

On sound transmission loss across a Helmholtz resonator in a low Mach number flow duct

S. K. Tang

Department of Building Services Engineering, The Hong Kong Polytechnic University, Hong Kong, China

(Received 19 October 2009; revised 31 March 2010; accepted 1 April 2010)

A simplified physical model, which described the generation of sound by a Helmholtz resonator upon flow excitation at its neck, was developed in the present investigation to study the drop in sound power transmission loss across such a resonator mounted on the wall of a duct conveying a low Mach number mean flow. Experiments were derived to validate the model. Mitigation methods derived according to the model were also tested experimentally. Results showed that the simplified model gave predictions which agreed with experimental observations. The proposed mitigation methods were also proved to be effective for building services application. It was also found that the sound intensity generated by the flow excited resonator scaled with approximately the ninth power of the flow velocity inside the duct.

© 2010 Acoustical Society of America. [DOI: 10.1121/1.3409481]

PACS number(s): 43.50.Gf, 43.28.Ra [LCS]

Pages: 3519–3525

I. INTRODUCTION

Ductlike components are very common in engineering for transporting fluids from one place to another. In a modern air conditioned building which is usually heavily serviced, the ventilation ducts are indispensable as they form the basic system for conveying treated fresh air from the air handling units into the interior of the building to maintain good air quality and thermal comfort condition for the building users.¹ However, the noise from the air handling units also propagates along these ducts. Silencing devices are therefore required to keep the indoor noise level suitable for human occupation. The dissipative silencers, which utilize porous materials to dissipate acoustical energy, are widely used worldwide because of their relatively simple configurations and low cost. However, these devices are bulky, inefficient for low frequency noise attenuation, and can result in large static pressure drop.² Reactive devices, such as the Helmholtz resonators³ and plenum chambers,⁴ can be used together with their dissipative counterparts to improve the low frequency attenuation in the ductworks.⁵

The Helmholtz resonator gives high sound power transmission loss within a narrow frequency bandwidth centered at its resonance frequency. Owing to its wide industrial application, there have been many studies on its properties in the past few decades (for instance, Ref. 6). The results of the recent works by Griffin *et al.*⁷ and Seo and Kim⁸ show that a broader attenuation bandwidth can be obtained by coupling resonators together. However, there are research studies indicating that the Helmholtz resonator can be excited by a flow across its mouth to generate sound (for instance, Refs. 9 and 10), and thus the use of the resonator as a noise mitigation measure in a flow duct is questionable. There is also research studying the responses of the resonator to a grazing flow,¹¹ flow separation,¹² and more theoretically to vortices.¹³ The results of Ffowcs Williams¹⁴ suggests the sound scattering property of apertures in the present of a turbulent flow. The work of Howe¹⁵ indicates the perforation

of a plate can result in the movement of vortical eddies toward the plate in the present of a mean flow and such eddy motions are sound producing. Dowling and Hughes¹⁶ showed that the vorticity generated by aperture flows can dissipate sound energy. One can therefore expect complicated aeroacoustical activities occur when the resonator interacts with flow turbulence.

In the building services practice, the dimensions of the Helmholtz resonator may not enable the kind of shear layer/vortex impingements studied in Dequand *et al.*¹² and Graf and Durgin¹⁷ to occur, especially when the mean flow is of very low Mach number and the resonator cavity is small. The theories adopted by Meissner¹¹ and Innes and Crighton¹⁸ assumed a continuous sinusoidal excitation at the mouth of the resonator by shear layer rollup/vortex shedding while the initiation of the oscillating flow was not explicitly shown in their formulations. In this study, an experiment was conducted to confirm the sound radiation characteristics of a Helmholtz resonator in a low Mach number flow duct. A simplified model, which was principally in agreement with experimental observations, was proposed for the sound radiation. Simple passive methods for attenuating the resonator sound generation were derived based on this simplified model and their effectiveness was tested experimentally.

II. RESONATOR EXCITATION BY A MEAN FLOW

It is well recognized that for a Helmholtz resonator of small aperture with damping, the equation of motion of the air mass within its neck is¹⁹

$$m\ddot{x} + R\dot{x} + Ap_{\text{in}} = Ap_{\text{out}}, \quad (1)$$

where x denotes the air mass motion into the resonator cavity, m the effective mass of the air mass, R the damping resistance, A the cross section area of the mouth, p_{in} and p_{out} the air pressures at the inner and outer sides of the air mass respectively, and the “.” represents time differentiation. A

mean flow of velocity U is introduced abruptly into the duct at time $t=t_o$ while the fluid inside the resonator cavity remains largely stagnant. The static pressure inside the duct is reduced resulting in the motion of the air mass in the resonator neck. Assuming such movement of the air mass is very small, the reduction of cavity volume is Ax and the equation of state then suggests (employing the binominal series expansion):

$$p_{\text{in}} = P \left(\frac{V}{V - Ax} \right)^\gamma = P \left(1 + \sum_{j=1}^{\infty} (Ax/V)^j \right)^\gamma \approx P \left(1 + \frac{A}{V} \gamma x \right) = P + \frac{\rho_o c^2 A}{V} x, \quad (2)$$

where c is the speed of sound, γ the specific heat ratio, V the resonator cavity volume, and ρ_o and P is the air density and stagnant air pressure inside the cavity, respectively (equals those inside the duct) before the introduction of the flow, and thus

$$p_{\text{in}} - p_{\text{out}} \sim \frac{\rho_o U^2}{2} H(t - t_o) + \frac{\rho_o c^2 A}{V} x, \quad (3)$$

where H is the Heaviside unit step function as the flow is introduced at t_o . Without loss of generality, t_o is set to zero in the foregoing analysis. In the absence of any vortex rollup or impingement and under weak turbulence condition, Eq. (1) becomes

$$m\ddot{x} + R\dot{x} + \frac{\rho_o c^2 A^2}{V} x = - \frac{\rho_o U^2 A}{2} H(t). \quad (4)$$

Equation (4) can be solved by using Fourier transformation and its inverse with the initial condition of vanishing x and \dot{x} at $t=0$. The solution for $t \geq 0$ is

$$x = - \frac{\rho_o U^2 A}{m\omega_o^2} + \frac{\rho_o U^2 A}{2m\omega_d\omega_o} e^{R/2mt} \cos \left[\omega_d t + \tan^{-1} \left(\frac{R}{2m\omega_d} \right) \right], \quad (5)$$

where $\omega_o = 2\pi f_o = \sqrt{\rho_o c^2 A^2 / (mV)}$ and $\omega_d = 2\pi f_d = \omega_o \sqrt{1 - [R/(2m\omega_o)]^2}$. f_o and f_d are the undamped and damped resonance frequencies, respectively, of the resonator. Equation (5) suggests that an oscillating flow within the resonator neck will be induced by the introduction of a mean flow with velocity, U . Sound at the damped resonance frequency of the resonator will then be generated, deteriorating the sound power transmission loss of the resonator. One should note that the factor R/m is usually very small and thus the time decay of x will be extremely slow. Equation (5) also illustrates that sound can be generated by the resonator in the absence of vortex shedding, which has been taken to be the driving excitation in the study of Meissner¹¹ and Innes and Crighton.¹⁸

Since damping is usually very small for a duct resonator, the oscillating flow induced at the mouth of the resonator, when U is sufficiently large, will have a velocity magnitude ($|\dot{x}|$) proportional to U^2 [Eq. (5)]. It is well known that the sound power generated by a low Mach number two-dimensional jet varies with the fifth power of its velocity.²⁰

Therefore, from Eq. (5), one can expect that the sound power produced by the “mean flow excited” resonator will vary with the tenth power of U .

In practice, the static pressure differential required to drive a flow through an aperture is proportional to the square of the flow velocity across the aperture, which can be factored into the air mass vibration equation as a force opposing the fluid movement. The following type of nonlinear equation should thus be more realistic for the above model:

$$m\ddot{x} + C|\dot{x}|\dot{x} + \frac{\rho_o c^2 A^2}{V} x = - \frac{\rho_o U^2 A}{2} H(t), \quad (6)$$

where C can be regarded as a positive constant. Equation (6) is basically a vibration equation with nonlinear air damping.²¹ Though Eq. (6) is not amenable to analytical solution, it still suggests the radiation of sound at the resonance frequency f_o for very small C . Assuming $|x| \sim U^\alpha$, $|\dot{x}| \sim U^\alpha$ as the frequency of x does not depend on U . One can observe that from Eq. (6) that $|\dot{x}|^2 \sim U^2$ when the damping effect becomes dominant as the acceleration and displacement terms in Eq. (6) will then be insignificant, and thus $\alpha \rightarrow 1$. For weak damping ($C \rightarrow 0$) which is the usual case in practice, α will be close to 2 [Eq. (5)]. This will be discussed further later together with the experimental results.

One should note that the intention of the present simplified theoretical consideration is to help develop methods which can attenuate the sound generated by the excited resonator. Equations (5) and (6) are not exact as the aeroacoustics and the possibility of feedback by the oscillating jet vortices²² have not been taken into account. However, there are many models in the existing literature, such as that of Nelson *et al.*,⁹ Innes and Crighton,¹⁸ and Mast and Pierce,²³ which adopt different dynamic equations to deal with the oscillating neck flow. A unified model for the oscillating resonator neck flow does not exist at least to the knowledge of the author. It is believed that the mechanism described by Eq. (6) initiates the oscillating neck flow, which is sustained/enforced later by the oscillating jet vortices through feedback while the system damping is expected to be weak. It should be noted that the strengths of these shed oscillating jet vortices should be proportional to the jet velocity.²⁴ Attenuating the initial neck flow will thus result in weaker sound radiation by the resonator.

The oscillating flow inside the neck results in vortex shedding,²² which can reduce the aerodynamic pressure on the side of the neck lip where the vortices are shed and thus enhancing the flow along the neck. Equation (6) can be modified to take care of such feedback from vortex excitation:

$$m\ddot{x} + C|\dot{x}|\dot{x} + \frac{\rho_o c^2 A^2}{V} x = - \frac{\rho_o U^2 A}{2} H(t) + k\dot{x}, \quad (7)$$

where k is a factor describing the pressure drop due to the induced velocities of the vortices which are expected to have strength proportional to the oscillating neck flow velocity under the ducted condition. Though Eq. (7) is probably a premature form of the interaction, it is closer to the one adopted by Mast and Pierce.²³ Since it is not the objective of

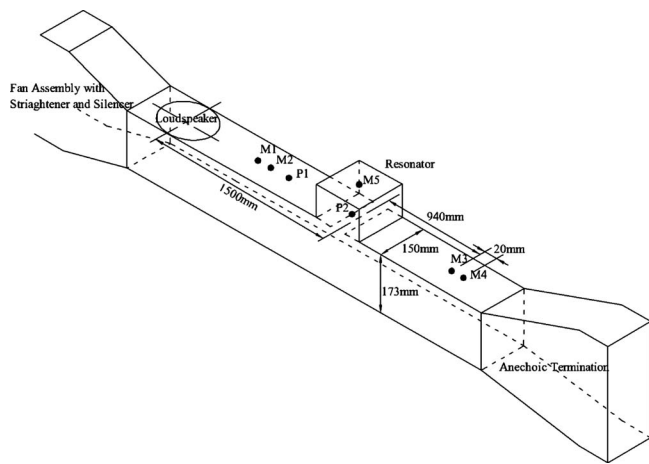


FIG. 1. The experimental setup (not drawn to scale) M: microphone locations; P: pressure transducer locations.

the present study to explore the exact solution of the resonator flow, Eq. (7) will be left to further investigation.

The result of the above theoretical consideration tends to suggest that the sound radiated by the resonator can be weakened if the static pressure differential across the resonator neck created by the abrupt introduction of the mean flow can be reduced. One can also expect that the magnitude of the neck flow oscillation can be lowered when the aerodynamic flow resistance at the resonator neck is increased.

III. EXPERIMENTAL SETUP

The experimental test rig used in the present study consisted of a fan, a flow straightening section, a converging section, a test section where the resonator was located, and an anechoic termination. Figure 1 shows the schematic of the test setup. The horizontal width of the main duct was 150 mm. The vertical width of the duct, d , was 173 mm, such that the first cut-off frequency of the duct test section was around 990 Hz (for a speed of sound of 343 m/s). The flow velocity in the present study was kept below 20 m/s to avoid vibration. This is also the common range of the flow magnitude adopted in building services engineering for ventilation duct design. The duct in the present study was made of 20 mm thick Perspex board with coincidence frequency well outside the frequency range of the present study. The duct walls could be assumed rigid to both acoustics and the flow.

The four microphone method for measuring sound power transmission loss was adopted. Details of this method can be found in the existing literature, such as Davies²⁵ and Tang and Li,²⁶ and thus are not repeated here. The Brüel & Kjær Type 4935 1/4 in. microphones were used in the study. The locations of these microphones (M1–M4), which were mounted flushed with the duct walls, were more than two duct widths away from the test section where the effect of the evanescent waves should be weak. M1 and M2 were separated by a distance of 20 mm as in Tang and Li²⁶ and the same applied to M3 and M4. Pressure transducers (Endevco Model 8507C-2 with amplifier model 136) were located at special locations (P1–P2) for capturing the pressure fluctuations at the duct wall and at the internal edge of the resonator

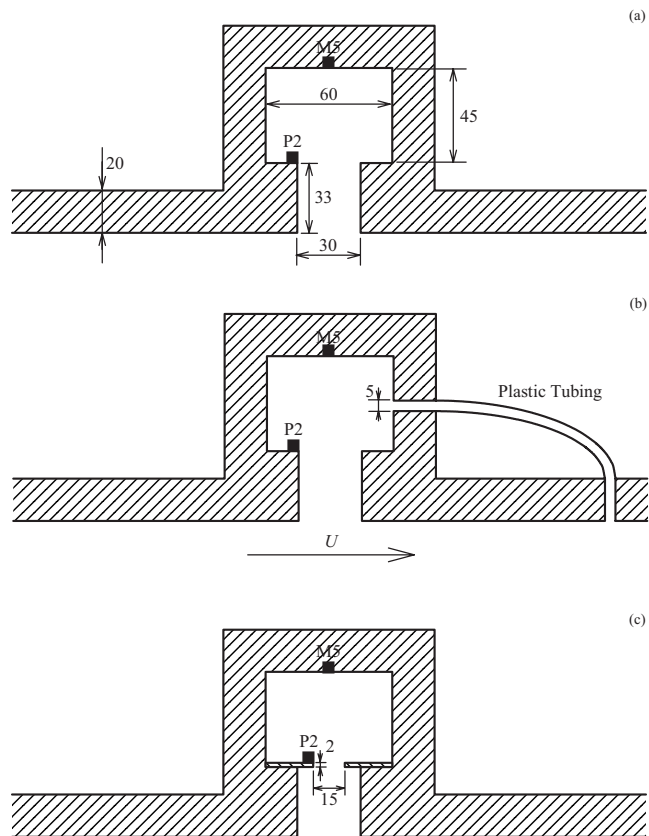


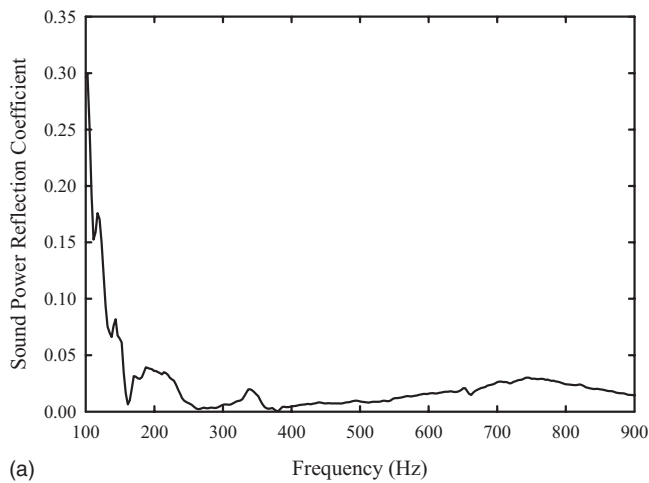
FIG. 2. Details of resonators used (flow from left to right, all dimensions in mm). (a) R0; (b) R1; (c) R2.

neck respectively during the tests. The microphone and pressure transducer signals were recorded by a SONY DAT recorder with a sampling rate of 12 000 samples/s per channel for later calculations.

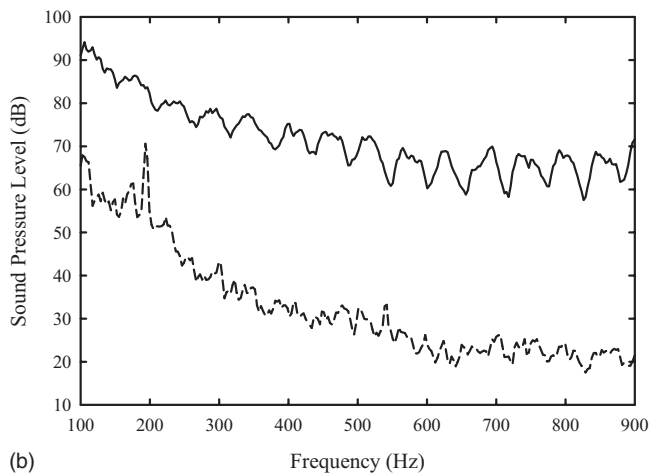
The resonator cross-sectional dimensions adopted in the present study are given in Fig. 2. The base resonator details are presented in Fig. 2(a). The aperture of the resonator was a slot of span equal to the spanwise width of the duct (and also the resonator), which was 150 mm. The width of the aperture opening was 30 mm and the neck length was 33 mm.

Two modifications to the base resonator, shown in Figs. 2(b) and 2(c), were made according to the abovementioned theoretical deduction in an attempt to improve the sound power transmission loss performance of the resonator in the presence of the mean flow. The first one was so designed to reduce the static pressure difference between the resonator cavity and the main duct [Fig. 2(b)]. A 5 mm diameter thin plastic tube connecting the cavity to the duct wall was used for the purpose. The other design increased the flow resistance at the neck of the resonator by adding two 2 mm thick rigid slabs at the entry of the neck (on the cavity side) to reduce the neck opening by 50% and at the same time to form an abrupt constriction [Fig. 2(c)]. For the sake of easy reference, the original resonator without modification (base resonator) is denoted by “R0,” the first modified version by “R1,” and the second by “R2” in the following discussions.

The anechoic termination was manufactured according to the design of Neise *et al.*²⁷ Figure 3(a) illustrates the



(a)



(b)

FIG. 3. Basic properties of test rig (no-flow case). (a) Performance of the anechoic termination; (b) acoustic forcing strength and background noise inside duct. —: Loudspeaker sound level; - - - -: background noise level.

sound power reflection coefficient of the anechoic ending adopted. The reflected power was less than 4% for frequencies between 200 and 900 Hz. This implies that the reflected sound powers at these frequencies were more than 14 dB below those created by the loudspeaker during the tests.

The upstream loudspeaker provided a strong acoustic forcing of constant magnitude throughout the experiments such that the noise from the fan and the background noise could not affect the measurement accuracy. This is also the general practice for sound power transmission loss measurement.²⁶ Figure 3(b) illustrates that the sound pressure levels created by the loudspeaker were at least 30 dB above the background noise level inside the test section over the frequency range from 200 to 900 Hz, which is the frequency range considered in the present experimental study. The background noise level inside the duct without the acoustic forcing and the mean flow dropped from ~ 50 dB at 200 Hz monotonically and gradually to ~ 20 dB at 900 Hz.

IV. OBSERVATIONS AND DISCUSSIONS

The effect of the mean flow on the sound power transmission loss of the resonator is shown in Fig. 4. The frequency spectra of the corresponding pressure fluctuations at P1 and P2, and those of the acoustic signals at M5 (inside

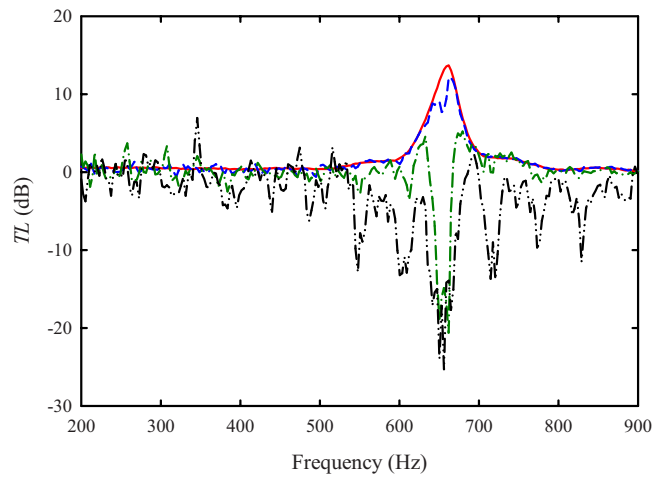


FIG. 4. (Color online) Sound power transmission loss of resonator R0. —: $U=0$ m/s; - - -: $U=8$ m/s; - · - ·: $U=12$ m/s; - - - -: $U=18$ m/s.

resonator cavity) are presented in Figs. 5(a)–5(c), respectively. The damped resonance frequency of R0, f_d , is ~ 660 Hz. The sound power transmission loss (TL) at or

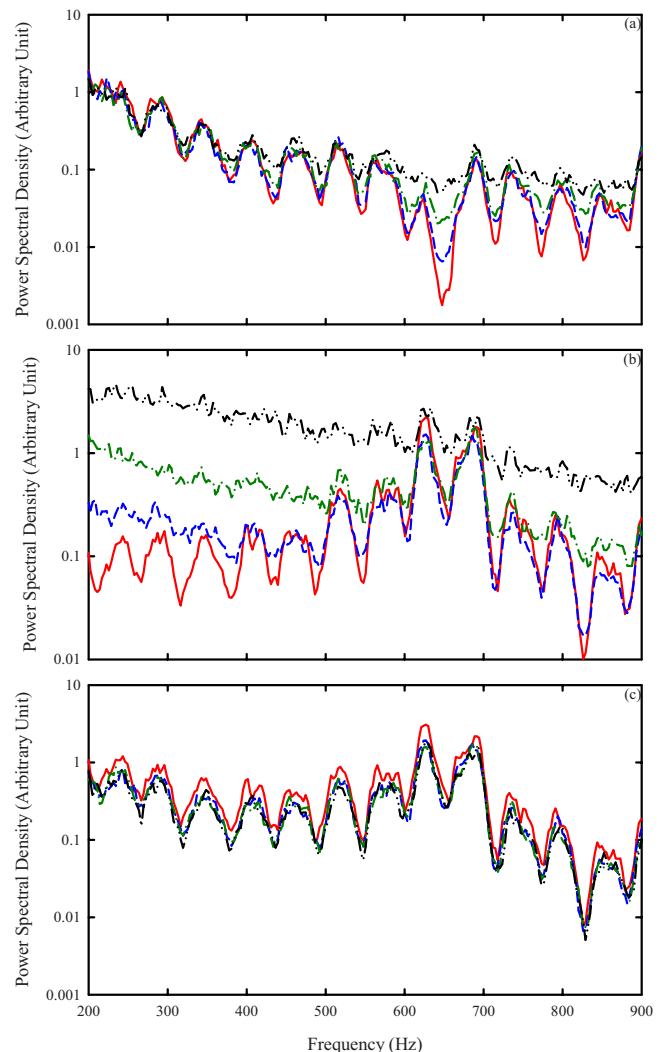


FIG. 5. (Color online) Pressure fluctuations under flow excitation of R0. (a) P1; (b) P2; (c) M5. Legends: same as those of Fig. 4.

near to this frequency is reduced with increasing mean flow velocity U in the duct and sound amplification is observed for $U \geq 10$ m/s. Such reduction in TL is due to the excitation of R0 by the mean flow. For lower U , this reduction is narrow-banded which agrees with Eq. (5). When U becomes large, spurious TL spectra with many sharp troughs are observed. This is believed to be the results of the nonlinear air damping effect [cf. Eq. (6)].

One can also notice that the Strouhal number of the resonator resonance frequency f_d (~ 660 Hz) based on the aperture slot width ($w=30$ mm) and a mean flow velocity U of 20 m/s is 0.99 ($=f_d w/U$), which is much higher than the expected fundamental frequency of the aerodynamic excitation, which has a Strouhal number of 0.25 as discussed in Nelson *et al.*⁹ A Strouhal number of 0.25 corresponds to a frequency of 167 Hz at $U=20$ m/s for the present resonators. This further suggests that the shear layer rollup as seen in Graf and Durgin¹⁷ was not likely to affect the results of the present study.

Figure 5(a) shows the pressure fluctuation spectra at P1 within the range of U tested. The sensor recorded the combined effects of the upstream acoustic forcing and the sound reflection from the resonator. A strong trough at the resonance frequency is observed for very low U , confirming the presence of a strong antiphase reflection created by the resonator under the acoustic excitation. The weakening of the trough upon the increase in U suggests the sound generation by the flow excited resonator.

The magnitude of the pressure fluctuation at P2 increases with increasing U in general, as shown in Fig. 5(b). There is a strong increase in the low frequency fluctuation energy, which is probably due to the oscillating shear flow within the neck (similar observation was made at the outer air jet boundary of Tang and Ko²⁸), but it has no bearing on the present study. It is also observed that the spectral energy of the pressure fluctuation at P2, near to the working frequency range of R0, increases with the mean flow speed for $U > 8$ m/s. Thus, the neck flow velocity oscillation is expected to grow in strength as the flow speed increases. This is in line with the deduction from the simplified model [Eq. (5)]. However, the acoustic pressure inside the resonator cavity, which should be uniform because of the small size of the cavity compared to the wavelength associated with the resonance frequency of R0, is only slightly reduced as the flow speed increases [Fig. 5(c)]. One should note that the pressure spectral peak at the resonator frequency observed in Nelson *et al.*⁹ is not observed inside the cavity here because of the presence of the strong upstream acoustic excitation having a spectrum with a local trough at the resonator frequency [Fig. 3(b)].

The sound level generated by R0 due to the mean flow excitation is equal to the drop in TL in the presence of the mean flow relative to that without the flow. Figure 6 shows the variation of the TL reduction resulted from the mean flow excitation with U . One should note that a certain static pressure differential across a flow constriction is in general required to overcome the head loss before a flow across the constriction can be driven.²⁹ This applies to the present study so that the resonator cannot generate much sound when U ,

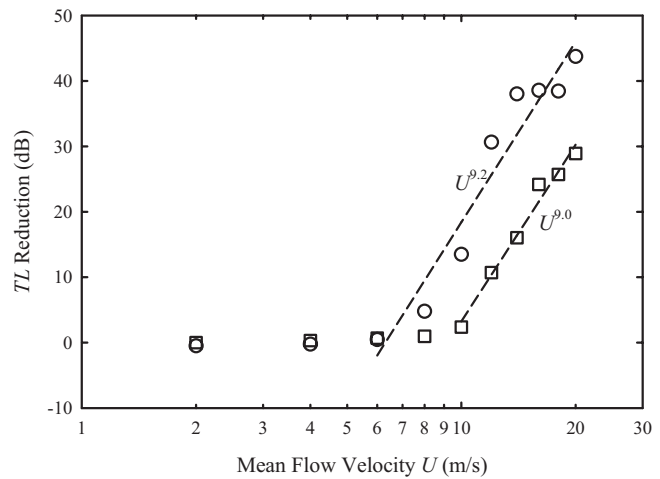


FIG. 6. Variation of sound level generated by excited resonators with mean flow velocity. ○: R0; □: R1. - - - : power law regression lines.

and thus the static pressure difference across the resonator neck, is low. The initial variation of TL reduction with U is thus very slow. For $U \geq 6$ m/s, the sound level varies approximately with $U^{0.2}$. (correlation coefficient of the regression, R^2 , is 0.93), confirming the deduced tenth power relationship in Sec. II. In this case, the power index α is approximately 1.84, suggesting that the air damping is relatively weak. Such scaling of the flow excited resonator sound level on the grazing flow velocity has not been observed or discussed in existing literature at least to the knowledge of the author. The experimental results of Nelson *et al.*⁹ show that the sound pressure level generated by a grazing flow excited resonator inside the resonator cavity varies approximately with $U^{7.3}$ or $16 \text{ m/s} < U \leq 20 \text{ m/s}$. According to the present simplified theoretical model, the corresponding α is 1.46, which is still within expectation. The larger cavity volume and the narrower slot width in Ref. 9 than those in the present study increase the relative significance of the air damping in the interaction between the resonator and the flow. A reduction of the power index α can thus be anticipated.

The effect of the mean flow on the TL of R1 is shown in Fig. 7(a). R1 was converted from R0 by connecting the resonator cavity to the duct wall by a 5 mm diameter plastic tubing [Fig. 2(b)]. The static pressure differential between the cavity and the mean flow should have been reduced and thus the acoustic radiation from the resonator can be attenuated. With such a reduction in the static pressure differential, the resonator is capable of producing reasonable TL even up to a flow speed of 14 m/s, while the working frequency range of the resonator is basically kept unchanged. The peak TL appears at a frequency of 660 Hz in the no-flow case as for the case of R0. The corresponding pressure fluctuation spectra are presented in Fig. 7(b). The much smaller increase in the low frequency spectral energy for R1 compared to that for R0 confirms that a much weaker oscillating air jet was created in R1. The acoustic spectra at M5 are basically similar to those shown in Fig. 5(c) and thus are not presented.

The sound generated by the flow excited R1 is also illustrated in Fig. 6. A larger U is required in this case to

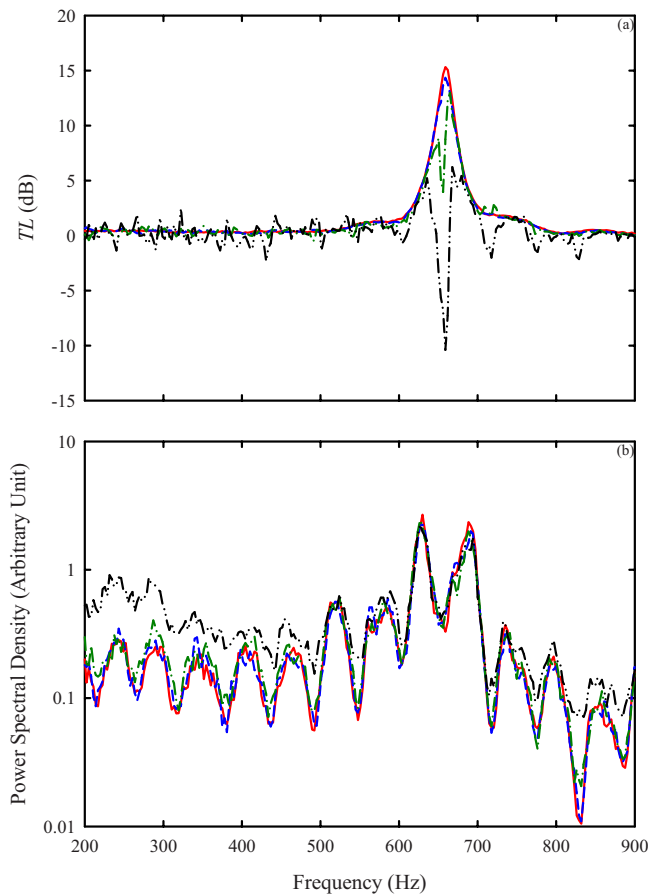


FIG. 7. (Color online) Sound transmission loss and pressure fluctuation spectra of R1. (a) Sound power transmission loss of resonator R1. (b) Pressure fluctuation spectra under flow excitation of R1 at P2. —: $U=0$ m/s; ---: $U=8$ m/s; -·-·: $U=12$ m/s; ···: $U=18$ m/s.

overcome the neck flow resistance before the oscillating jet can be of significant magnitude to effect a strong sound generation. The critical U here is 10 m/s (cf. 6 m/s for R0). For $U \geq 10$ m/s, the sound power of the flow excited R1 varies with $U^{9.0}$ ($R^2=0.98$) which is again in line with the deduction in Sec. II.

One can observe from Fig. 8 that there is a slight down-

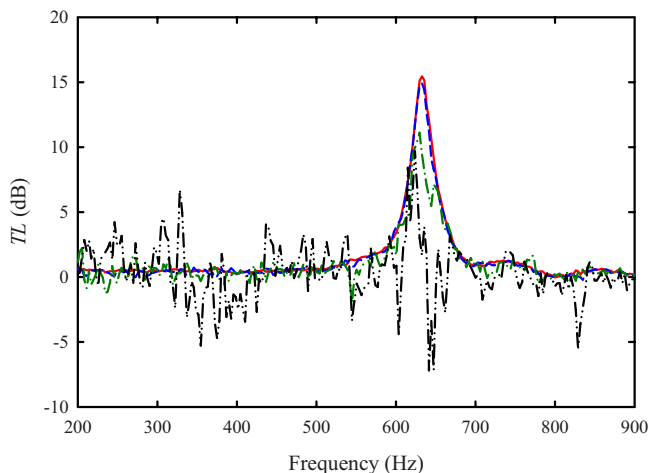


FIG. 8. (Color online) Sound power transmission loss of resonator R2. Legends: same as those of Fig. 7.

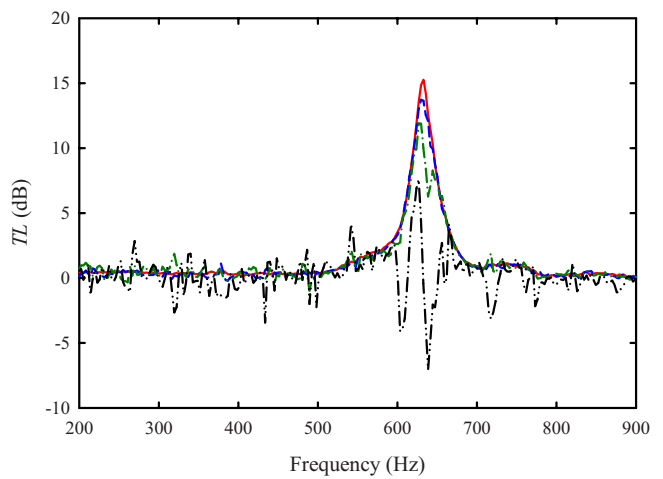


FIG. 9. (Color online) Sound power transmission loss of resonator formed by combining R1 and R2. Legends: same as those of Fig. 7.

ward shift of resonance frequency to 630 Hz for the resonator R2 which is formed by introducing two thin slabs at the entry inside the resonator R0. This design, which increases the neck flow resistance, is capable of keeping reasonable sound attenuation even up to 18 m/s. The spectra at M5 and P2 for this case do not show any significant dependence on the mean flow velocity and thus they are not presented. The absence of the low frequency energy at P2, in this case at increased U , suggests that the oscillating jet at the resonator neck due to the excited oscillating neck flow is very weak compared to those in R0 and R1, which is again expected.

Figure 9 illustrates the TL of a resonator formed by combining R1 and R2. One can observe that the magnitudes of the highest TL are basically similar to those of the R2. At higher flow velocity, the present setting also results in less spurious TL spectra compared to that of R2, indicating that the oscillating neck flow velocity is further reduced such that the nonlinear air damping is weakened further.

V. CONCLUSIONS

A simplified model for the sound radiation by a wall-mounted Helmholtz resonator excited by a low Mach number duct flow was developed in the present study in an attempt to explain the effect of such a flow on the sound power transmission loss of the resonator. Experiments were derived to confirm the validity of the simplified model. The resonance frequency of the resonator in the present study was much higher than the expected fundamental frequency of the shear layer at the mouth of the resonator. The acoustic excitation was achieved by a loudspeaker sufficiently upstream of the resonator. The strength of this excitation was kept unchanged throughout the experiment.

The simplified model indicates that the resonator will radiate sound at the damped resonance frequency of the resonator when a static pressure differential is applied across its neck, suggesting that the sound power transmission loss of a resonator will be reduced by the introduction of a grazing flow across its mouth in the ducted condition. This has been confirmed by the experimental results, which show that the

increase in the grazing flow velocity leads to a larger reduction of the sound power transmission loss once the grazing flow velocity exceeds a certain threshold.

Two passive mitigation modifications to the resonator were then employed. One of them was to bridge the resonator cavity with the downstream duct section, such that the static pressure differential across the resonator neck can be expected to have been reduced. The other one was to introduce an additional flow resisting device into the resonator such that the neck flow velocity can be reduced under the same mean flow velocity in the duct. These methods were proved to be effective in maintaining the sound power transmission loss of the resonator at a satisfactory level within the practical ventilation duct flow velocity range used experimentally.

One important finding here is that the sound power level produced by the flow excited resonator in a ducted condition, which is equal to the reduction of resonator sound power transmission loss in the presence of the mean flow, increases with the mean flow velocity according to a power law with a power index of approximately 9 when the mean flow velocity exceeds a critical value. By comparing with data in existing literature, this power index is believed to depend on the configuration of the resonator, which affects the resonance frequency, the cavity stiffness and the damping. This is left to further investigation.

ACKNOWLEDGMENTS

This work was supported by grants from the Research Committee, the Hong Kong Polytechnic University under the Project No. G-U403.

¹A. Fry, *Noise Control in Building Services* (Pergamon, Oxford, 1988).

²C. M. Harris, *Handbook of Noise Control* (McGraw-Hill, New York, 1979).

³U. Ingard, "On the theory and design of acoustic resonators," *J. Acoust. Soc. Am.* **25**, 1037–1061 (1953).

⁴M. L. Munjal, *Acoustics of Ducts and Mufflers* (Wiley, New York, 1987).

⁵A. Selamet, I. J. Lee, and N. T. Huff, "Acoustic attenuation of hybrid silencers," *J. Sound Vib.* **262**, 509–527 (2003).

⁶A. S. Hersh, B. E. Walker, and J. W. Celano, "Helmholtz resonator impedance model, Part I: Nonlinear behavior," *AIAA J.* **41**, 795–808 (2003).

⁷S. Griffin, S. A. Lane, and S. Huybrechts, "Coupled Helmholtz resonators

for acoustic attenuation," *Trans. ASME, J. Vib. Acoust.* **123**, 11–17 (2001).

⁸S. H. Seo and Y. H. Kim, "Silencer design by using array resonators for low frequency band noise reduction," *J. Acoust. Soc. Am.* **118**, 2332–2338 (2005).

⁹P. A. Nelson, N. A. Halliwell, and P. E. Doak, "Fluid dynamics of a flow excited resonance, Part I: Experiment," *J. Sound Vib.* **78**, 15–38 (1981).

¹⁰R. L. Panton and J. M. Miller, "Excitation of a Helmholtz resonator by a turbulent boundary layer," *J. Acoust. Soc. Am.* **58**, 800–806 (1975).

¹¹M. Meissner, "Excitation of Helmholtz resonator by grazing air flow," *J. Sound Vib.* **256**, 382–388 (2002).

¹²S. Dequand, S. Hulshoff, H. van Kuijk, J. Willems, and A. Hirschberg, "Helmholtz-like resonator self-sustained oscillations, Part II: Detailed flow measurements and numerical simulations," *AIAA J.* **41**, 416–423 (2003).

¹³M. S. Howe, "On the Helmholtz resonator," *J. Sound Vib.* **45**, 427–440 (1976).

¹⁴J. E. Ffowcs Williams, "The acoustics of turbulence near sound-absorbent liners," *J. Fluid Mech.* **51**, 737–749 (1972).

¹⁵M. S. Howe, "A note on the interaction of unsteady flow with an acoustic liner," *J. Sound Vib.* **63**, 429–436 (1979).

¹⁶A. P. Dowling and I. J. Hughes, "Sound absorption by a screen with a regular array of slits," *J. Sound Vib.* **156**, 387–405 (1992).

¹⁷H. R. Graf and W. W. Durgin, "Measurement of the nonsteady flow field in the opening of a resonating cavity excited by grazing flow," *J. Fluids Struct.* **7**, 387–400 (1993).

¹⁸D. Innes and D. G. Crighton, "On a non-linear differential equation modelling Helmholtz resonator response," *J. Sound Vib.* **131**, 323–330 (1989).

¹⁹L. E. Kinsler and A. R. Frey, *Fundamentals of Acoustics* (Wiley, New York, 1962).

²⁰J. E. Ffowcs Williams, "Hydrodynamic noise," *Annu. Rev. Fluid Mech.* **1**, 197–222 (1969).

²¹D. J. Inman, *Engineering Vibration* (Prentice-Hall, Englewood Cliffs, NJ, 1994).

²²C. K. W. Tam, H. Ju, M. G. Jones, W. R. Watson, and T. L. Parrott, "A computational and experimental study of slit resonators," *J. Sound Vib.* **284**, 947–984 (2005).

²³T. D. Mast and A. D. Pierce, "Describing-function theory for flow excitation of resonators," *J. Acoust. Soc. Am.* **97**, 163–172 (1995).

²⁴P. G. Saffman, *Vortex Dynamics* (Cambridge University Press, Cambridge, 1992).

²⁵P. O. A. L. Davies, "Measurement of plane wave acoustic fields in flow ducts," *J. Sound Vib.* **72**, 539–542 (1980).

²⁶S. K. Tang and F. Y. C. Li, "On low frequency sound transmission loss of double sidebranches: A comparison between theory and experiment," *J. Acoust. Soc. Am.* **113**, 3215–3225 (2003).

²⁷W. Neise, W. Frommhold, F. P. Mechel, and F. Holste, "Sound power determination in rectangular flow ducts," *J. Sound Vib.* **174**, 201–237 (1994).

²⁸S. K. Tang and N. W. M. Ko, "A study on the noise generation mechanism in a circular air jet," *ASME Trans. J. Fluids Eng.* **115**, 425–435 (1993).

²⁹V. L. Streeter and E. B. Wylie, *Fluid Mechanics* (McGraw-Hill, New York, 1981).