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Fractal dynamics analysis of the VHF radiation pulses during initial breakdown process of lightning

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[1] This paper presents the first analysis of fractal dynamics of lightning initiation process with a coherent approach. Using experimental data obtained from a broadband interferometer system operating at a bandwidth of 25-100 MHz, the sequences of inter-pulse time interval of the VHF radiation pulses emitted during the lightning initial breakdown process were analyzed with the multifractal detrended fluctuation analysis (MF-DFA) method. For 63 lightning flashes analyzed, it was found that there is an apparent long-range time correlated and nonlinear cascade behavior within the initiation breakdown process of the lightning. This result suggests that the initiation of lightning may be associated with progressively building up of correlated strong electrical field regions by self-similar scaling up (inverse cascading) of discharges across scales from small to large. The result may be promising toward our understanding of lightning initiation. **Citation:** Gou, X., M. Chen, Y. Du, and W. Dong (2010), Fractal dynamics analysis of the VHF radiation pulses during initial breakdown process of lightning, *Geophys. Res. Lett.*, *37*, L11808, doi:10.1029/2010GL043178.

1. Introduction

[2] Comparing with a laboratory long spark discharge, a unique and puzzling process about the lightning is its initiation. As the apparent discrepancy between the observed electric field in thunderclouds and the expected conventional breakdown field, most discussions on the lightning initiation mechanism, such as the hydrometeor-initiated positive streamer and the cosmic ray-initiated runaway breakdown (see the review by *Petersen et al.* [2008, and references therein]), focus on the local electric field intensification in thunderclouds. However even if such local electric fields already exist, they are unlikely to transform necessarily and directly into a lightning leader system. They may only provide the "lightning seed", something like a laboratory scale (a few to about hundred meters) spark discharge.

[3] Experimental observation of lightning initiation is characterized with the burst of numerous VHF radiation pulses, which is generated primarily by negative breakdown processes [*Mazur*, 2002]. Interpretation of the VHF radiation map source is usually associated with bidirectional dynamics of streamer/leader [*Mazur*, 2002; *Williams*, 2006]. The absence of the VHF pulses for positive breakdown arises from the order-of-magnitude polarity asymmetry in the two ends of the streamer/leader [*Thomas et al.*, 2001; *Williams*, 2006]. The initiation of lightning seems to be more associated with the intermittent type breakdown, which may be associated with the successive screening of competitive growing branches of evolutionary lightning tree [*Mazur*, 2002] and has apparently fractal characteristic [*Riousset et al.*, 2007]. In our opinion, such burst and fractal behavior of the VHF pulses, which may be the key to the understanding of lightning initiation, could be well-explored using the self-organized and critical model for catastrophic event, which is widely accepted in geophysical events, such as earthquake, landslides, forest fire and flood [*Sornette*, 2002; *Turcotte et al.*, 2002].

[4] In fact, as far as a nonlinear dynamical system goes, the existence of critical points is more the rule than the exception [*Sornette*, 2002]. The fractal and burst behavior may be the two sides of the same coin, manifesting the self-organized evolution to the criticality [*Bak et al.*, 1987; *Turcotte et al.*, 2002; *Sornette*, 2002]. On the critical point view, a large-scale discharge is possible only in a strong electrical field region correlated to sufficient length. Small breakdown events may occur randomly in short-correlated regions, however, they become cooperative with the increase of the correlated length and associated susceptibility. Once triggered, they can jump barriers by formatting the event chains (like domino effect) and cascade into a large-scale discharge. The small breakdown events serve as the agent by which long correlations are built up progressively in a self-similar inverse cascaded way. The inverse cascade behavior can be illustrated by the forest-fire model, in which individual trees are planted and small clusters expand and coalesce to form larger clusters, and so on. At the same time, the clusters of all size are lost due to forest fires, giving the fractal behavior of clusters [*Turcotte et al.*, 2002]. Similar ideas were suggested by *Iudin et al.* [2003] and *Hayakawa et al.* [2008] also, they thought that the spreading of small-scale breakdowns accompanied by breakdown cluster length growth can lead to the stepped leader formation at the final stage. Such a process is typical for percolation fractals with self-organized criticality behavior.

[5] The above arguments suggest that study of the intermittent VHF pulses and their scaling behavior is promising in understanding the underlying process of lightning initiation. In this paper, by using multifractal detrended fluctuation analysis (MF-DFA) methods [*Kantelhardt et al.*, 2002], the correlation and cascade properties of the inter VHF pulses time interval (IVPTI) sequence of the lightning VHF

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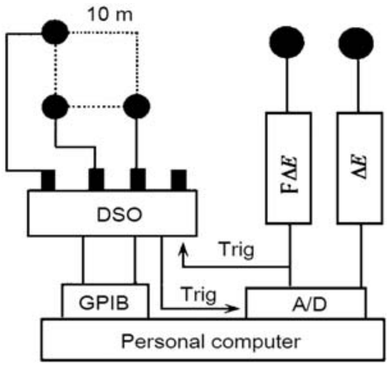

Figure 1. System configuration of the broad band interferometer.

pulses emitted during the initial breakdown process were investigated.

2. MF-DFA Method

[6] The nonlinear and cascade process is usually associated with multifractality [*Davis et al.*, 1994], the idea of which is to classify the singularities of a fractal signal by their degree of effect on the global structure. One of a widely used method of quantifying multifractal properties of time series is MF-DFA, which involves following four steps.

[7] Step 1: For a given time series *x(i), i = 1,…,N,* determine the "profile"

$$Y(k) = \sum_{i=1}^{k} (x(i) - <x>), \tag{1}$$

where $k = 1,\ldots,N$, and $\langle\ \rangle$ denotes the mean of *x(i)* for $i = 1,\ldots, N$

[8] Step 2: Divide *Y(k)* into $M_s = int(N/s)$ non-overlapping segments of length *s* starting from both the beginning and the end of the time series (i.e., $2M_s$ such segments in total); For each segment $Y_v(k)$, $(v = 1,\ldots,2Ms$ and $k = 1,\ldots,s)$, calculate the root mean square fluctuation function *F(v,s)*

$$F(\nu, s) = \sqrt{\frac{1}{s}\sum_{k=1}^{s}(Y_\nu(k) - Y_{v,n}(k))^2}, \tag{2}$$

where $Y_{v,n}$ *(k)* is the polynomial fitting value of the *v-th* segment $Y_v(k)$.

[9] Step 3: Calculate the *q-th* order fluctuation function

$$F_q(s) = (\frac{1}{2M_s}\sum_{\nu=1}^{2M_s}|F(\nu, s)|^q)^{1/q}. \tag{3}$$

[10] Step 4: Determine the scaling behavior of the *q-th* order fluctuation

$$F_q(s) \sim s^{h(q)}, \tag{4}$$

where *h(q)* is called generalized Hurst exponents. For monofractal time series, *h(q)* is independent of *q*, whereas for multifractal time series, *h(q)* varies with *q*. Specifically, *h(2)* is assumed to be identical to the Hurst exponent *H*, which quantifies the strength of the long-range correlations of the signal. $H > 0.5$ indicates the presence of persistent long-range correlations, meaning that a large (compared to the average) value is more likely to be followed by a large value and vice versa. On the contrary, $H < 0.5$ indicates the presence of anti-persistent correlations.

[11] In comparison with the traditional multifractal method, *h(q)* is directly related to Renyi exponents $\tau(q)$ as the relation $\tau(q) + 1 = qh(q)$. The singularity spectrum $f(\alpha)$ is related to $\tau(q)$ as

$$\begin{cases} \alpha(q) = \frac{d\tau(q)}{dq} \\ f(\alpha) = q\alpha(q) - \tau(q) \end{cases}, \tag{5}$$

where α is called Hölder exponent. The strength of multifractality of time series can be quantified by width of the singularity spectrum $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$, the wider the exponent is, the stronger the multifractality.

3. Data Acquisition

[12] The data used were acquired in a comprehensive lightning observation campaign conducted in Tibet Plateau of China in the summer of 2002 with a broadband interferometer system. The system consists of three antennas equipped at the apexes of a right triangle with two 10 m sides (see Figure 1). The antenna was a flat-plate type that can measure the vertical electrical field in a broad band of 25 MHz to 100 MHz. The broadband pulses were digitized at a sample rate of 1GS/s and 8-bit resolution by a 4-channel Digital Storage Oscilloscope (DSO). The memory of each channel was divided into 2000 segments maximum and each segment can record a broadband EM pulse for 1.0μs. The minimum time between two consecutive radiation sources is about 80 μs due to instrumentally dead time. Along with broadband interferometer, a fast field change antenna with a time constant of 2 ms and a slow field change antenna with a time constant of 6 s were also equipped to measure E-field changes (more detail of the system, see the introduction by *Dong et al.* [2001]). The occurring time of VHF pulse sequences can be considered as a temporal point process t_i for $i = 1,\ldots, N+1$. The method in Section 2 was applied to the inter VHF pulse time interval (IVPTI) sequences $X_i = t_{i+1} - t_i$ for $i = 1,\ldots, N$. Figure 2 shows an example of the IVPTI

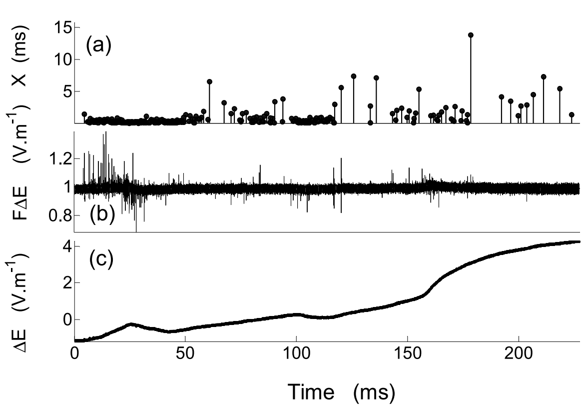

Figure 2. (a) The IVPTI sequences (X), (b) the fast (FΔE) and (c) the slow (ΔE) electric field changes of an intracloud flash (No. nq080132).

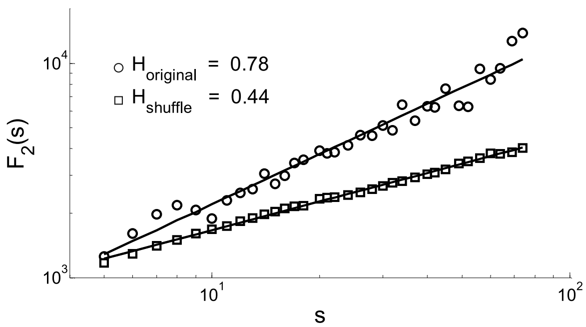

Figure 3. Time correlation analysis for the IVPTI sequence shown in Figure 2a (circles) and that of its shuffled data (squares). The thick solid lines are the linear least square fitting on their log-log plots.

sequences (Figure 2a), fast antenna (FΔE (Figure 2b)) and slow antenna (ΔE (Figure 2c)) signals, respectively, for a intra-cloud flash observed.

[13] During the campaign more than one hundred lightning discharges were captured. Among them, 19 cloud-to-ground (CG) and 44 intra-cloud (IC) lightning discharges whose distances were about 4 – 6 km to the observation site were chosen for analyses. In the analyses, focuses were given on the scaling behavior of the IVPTI sequences during the initiation breakdown process for both IC and CG flashes.

4. Results

[14] For the analysis of the temporal correlation behavior of the IVPTI sequences, a surrogate data test [*Theiler et al.*, 1992] was performed to evaluate the statistical significance of the results for long-range correlations. In the analysis, the shuffled version of surrogate data (marked as "shuffle" in Figure 3) was used to destroy long-range correlation by randomly reordering the original data in the time space. For each IVPTI sequence for a flash, a set of 100 shuffled series were generated. Then, the scaling exponents for both the original data and the 100 sets of shuffled data were calculated with the DFA method, and were compared. Let H being the scaling exponent of the original sequence, and H_{shuf}, Δ_{shuf} the mean and the standard deviation of the scaling exponents for the shuffled sequences respectively, the difference between the scaling exponents before and after the shuffled data could be quantified as $\sigma = |H - H_{shuf}|/\Delta_{shuf}$. A larger σ indicates stronger correlation. The significance level p is calculated as $p = erfc(\sigma/\sqrt{2})$ [*Theiler et al.*, 1992], which results in a confidence level of $1\text{-}p$.

[15] Figure 3 shows the correlation analysis of the signal shown in Figure 2a. The good linearity of the log-log plot of the fluctuation versus scale (the Pearson correlation coefficients are as high as 0.99) and the clear difference between the exponent H and H_{shuf} imply an apparent long-range persisted behavior of the IVPTI sequences.

[16] Figure 4 shows the Hurst exponent and corresponding sigma value σ for all the 63 flashes analyzed. The horizontal dot-line is present $\sigma = 1.96$ (corresponding to confidence level 95%). It was found that all the Pearson correlation coefficients were great than 0.9, and the Hurst exponent ranges from 0.5 to 1.0 with an average of 0.8. For nearly all cases the sigma value $\sigma > 1.96$ with the confidence level

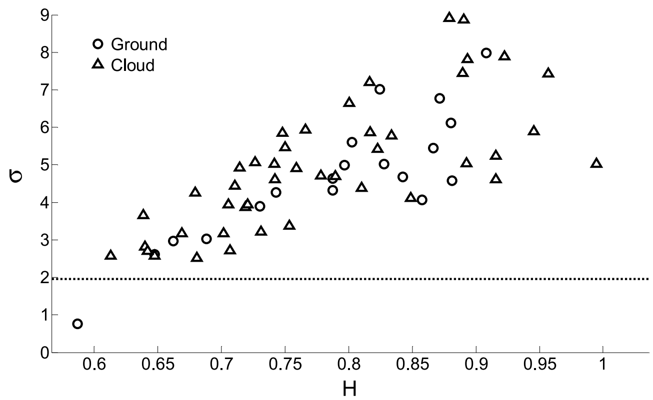

Figure 4. Hurst exponents H versus sigma value σ for 63 flashes analyzed, the horizontal dot-line corresponding to $\sigma = 1.96$ (p = 0.05).

better than 95%, indicating that the long-range persisted behavior exists at least for the IVPTI sequence for the lightning initial breakdown process.

[17] For the multifractal analysis, another type of the surrogate data (marked as "surrogate" in Figure 5) was used to check the nonlinear nature of the signal. The surrogate data have the same Fourier spectrum as the original data, but nonlinearities were destroyed by randomizing the Fourier phase [*Theiler et al.*, 1992]. For the Renyi exponents τ (q), it was fitted with the formula $\tau(q) = -\ln(a^q + b^q)/\ln 2$, which was derived from a modification of the binomial multiplicative cascade multifractal model [*Kantelhardt et al.*, 2002].

[18] Figure 5 shows the result of multifractal analysis of the same IC shown in Figure 2a. The Renyi exponents were determined by fitting in the regime $4 < s < N/4$ for q varying between −5 and 5, where N was the length of the signal. It is showed that the Renyi exponent $\tau(q)$ was curved and the multifractal spectrum $f(\alpha)$ was wide for the original data, while its surrogate data exhibited monofractal behavior. This has confirmed the apparent multifractal and nonlinear nature of IVPTI sequences in lightning breakdown process. The fitting of $\tau(q)$ and obtained parameters a, b were also shown in Figure 5a (solid line). It can be seen that the fitting are surprisingly well in the whole q-range, indicating the possible cascade process in the lightning. The result of multifractal analysis for the 63 lightning flashes is similar. The width of the multifractal spectrum is ranged from 0.7 to

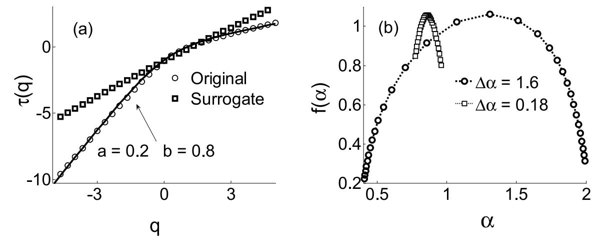

Figure 5. Multifractal analysis of the signal in Figure 2a: (a) Renyi exponent $\tau(q)$ and (b) multifractal spectrum $f(\alpha)$ for the original (circles) and surrogate (squares) data. The fitting curve of $\tau(q)$ (thick solid line in Figure 5a) and obtained parameters a and b are also shown.

2.1 with a mean of 1.4, further implying the nonlinear and cascade nature of the lightning.

5. Discussion and Conclusions

[19] In this paper, the lightning initiation was associated with a self-organized and critical phenomenon by a coherent fractal approach. The lightning initiation is possible only when the strong electric field region is correlated and cascaded up to a sufficient length to produce them.

[20] Using multifractal detrended fluctuation analysis method, the fractal correlation and cascade characteristics of the IVPTI sequences emitted in initiation period of lightning were investigated. For 63 lightning flashes analyzed, the apparent long-range persistence and nonlinear cascade properties in the IVPTI sequences of the lightning initiation were found. The Hurst exponent ranges from 0.6 to 1.0 with an average of 0.8. The multifractal spectrum can be described by modified version of the binomial cascade multifractal model, with the spectrum width ranges from 0.7 to 2.1 with a mean of 1.4.

[21] There are several observed evidence of growing correlated length in the lightning initiation, such as the appearance of the VHF/HF train [e.g., *Proctor*, 1997], the usually delayed recoil breakdown [*Mazur*, 2002; *Saba et al.*, 2008] and the long-wave (LF/VLF) pulse [*Betz et al.*, 2008]. Other observed evidence include the increasing in the frequency of intermediate-magnitude discharge event, the acceleration of the electrical moment release rate during lightning, etc. It is considered that the competition ingredient, short wavelength instabilities of the Mullins-Sekerka type [*Mullins and Sekerka*, 1963] and a strong screening effect provide a general scenario for the spontaneous formation of hierarchy structures of criticality by cascade process [*Huang et al.*, 1997]. We believe that the screening or shielding effect is similar to the mechanism of breakdown event described by *Mazur* [2002] in bidirectional model of lightning. Meanwhile, the so-called Mullins-Sekerka instability is nothing but the "lightning-rod-effect" in electrostatics (the effect of electric field enhancement at the sharp head of a lightning rod); however, detailed consideration of these features is a subject of further investigations. We conclude that the fractal nature of VHF pulses is very promising in reveal the fractal dynamics of lightning initiation. The framework presented in this paper may provide a first step in this direction.

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