Critical Behavior of the Random-Bond Clock Model

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Abstract. The critical behavior of the clock model in two-dimensional square lattice is studied numerically using Monte Carlo method with Wolff algorithm. The Kosterlitz-Thouless (KT) transition is observed in the 8-state clock model, where an intermediate phase exists between the low-temperature ordered phase and the high-temperature disordered phase. The bond randomness is introduced to the system by assuming a Gaussian distribution for the coupling coefficients with the mean $\mu = 1$ and different values of variance: from $\sigma^2 = 0.1$ to $\sigma^2 = 3.0$. An abrupt jump in the helicity modulus at the transition, which is the key characteristic of the KT transition, is verified with a stability argument. Our results show that, a small amount of disorder (small σ) reduces the critical temperature of the system, without altering the nature of transition. However, a larger amount of disorder changes the transition from the KT-type into that of non-KT-type.

Keywords: random-bond clock model, Kosterlitz-Thouless transition, Gaussian distribution
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INTRODUCTION

Unlike the phase transitions manifested in most spin models, Kosterlitz-Thouless (KT) transition is a specific phase transition observed in the critical behavior of the superfluid systems and it can be described by the two-dimensional *XY* model [1-3]. On the other hand, the q-state clock model is a discrete version of the *XY* model. It is expected to have various critical behavior under different q values. Extensive studies [4-10] on the clock model had shown that, for $q \leq 4$, the phase transition is Ising-like, and for $q \geq 6$, it is *XY*-like.

The presence of defects interrupts the periodic structure of crystalline materials and the systems become disordered when the quantity of interruptions is large. It can be visualized by a random distribution of coupling coefficients between neighboring spins. In statistical point of view, it is natural to consider a Gaussian distribution for the coupling coefficients. It is our motivation to study the effect of disorder on the phase transition in the clock model. Since the critical behavior for the q-state clock model does not change appreciably on varying q values when q is large. For simplicity, we can use the random-bond 8-state clock model ($q = 8$) to study the KT transition.

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MODEL

We consider a system of spins on a square lattice with size $N = L \times L$. In the q-state clock model, the spins are confined on a plane with q different states each of which is specified by a phase angle $\theta_n = n\left(2\pi/q\right)$, where $n = 0,1,2,...,q-1$. The spin state at each site can be defined by the spin vector $\mathbf{S}_n = \left(\sin\theta_n, \cos\theta_n\right)$. The Hamiltonian is given by

$$H = -\sum_{\langle ij\rangle} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \,, \tag{1}$$

For simplicity, the summation is restricted to the nearest neighbors only. The coupling coefficients J_{ij} are assumed to follow the Gaussian distribution

$$P(J) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(J-\mu)^2}{2\sigma^2}\right], \tag{2}$$

where $\mu = 1$ is the mean and σ^2 is the variance of the distribution. The bond-randomness is reflected by the parameter σ, in particular, $\sigma^2 = 0$ represents a pure system with no disorder and with constant coupling coefficient $J = 1$.

The energy per spin E of the system is given by

$$E = -\frac{1}{N}\left\langle \sum_{\langle ij\rangle} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \right\rangle, \tag{3}$$

where $\langle \ldots \rangle$ denoted the ensemble average of the quantities. The magnetization per spin $\mathbf{m}$ is given by

$$\mathbf{m} = \left(\left\langle \frac{1}{N}\sum_i \sin\theta_i \right\rangle, \left\langle \frac{1}{N}\sum_i \cos\theta_i \right\rangle \right), \tag{4}$$

and its magnitude is denoted by m. Furthermore, the susceptibility per spin χ can be obtained from the fluctuations of m and is given by

$$\chi = \frac{N}{k_B T}\left(\langle m^2 \rangle - \langle m \rangle^2 \right). \tag{5}$$

METHODS

The helicity modulus per spin Υ is a measure of the resistance to an infinitesimal spin twist Δ across the system along one direction. It is given by

$$\Upsilon = -\frac{1}{2}\langle E \rangle - \frac{N}{T}\langle s^2 \rangle \,, \tag{6}$$

where $s \equiv \sum_{\langle ij\rangle} J_{ij} \sin(\theta_i - \theta_j)(\mathbf{e}_{ij} \cdot \mathbf{x}) \big/ N$ and $\mathbf{e}_{ij}$ is a unit vector pointing from lattice site i to j. The abrupt jump in the helicity modulus from the finite value $(2/\pi)k_B T_c$ to zero at the critical temperature T_c in the thermodynamic limit at the critical temperature is the key feature of the KT transition [2]. The fourth-order helicity modulus can be expressed as

$$\langle \Upsilon_4 \rangle = \frac{1}{2N}\langle E\rangle + \frac{3}{4T}\langle E\rangle^2 - \frac{3}{4T}\langle E^2\rangle + \frac{4}{T}\langle s^2\rangle + \frac{3N^2}{T^3}\langle s^2\rangle^2 + \frac{3N}{T^2}\langle E\rangle\langle s^2\rangle . \quad (7)$$

Supposed Υ_4 is negative at the transition, then Υ cannot approach zero continuously but must make a discontinuous jump toward zero at the transition instead [11]. Thus, we can identify the KT transition.

RESULTS AND DISCUSSION

The magnetization and susceptibility against temperature for the random-bond 8-state clock model with various σ are given in Fig. 1 In the pure case ($\sigma^2 = 0$), the system undergoes two transitions at two critical temperatures instead of a single one. There exists an intermediate phase called the KT phase (or the massless phase) between the low-temperature ordered phase and the high-temperature disordered phase. The results for the susceptibility show the double-peak feature of the KT transition. As σ increases, the critical temperature of the system decreases and the distance between two peaks in the susceptibility graph decreases. The results show the shrinkage of the intermediate phase as the amount of disorder increases.

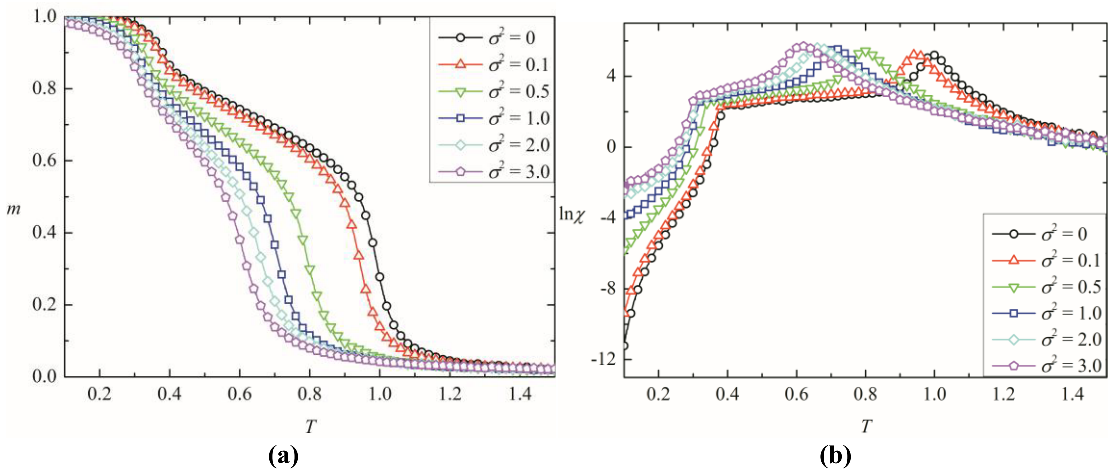

FIGURE 1. (a) The magnetization (b) The susceptibility against temperature for the random-bond 8-state clock model with lattice size $N = 128\times128$ and various σ : 0, 0.1, 0.5, 1.0, 2.0, and 3.0.

In order to demonstrate the disappearance of the KT transition under a large amount of disorder, we have calculated the fourth-order helicity modulus of the systems and the results are given in Fig. 2. For $\sigma^2 = 0$ and $\sigma^2 = 0.5$, the fourth-order helicity modulus at the critical temperature are clearly negative, implying the system undergoes the KT transition. However, despite the noisy nature, the depth of the trough of the fourth-order helicity modulus reduces as σ increases. For $\sigma^2 = 2.0$, the trough is no longer observable and the helicity modulus does not jump discontinuously. Hence, we conclude that the transition is no longer of the KT-type and this is consistent with our above results.

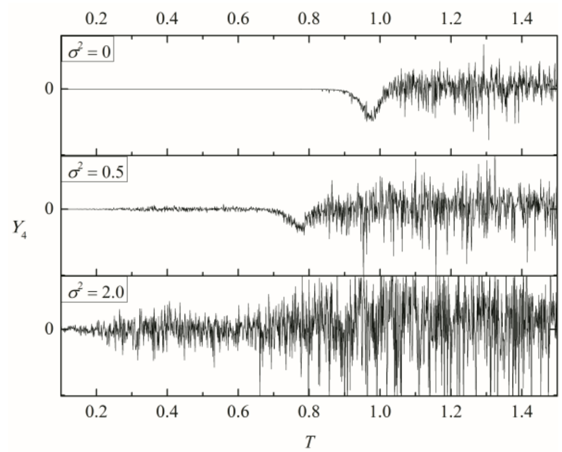

FIGURE 2. The fourth-order helicity modulus against temperature for the random-bond 8-state clock model with lattice size $N = 128\times128$ and various σ : 0, 0.5, and 2.0.

Our results show that, a small amount of disorder (small σ) reduces the critical temperature of the system, without altering the nature of transition. However, a larger amount of disorder changes the transition from the KT-type into that of non-KT-type.

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REFERENCES

1. J. M. Kosterlitz, and D. J. Thouless, *J. Phys. C* **6**, 1181-1203 (1973).
2. J. M. Kosterlitz, *J. Phys. C* **7**, 1046-1060 (1974).
3. D. R. Nelson, and J. M. Kosterlitz, *Phys. Rev. Lett.* **39**, 1201-1205 (1977).
4. S. Miyashita, H. Nishimori, A. Kuroda, and M. Suzuki, *Prog. Theor. Phys.* **60**, 1669-1685 (1978).
5. J. Tobochnik, *Phys. Rev. B* **26**, 6201-6027 (1982).
6. M. S. S. Challa, and D. P. Landau, *Phys. Rev. B* **33**, 437-443 (1986).
7. A. Yamagata, and I. Ono, *J. Phys. A* **24**, 265-275 (1991).
8. Y. Tomita, and Y. Okabe, *Phys. Rev. B* **65**, 184405 (2002).
9. S. K. Baek, P. Minnhagen, and B. J. Kim, *Phys. Rev. E* **81**, 063101 (2010).
10. S. K. Baek, and P. Minnhagen, *Phys. Rev. E* **82**, 031102 (2010).
11. P. Minnhagen, and B. J. Kim, *Phys. Rev. B* **67**, 172509 (2003).

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