919	0.006
				x	x	x		x	0.826	0.055
				x	x	x	x	x	0.967	0.008
	x	x	x	x	x	0.874	0.020			

In detail, the table reports the size of the network (first two columns), the formula for the discharge coefficient (C_i) used for training, the number of considered parameters (the symbol x indicates whether a certain parameter has been included in the derivation of the transfer function), the average coefficient of determination (R^2) between C_i and C^* and the variance of R^2 , calculated over a minimum of four different trainings of the network, in order to avoid the problem of finding a solution characterized by a local minimum.

Table 1 indicates that with the use of C_D and C_S expressions is possible to obtain satisfactory results ($R^2 > 0.97$) even with a small network (two layers with two neurons), while for C_1 and C_M it is necessary to increase the size (i.e., seven and five neurons in the first and second layer, respectively) in order to obtain acceptable R^2 values. In addition, the networks trained with C_D and C_S , in spite of their smaller size, provide more accurate results, characterized also by a small variance, which is indicative of the existence of a simple functional relationship. Interestingly, for C_D and C_S the R^2 values are constantly high even when the number of the input parameters is decreased (i.e. excluding Fr , i and χ or a combination of them), while with the C_1 coefficient it is necessary to use a much more complex network, by including in the analysis all the selected parameter, in order to get a comparable accuracy.

The performances of the networks trained with C_M demonstrate instead that channel and friction slope parameters are very important, more than Froude number, for the accuracy of the transfer functions: indeed, R^2 values range from 0.987, calculated for the networks trained with all the parameters, to 0.967, when excluding only Fr , up to 0.919 and 0.878, when excluding χ or i , respectively; in particular, Table 1 also shows an increase in the variability of the results, highlighted by larger values of the standard deviation of R^2 , when channel slope is not considered among the input parameters.

4 CONCLUSIONS

This study originated from the impossibility to derive an explicit equation for the assessment of the discharge coefficient to be used in the De Marchi's model for the design of inclined side weirs (i.e., the definition of the length of the structure to reduce the incoming flow Q_1 to a target Q_2 value) and analyzed the feasibility of using other approaches, by quantifying the expected error that these can introduce in the estimates.

The results showed that the use of Dominguez's or Schmidt's approaches (C_D and C_S) can be an effective solution, especially for downward inclined side weirs (maximum errors up to 15%), while important differences (errors up to more than one order of magnitude) have been found for all the tested geometric configurations with the use of the C_1 formula, which in many studies in the literature is erroneously claimed to be the "De Marchi's formula". Instead, for upward configurations, it was found that also C_D and C_S generally provide a not negligible overestimation of the discharge coefficient, especially for long lateral weirs, with errors up to 60-80% (for $L/B > 4$).

Based on these results, multilayer perceptron networks were implemented to identify the functions for transforming the coefficients evaluated with the different methods into the values to be used in the extended De Marchi's model, for any weir's crest angle. The sensitivity analysis of the resulting transfer functions from C_D or C_S to the reference C^* coefficient indicated the existence a simple functional relationship, which is also not particularly sensitive to the number of input parameters considered in the training of the network. Instead, the results concerning the transfer function from C_M to C^* (i.e., the function that evaluates the influence of De Marchi's assumption of energy conservation over the weir length) highlighted the importance of including channel and friction slopes for a reliable estimation of the discharge coefficient; however, it is important to stress that the results of this last point on C_M refer only to the influence of the constant-energy hypothesis, while a further experimental analysis would be required for a global assessment of the overall De Marchi's model.

REFERENCES

- Aghayari, F., Honar, T., and Keshavarzi, A. (2009). A study of spatial variation of discharge coefficient in broad-crested inclined side weirs. *Irrigation and Drainage*, 58(2), 246-254.
- Bagheri, S., Kabiri-Samani, A., and Heidarpour, M. (2014a). Discharge coefficient of rectangular sharp-crested side weirs - part I: Traditional weir equation. *Flow Measurement and Instrumentation*, 35, 109–115.
- Bagheri, S., Kabiri-Samani, A. and Heidarpour, M. (2014b). Discharge coefficient of rectangular sharp-crested side weirs - part II: Domínguez's method. *Flow Measurement and Instrumentation*, 35, 116–121.

- Coleman, G.S. and Smith, D. (1917). The discharging capacity of side weirs, Institution of Civil Engineers, *Selected Engineering Papers*, n.6, London, UK.
- De Marchi, G. (1934). Saggio di teoria del funzionamento degli stramazzi laterali. *L'Energia Elettrica*, 11, 849-860 (in Italian).
- Dominguez F.J. (1935) *Hidráulica*. Nascimento Editor, Santiago, Chile (in Spanish).
- Emiroglu, M.E., Agaccioglu, H., and Kaya, N. (2011). Discharging capacity of rectangular side weirs in straight open channels. *Flow Measurement and Instrumentation*, 22(4), 319–330.
- Emiroglu, M. and Ikinogullari, E. (2016). Determination of discharge capacity of rectangular side weirs using Schmidt approach. *Flow Measurement and Instrumentation*, 50, 158–168.
- Engels, H. (1917). *Forschungsarbeiten auf dem gebiete des ingenieurwesens*. Vol. 200-201, Berlin, Germany, p.55 (in German).
- Ghodsian, M. (2003). Supercritical flow over a rectangular side weir. *Canadian Journal of Civil Engineering*, 30(3), 596–600.
- Hager, W. (1987). Lateral outflow over side weirs. *Journal of Hydraulic Engineering*, 113(4), 491-504.
- Marquardt, D.W. (1963). An algorithm for least-squares estimation of nonlinear parameters. *Journal of the society for Industrial and Applied Mathematics*, 11(2), 431-441.
- Poleni, G. (1717) *De motu aquae mixto libri duo. Quibus multa nova pertinentia ad aestuaria, ad portus, atque ad flumina continentur*, Padua, Italy (in Latin).
- Schmidt, M. (1954). *Zur frage des abflusses uber streichwehre: eine kritische betrachtung der bekanntesten berechnungsverfahren und versuche im zusammenhang mit eigenen versuchen*. Technische Universitat Charlottenburg, Germany (in German).
- Shariq, A., Hussain, A., and Ansari, M.A. (2018). Lateral flow through the sharp crested side rectangular weirs in open channels. *Flow Measurement and Instrumentation*, 59, 8-17.
- Singh, R., Manivannan, D., and Satyanarayana, T. (1994). Discharge coefficient of rectangular side weirs. *Journal of Irrigation and Drainage Engineering*, 120(4), 814–819.
- Velatta, M. (1934). Contributo sperimentale allo studio degli sfioratori laterali. *L'Energia Elettrica*, 7(11) (in Italian).
- Wang, Y., Wang, W., Hu, X., and Liu, F. (2018). Experimental and numerical research on trapezoidal sharp-crested side weirs. *Flow Measurement and Instrumentation*, 64, 83-89.