# A New Intuitionistic Fuzzy Rough Set Approach for Decision Support

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**Abstract.** The rough set theory was proved of its effectiveness in dealing with the imprecise and ambiguous information. Dominance-based Rough Set Approach (DRSA), as one of the extensions, is effective and fundamentally important for Multiple Criteria Decision Analysis (MCDA). However, most of existing DRSA models cannot directly examine uncertain information within rough boundary regions, which might miss the significant knowledge for decision support. In this paper, we propose a new believe factor in terms of an intuitionistic fuzzy value as foundation, further to induce a kind of new uncertain rule, called believable rules, for better performance in decision-making. We provide an example to demonstrate the effectiveness of the proposed approach in multicriteria sorting and also a comparison with existing representative DRSA models.

**Keywords:** Multicriteria decision analysis; Rough set; Intuitionistic fuzzy set; Rule-based approach; Sorting.

## 1 Introduction

Rough set methodology is an effective mathematical tool for Multicriteria Decision Analysis (MCDA) because of its strength in data analysis and knowledge discovery from imprecise and ambiguous data. The classical Pawlak's rough set had been successfully applied in medical diagnosis [13], supplier selection [5], etc. However, it cannot deal with the preference-ordered data. With substitution of indiscernibility relations by dominance relations, Classical Dominance-based Rough Set Approach (C-DRSA) was firstly generated by Greco et al. [8]. Compared with Pawlak's Rough Set, the key idea of C-DRSA is mainly in two aspects: (1) the knowledge granules generated from multiple criteria are dominance cones rather than the concept of indiscernibility; (2) the objective sets of rough approximations are the upward and downward unions of preference-ordered classes, rather than the binary-relation-based non-preference classes. Such properties let C-DRSA be a suitable means for decision supports, particularly with respect to multicriteria ranking, sorting and choice.

C-DRSA is the core procedure for calculation of rough approximations, in which consistency data are assigned to lower approximations and inconsistency data are put into the rough boundary regions. The purpose of applying DRSA models is to induce decision rules and then employ them for providing assignments to pre-defined decision classes. Various extensions of DRSA models also appeared. Variable-Precision DRSA (VP-DRSA) [9] defined a threshold called the precision to control the membership of inconsistent objects into the lower approximations. Quasi-DRSA [6] hybridized Pawlak's rough set and C-DRSA for lower error rates in natural selection. Chai and Liu [3] provided a class-based rough approximation model and studied the reducts preserving the singleton class rather than the traditional class unions.

However, most of previous DRSA models aim to generate a minimal rule set, which might neglect valuable uncertain information within rough boundary regions [8]. Even though such possible rules and approximate rules as uncertain rules are able to extract uncertain information, they rarely can be employed in real world. A significant extension of C-DRSA is Variable-Consistency DRSA (VC-DRSA) [7] that relaxes the strict dominance principle and hence admits several inconsistent objects to the lower approximations. This approach indeed enhances the opportunity of discovering the strong rule patterns, and is particularly useful for large datasets. Yet, it is still far from satisfactory.

In this paper, we develop a new DRSA model through inducing a new kind of uncertain rule called believable rule, in order to better extract valuable uncertain information. To this end, we introduce a new believe factor in terms of the concept of intuitionistic fuzzy value [4], [11]. Three related measurements are generated for exploring rough boundary region. Finally, aided by the proposed believe factor, we define a new kind of uncertain rule, called believable rule, for better examination of uncertain information within rough boundary regions. Through comparing with previous representative DRSA models, an example is provided to verify the capability of the proposed model in solving sorting problems.

The rest of this paper is organized as follows. Section 2 provides the preliminaries, including the principles of DRSA methodology and intuitionistic fuzzy theory. Section 3 presents believable rule induction aided by believe factor. In section 4, we demonstrate the capability of the proposed model via an illustrative example with a comparison. Finally, we draw the conclusion and outline the future work in Section 5.

## 2 Preliminaries

### 2.1 Dominance-based Rough Set Approach

An information table can be transferred to a *decision table* via distinguishing condition criteria and decision criteria. Formally, a decision table is the 4-tuple $S = \langle U, Q, V, f \rangle$ , which includes (1) a finite set of objects denoted by $U$, $x \in U = \{x_1, ..., x_m\}$; (2) a finite set of criteria denoted by $Q = C \bigcup D$, where condition criteria set $C \neq \emptyset$, decision criteria set $D \neq \emptyset$ (usually the singleton set $D = \{d\}$), and $q \in Q = \{q_1 ..., q_n\}$; (3) the scale of criterion $q$ denoted by $V_q$, where $V = \bigcup_{q \in Q} V_q$; (4) information function denoted by $f_q(x) : U \times Q \to V$, where $f_q(x) \in V_q$ for each $q \in Q$, $x \in U$. In addition, each object $x$ from $U$ is described by a vector called *decision description* in terms of the decision information on the criteria, denoted by $Des_Q(x) = [f_{q_1}(x), ..., f_{q_n}(x)]$. As such, information function $f_q(x)$ also can be called *decision values* in MCDA.

The objective sets of dominance-based rough approximations are the upward or downward unions of predefined decision classes. Suppose the decision criterion $d$ makes a partition of $U$ into a finite number of classes $CL = \{Cl_t, t = 1, ..., l\}$. We assume that $Cl_{t+1}$ is superior to $Cl_t$ according to DM's preference. Each object $x$ from $U$ belongs to *one and only one* class $Cl_t$. The upward and downward unions of classes are represented respectively as: $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$, where $t = 1, ..., l$.

Then, the following operational laws are valid: $Cl_1^{\leq} = Cl_1; Cl_l^{\geq} = Cl_l; Cl_t^{\geq} = U - Cl_{t-1}^{\leq}; Cl_t^{\leq} = U - Cl_{t+1}^{\geq}; Cl_1^{\geq} = Cl_l^{\leq} = CL; Cl_0^{\leq} = Cl_{l+1}^{\geq} = \emptyset$.

The granules of knowledge in DRSA theory are *dominance cones* with respect to values space of the considered criteria. If two decision values are with the dominance relation like $f_q(x) \geq f_q(y)$ for every considered criterion $q \in P \subseteq C$, we say $x$ *dominates* $y$, denoted by $xD_py$. The dominance relation is reflexive and transitive. With this in mind, the *dominance cone* can be represented by: P-dominating set $D_P^+(x) = \{y \in U : yD_px\}$; P-dominated set $D_P^-(x) = \{y \in U : xD_py\}$.

The key concept in DRSA theory is the *Dominance Principle*: if the decision value of object $x$ is no worse than that of object $y$ on all considered condition criteria (saying $x$ is dominating $y$ on $P \subseteq C$), object $x$ should also be assigned to a decision class no worse than that of object $y$ (saying $x$ is dominating $y$ on $D$). Founded on such dominance principle, the definitions of rough approximations are given in the following.

P-lower approximations denoted as $\underline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq})$, are represented as: $\underline{P}(Cl_t^{\geq}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\}; \underline{P}(Cl_t^{\leq}) = \{x \in U : D_P^-(x) \subseteq Cl_t^{\leq}\}$.

P-upper approximations denoted as $\overline{P}(Cl_t^{\geq})$ and $\overline{P}(Cl_t^{\leq})$, are represented as: $\overline{P}(Cl_t^{\geq}) = \{x \in U : D_P^-(x) \cap Cl_t^{\geq} \neq \emptyset\}$; $\overline{P}(Cl_t^{\leq}) = \{x \in U : D_P^+(x) \cap Cl_t^{\leq} \neq \emptyset\}$.

VC-DRSA model accepts a limited number of inconsistent objects which are controlled by the predefined threshold called *consistency level*. For $P \subseteq C$, the P-lower approximations of VC-DRSA can be represented as: $\underline{P}^l(Cl_t^{\geq}) = \{x \in Cl_t^{\geq} : \frac{|D_P^+(x) \bigcap Cl_t^{\geq}|}{|D_p^+(x)|} \geq l\}$ ; $\underline{P}^l(Cl_t^{\leq}) = \{x \in Cl_t^{\leq} : \frac{|D_P^-(x) \bigcap Cl_t^{\leq}|}{|D_p^-(x)|} \geq l\}$ , where consistency level $l$ means that object $x$ from $U$ belongs to the class union $Cl_t^{\geq}$ (or $Cl_t^{\leq}$) with no ambiguity at level $l \in (0, 1]$.

### 2.2 Intuitionistic Fuzzy Theory

This section revisits the principles of intuitionistic fuzzy theory as one of our preliminaries. Atanassov [1] extended Zadeh's fuzzy set employed by a membership function, and defined the notion of intuitionistic fuzzy set (IFS) via further considering a non-membership function. An IFS $A$ in a finite set $X$ can be written as: $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ s.t. $0 \le \mu_A + \nu_A \le 1, x \in X$; with $\mu_A : X \to [0,1], x \in X \to \mu_A(x) \in [0,1]$; $\nu_A : X \to [0,1], x \in X \to \nu_A(x) \in [0,1]$. The hesitation degrees [10] can be defined as: $\pi_A = 1 - \mu_A - \nu_A$.

Xu [11] extracted the basic element from IFS as the Intuitionistic Fuzzy Value (IFV) denoted as $a = (\mu_a, \nu_a, \pi_a)$, where the membership degree $\mu_a \in [0,1]$, the non-membership degree $\nu_a \in [0,1]$, and the hesitation degree $\pi_a \in [0,1]$ with $\pi_a = 1 - \mu_a - \nu_a$. Let $a_1$ and $a_2$ be two IFVs. The related operations [12] are revisited in the following. Complement: $\overline{a} = (\nu_a, \mu_a)$; Addition: $a_1 \oplus a_2 = \{\mu_{a_1} + \mu_{a_2} - \mu_{a_1}\mu_{a_2}, \nu_{a_1}\nu_{a_2}\}$; Multiplication: $a_1 \otimes a_2 = \{\mu_{a_1}\mu_{a_2}, \nu_{a_1} + \nu_{a_2} - \nu_{a_1}\nu_{a_2}\}$; Multiple law: $\lambda a = (1 - (1 - \mu_a)^\lambda, \nu_a^\lambda)$, $\lambda > 0$; Exponent law: $a^\lambda = (\mu_a^\lambda, 1 - (1 - \nu_a)^\lambda)$, $\lambda > 0$; The Score Function: $s(a) = \mu_a - \nu_a$; The Accuracy Function: $h(a) = \mu_a + \nu_a$. The method for comparing two intuitionistic fuzzy values through using $s(a)$ and $h(a)$ is presented: If $s(a_1) < s(a_2)$, then $a_1 < a_2$ . If $s(a_1) = s(a_2)$ , then, 1) If $h(a_1) = h(a_2)$, then $a_1 = a_2$; 2) If $h(a_1) < h(a_2)$, $a_1 < a_2$; 3) If $h(a_1) > h(a_2)$, then $a_1 > a_2$.

## 3 Uncertain Rule Induction

### 3.1 Believe Factor

Considering the assignment of object $x \in U$, dominance cones $D_P^+(x)$ and $D_P^-(x)$ can be divided into three subsets, denoted as $X_1$, $X_2$ and $X_3$: (a) for $D_P^+(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$; (b) for $D_P^-(x)$, we have $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$. With respect to the objects belonging to the class unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$ but failing to be assigned to the corresponding lower approximations, the following assertions are valid: (1) For $t = 2, ..., l$, we have $Bn_P(Cl_t^{\geq}) = Bn_P(Cl_{t-1}^{\leq}) = (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) \bigcup (Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq}))$. (2) For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $t = 2, ..., l$, we have $D_P^+(x) = X_1 \bigcup X_2 \bigcup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\geq})$, $X_2 \subseteq Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, $X_3 \subseteq Cl_{t-1}^{\leq}$ . (3) For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $t = 1, ..., l-1$, we have $D_P^-(x) = X_1 \bigcup X_2 \bigcup X_3$ subject to $X_1 \subseteq \underline{P}(Cl_t^{\leq})$, $X_2 \subseteq Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $X_3 \subseteq Cl_{t+1}^{\geq}$.

**Lemma 1.** *For $x \in Bn_P(Cl_t^{\geq})$(or $x \in Bn_P(Cl_t^{\leq})$), the following assertions are valid:*

$$\text{(a) } |X_1| \ge 0; \text{ (b) } |X_2| \ge 1; \text{ (c) } |X_3| \ge 1,$$

*where the number of objects in a set is denoted by $| \bullet |$.*

*Proof.* We take $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ as example. For (a), it is given by nature. For (b), assuming $|X_2| = 0$, we get $D_P^+(x) \bigcap (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) = \emptyset$. Since we held $x \in D_P^+(x)$, we then infer $x \notin Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, which is contradictory to our premises: $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Therefore, the assumption $|X_2| = 0$ does not hold. Finally, we obtain $|X_2| \ge 1$. For (c), assuming $|X_3| = 0$, we get

$D_P^+(x) \bigcap Cl_{t-1}^{\leq} = \emptyset$. Since we held $U - Cl_{t-1}^{\leq} = Cl_t^{\geq}$, we then get $D_P^+(x) \subseteq Cl_t^{\geq}$. According to the definition of $\underline{P}(Cl_t^{\geq})$, we then hold $x \in \underline{P}(Cl_t^{\geq})$, which is contradictory to our premises : $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$. Therefore, the assumption $|X_3| = 0$ does not hold. Finally, we hold $|X_3| \geq 1$. For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, the proof is in the similar processing.

Based on these observations, we propose a new coefficient, called Believe Factor of upward and downward unions (*Believe Factor* for short). The definition is given as follows.

**Definition 1.** *For $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq}), t = 2, ..., l$, we have the believe factor of upward union of decision classes (upward believe factor for short):*

$$\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x), \nu_t^{\geq}(x), \pi_t^{\geq}(x)) \ s.t.\ \mu_t^{\geq}(x) = \frac{|D_P^+(x) \bigcap \underline{P}(Cl_t^{\geq})|}{|D_P^+(x)|},$$
$$\nu_t^{\geq}(x) = \frac{|D_P^+(x) \bigcap Cl_{t-1}^{\leq}|}{|D_P^+(x)|},\ \pi_t^{\geq}(x) = \frac{|D_P^+(x) \bigcap (Cl_t^{\geq} - \underline{P}(Cl_t^{\geq}))|}{|D_p^+(x)|}\ .$$

**Definition 2.** *For $x \in Cl_t^{\leq} - \underline{P}(Cl_t^{\leq})$, $t = 1, ..., l-1$, we have the believe factor of downward union of decision classes (downward believe factor, for short):*

$$\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x), \nu_t^{\leq}(x), \pi_t^{\leq}(x)) \ s.t.\ \mu_t^{\leq}(x) = \frac{|D_P^-(x) \bigcap \underline{P}(Cl_t^{\leq})|}{|D_P^-(x)|},$$
$$\nu_t^{\leq}(x) = \frac{|D_P^-(x) \bigcap Cl_{t+1}^{\geq}|}{|D_P^-(x)|},\ \pi_t^{\leq}(x) = \frac{|D_P^-(x) \bigcap (Cl_t^{\leq} - \underline{P}(Cl_t^{\leq}))|}{|D_p^-(x)|}\ .$$

Remark that the symbol "$\to$" in $\beta(x \to Cl_t^{\geq})$ and $\beta(x \to Cl_t^{\leq})$ can be understood as "be assigned to" or "belongs to". For object $x \in U$, $\mu(x)$ (including $\mu_t^{\geq}(x)$ and $\mu_t^{\leq}(x)$) is called *positive score* ; $\nu(x)$ (including $\nu_t^{\geq}(x)$ and $\nu_t^{\leq}(x)$) is called *negative score*; $\pi(x)$ (including $\pi_t^{\geq}(x)$ and $\pi_t^{\leq}(x)$) is called *hesitancy score*. The forms of upward/downward believe factors can be regarded as intuitionistic fuzzy values [4], [11].

**Lemma 2.** *For object $x \in Cl_t, t = 1, ..., l$, the following assertions are valid:*

$$\mu_t^{\geq}(x) + \nu_t^{\geq}(x) + \pi_t^{\geq}(x) = 1;\ \mu_t^{\leq}(x) + \nu_t^{\leq}(x) + \pi_t^{\leq}(x) = 1\ .$$

*Proof.* It can be easily proved according to definition 1 and definition 2.

**Lemma 3.** $\beta(x \to Cl_t^{\geq}) = (\mu_t^{\geq}(x), \nu_t^{\geq}(x), \pi_t^{\geq}(x)) = (1, 0, 0)$ *is valid for* $x \in \underline{P}(Cl_t^{\geq})$. $\beta(x \to Cl_t^{\leq}) = (\mu_t^{\leq}(x), \nu_t^{\leq}(x), \pi_t^{\leq}(x)) = (1, 0, 0)$ *is valid for* $x \in \underline{P}(Cl_t^{\leq})$.

*Proof.* It can be easily proved according to definition 1 and definition 2.

### 3.2 Measurements

We introduce three measurements related to believe factor for uncertain rule induction.

**Definition 3.** *(Confidence degree) For object $x \in U$, the confidence degree of believe factor, denoted by $L(x)$, is defined by: $L(x) = \mu(x) + \pi(x)$, where $\mu(x)$ is positive score and $\pi(x)$ is hesitancy score. Specifically, we hold:*

$$L(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) + \pi_t^{\leq}(x); L(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) + \pi_t^{\geq}(x).$$

**Definition 4.** *(Believe degree) For object $x \in U$, the believe degree of believe factor, denoted by $S(x)$, is defined by: $S(x) = \mu(x) - \nu(x)$, where $\mu(x)$ is positive score and $\nu(x)$ is negative score. Specifically, we hold:*

$$S(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) - \nu_t^{\leq}(x); S(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) - \nu_t^{\geq}(x).$$

**Definition 5.** *(Accuracy degree) For object $x \in U$, the accuracy degree of believe factor, denoted by $H(x)$, is defined by: $H(x) = \mu(x) + \nu(x)$, where $\mu(x)$ is positive score and $\nu(x)$ is negative score. Specifically, we hold:*

$$H(x \to Cl_t^{\leq}) = \mu_t^{\leq}(x) + \nu_t^{\leq}(x); H(x \to Cl_t^{\geq}) = \mu_t^{\geq}(x) + \nu_t^{\geq}(x).$$

### 3.3 Believable Rule Induction

Given a decision table, each object $x$ from $U$ has a *decision description* in terms of the evaluations on the considered criteria: $Des_P(x) = [f_{q_1}(x), ..., f_{q_n}(x)]$, where information function $f_q(x) \in V_q$, for $V = \bigcup_{q \in P} V_q$, $q \in P \subseteq C$. We say each $Des_P(x)$ is able to induce an uncertain rule based on *cumulated preferences.* Considering $Des_P(x)$ of boundary object $x$ which is coming from $Bn_P(Cl_t^{\geq})$, there are two kinds of decision descriptions in the separated rough boundary regions as: $Des_P(x) = [r_{q_1}^{\geq}, r_{q_2}^{\geq}, ..., r_{q_n}^{\geq}]$, for $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$ ; $Des_P(x) = [r_{q_1}^{\leq}, r_{q_2}^{\leq}, ..., r_{q_n}^{\leq}]$, for $x \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$ ; where $(Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})) + (Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})) = Bn_P(Cl_t^{\geq}) = Bn_P(Cl_{t-1}^{\leq})$.

With this in mind, the boundary objects carry the *valuable* uncertain information for decision making on the following conditions: (1) Considering the believe factor of object $x \in Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, if believe degree $S(x \to Cl_t^{\geq}) > 0$, we say object $x$ carries the *believable* decision information as: Providing the assignment to class union $Cl_t^{\geq}$ in some degree. (2) Considering the believe factor of object $x \in Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$, if believe factor $S(x \to Cl_{t-1}^{\leq}) > 0$, we say object $x$ carries the *believable* decision information as: Providing the assignment to class union $Cl_{t-1}^{\leq}$ in some degree.

The boundary objects satisfying the above conditions are called *valuable* objects. The induced uncertain rules on the basis of these *valuable* objects are called *believable rules.* In the following, the strategies are given in order to induce a set of believable rules.

**Strategy I** (Upward believable rule): Considering the object $x_i$ from the separated boundary region $Cl_t^{\geq} - \underline{P}(Cl_t^{\geq})$, if $S(x_i \to Cl_t^{\geq}) = \mu_t^{\geq}(x_i) - \nu_t^{\geq}(x_i) > 0$ is satisfied, we then induce an upward believable rule $BR_t^{\geq}$ based on the decision description $Des_P(x_i) = [r_{q_1}^{\geq}, r_{q_2}^{\geq}, ..., r_{q_n}^{\geq}]$: If $f_{q_1}(x) \geq r_{q_1}^{\geq}$ and $f_{q_2}(x) \geq r_{q_2}^{\geq} ... f_{q_n}(x) \geq r_{q_n}^{\geq}$, then $x \in Cl_t^{\geq}$, which is with three measuring degrees: $L(x_i \to$

$Cl_t^{\geq}$), $S(x_i \to Cl_t^{\geq})$ and $H(x_i \to Cl_t^{\geq})$.
**Strategy II** (Downward believable rule): Considering the object $x_i$ from the separated boundary region $Cl_{t-1}^{\leq} - \underline{P}(Cl_{t-1}^{\leq})$, if $S(x_i \to Cl_{t-1}^{\leq}) = \mu_{t-1}^{\leq}(x_i) - \nu_{t-1}^{\leq}(x_i) > 0$ is satisfied, we then induce a downward believable rule $BR_{t-1}^{\leq}$ based on the decision description $Des_P(x_i) = [r_{q_1}^{\leq}, r_{q_2}^{\leq}, ..., r_{q_n}^{\leq}]$: If $f_{q_1}(x) \leq r_{q_1}^{\leq}$ and $f_{q_2}(x) \leq r_{q_2}^{\leq} ... f_{q_n}(x) \leq r_{q_n}^{\leq}$, then $x \in Cl_{t-1}^{\leq}$, which is with three measuring degrees: $L(x_i \to Cl_{t-1}^{\leq})$, $S(x_i \to Cl_{t-1}^{\leq})$ and $H(x_i \to Cl_{t-1}^{\leq})$.

## 4 Illustrative Example

### 4.1 Decision Table and Rough Approximations

In this section, we use an example to illustrate the application of believable rule for multicriteria sorting (also known as ordinal classification). We use synthetic data set as shown in Table 1. We consider that the decision table is monotonic, which means a better decision value on condition criteria tends to contribute a better assignment in decision class, rather than the worse one, or vice versa. The decision information is summarized below: object set $\{S1, S2, ..., S50\}$; condition criterion set $\{A, B, C\}$; single decision criterion $\{D\}$; decision values scale $[1, 2, 3, 4, 5]$, where the larger number is superior to the smaller one according to DM's preference; decision class scale [III, II, I], where Class I is superior than Class II, and then Class III, denoted by Class I $=Cl_3$; Class II$=Cl_2$; Class III$=Cl_1$.

**Table 1.** Decision table

| Object | A | B | C | D | Object | A | B | C | D | Object | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S 1 | 3 | 4 | 3 | I | S 18 | 3 | 4 | 3 | I | S 35 | 2 | 3 | 3 | III |
| S 2 | 4 | 3 | 3 | I | S 19 | 5 | 2 | 4 | II | S 36 | 1 | 2 | 3 | III |
| S 3 | 5 | 3 | 4 | I | S 20 | 3 | 4 | 2 | II | S 37 | 2 | 3 | 3 | III |
| S 4 | 5 | 3 | 4 | I | S 21 | 4 | 2 | 3 | II | S 38 | 2 | 2 | 3 | III |
| S 5 | 5 | 4 | 3 | I | S 22 | 5 | 2 | 4 | II | S 39 | 1 | 3 | 2 | III |
| S 6 | 3 | 4 | 3 | I | S 23 | 5 | 2 | 4 | II | S 40 | 1 | 2 | 3 | III |
| S 7 | 5 | 3 | 3 | I | S 24 | 1 | 4 | 2 | II | S 41 | 2 | 3 | 2 | III |
| S 8 | 1 | 3 | 3 | I | S 25 | 1 | 4 | 2 | II | S 42 | 1 | 3 | 2 | III |
| S 9 | 4 | 3 | 4 | I | S 26 | 2 | 4 | 3 | II | S 43 | 1 | 3 | 2 | III |
| S 10 | 4 | 3 | 4 | I | S 27 | 3 | 4 | 2 | II | S 44 | 2 | 3 | 2 | III |
| S 11 | 4 | 4 | 3 | I | S 28 | 1 | 4 | 2 | II | S 45 | 3 | 2 | 3 | III |
| S 12 | 1 | 3 | 3 | I | S 29 | 1 | 4 | 2 | II | S 46 | 4 | 2 | 3 | III |
| S 13 | 3 | 4 | 3 | I | S 30 | 2 | 1 | 3 | II | S 47 | 3 | 2 | 3 | III |
| S 14 | 4 | 3 | 3 | I | S 31 | 3 | 2 | 4 | II | S 48 | 5 | 3 | 3 | III |
| S 15 | 5 | 3 | 4 | I | S 32 | 3 | 2 | 4 | II | S 49 | 3 | 2 | 3 | III |
| S 16 | 5 | 3 | 4 | I | S 33 | 5 | 2 | 4 | II | S 50 | 3 | 2 | 3 | III |
| S 17 | 5 | 4 | 3 | I | S 34 | 1 | 3 | 2 | III | | | | | |

Each object belongs to *one and only one* decision class. The upward and downward unions of decision classes are given as: $Cl_1^{\leq} = Cl_1; Cl_2^{\leq} = Cl_1 \bigcup Cl_2; Cl_2^{\geq} = Cl_3 \bigcup Cl_2; Cl_3^{\geq} = Cl_3; Cl_1^{\geq} = Cl_3^{\leq} = Cl_1 \bigcup Cl_2 \bigcup Cl_3$.

According to the strict dominance principle, we can obtain the $C$-lower approximation. Then, we can further obtain the separated boundary regions as: $Cl_3^{\geq} - \underline{C}(Cl_3^{\geq})$=$\{S2; S7; S8; S12; S14\}$; $Cl_2^{\geq} - \underline{C}(Cl_2^{\geq})$=$\{S2; S7; S8; S12; S14; S30; S21\}$; $Cl_1^{\leq} - \underline{C}(Cl_1^{\leq})$=$\{S35; S37; S48; S50; S49; S47; S46; S45; S38\}$; $Cl_2^{\leq} - \underline{C}(Cl_2^{\leq})$ =$\{S35; S37; S48; S26\}$.

### 4.2 Believable Rule Induction

We calculate the believe factor of each object which is from the rough boundary regions, as shown in Table 2. In this table, the believe degree of S46 is equal to zero rather than a positive value. Thus, S46 is not a valuable object, and it is unable to provide any assignment for decision-making. Excluding S46, other objects are all valuable objects and are able to induce believable rules. According to Strategy I and Strategy II in section 3.3, we generate the believable rules together with their measurements, as shown in Table 3.

**Table 2.** Believe factor of rough boundary objects

| Regions | Boundary objects | Believe factors | | | Measurements | | |
|---|---|---|---|---|---|---|---|
| | | $\mu(x)$ | $\pi(x)$ | $\nu(x)$ | $S(x)$ | $H(x)$ | $L(x)$ |
| $Cl_3^{\geq} - \underline{P}(Cl_3^{\geq})$ | S2; S14 | 9/13 | 3/13 | 1/13 | 8/13 | 10/13 | 12/13 |
| | S7 | 6/8 | 1/8 | 1/8 | 5/8 | 7/8 | 7/8 |
| | S8; S12 | 13/22 | 5/22 | 4/22 | 9/22 | 17/22 | 18/22 |
| $Cl_2^{\geq} - \underline{P}(Cl_2^{\geq})$ | S2; S14 | 9/13 | 3/13 | 1/13 | 8/13 | 10/13 | 12/13 |
| | S7 | 6/8 | 1/8 | 1/8 | 5/8 | 7/8 | 7/8 |
| | S8; S12 | 14/22 | 5/22 | 3/22 | 11/22 | 17/22 | 19/22 |
| | S21 | 13/19 | 4/19 | 2/19 | 11/19 | 15/19 | 17/19 |
| | S30 | 20/34 | 5/34 | 9/34 | 11/34 | 29/34 | 25/34 |
| $Cl_1^{\leq} - \underline{P}(Cl_1^{\leq})$ | S35; S37 | 8/14 | 3/14 | 3/14 | 5/14 | 11/14 | 11/14 |
| | S38 | 2/4 | 1/4 | 1/4 | 1/4 | 3/4 | 3/4 |
| | S45; S47; S49; S50 | 2/8 | 5/8 | 1/8 | 1/8 | 3/8 | 7/8 |
| | **S46** | **2/10** | **6/10** | **2/10** | **0/10** | **4/10** | **8/10** |
| | S48 | 8/24 | 9/24 | 7/24 | 15/24 | 15/24 | 17/24 |
| $Cl_2^{\leq} - \underline{P}(Cl_2^{\leq})$ | S26 | 14/19 | 3/19 | 2/19 | 12/19 | 16/19 | 17/19 |
| | S35; S37 | 10/14 | 2/14 | 2/14 | 8/14 | 12/14 | 12/14 |
| | S48 | 16/24 | 3/24 | 5/24 | 11/24 | 21/24 | 19/24 |

### 4.3 Verification of Sorting Capability

This section aims to verify the sorting capability of our induced rules. We choose existing representative DRSA models as competitors, including C-DRSA model

**Table 3.** Induction of believable decision rules

| Believable rules | Conditional criteria A | B | C | Assign- ments | Confidence degree | Accuracy degree | Base(s) of rules |
|---|---|---|---|---|---|---|---|
| [$B$1] | ≥4 | ≥3 | ≥3 | ≥I | 0.9231 | 0.7692 | S2; S14 |
| [$B$2] | ≥5 | ≥3 | ≥3 | ≥I | 0.8750 | 0.8750 | S7 |
| [$B$3] | ≥1 | ≥3 | ≥3 | ≥I | 0.8182 | 0.7727 | S8; S12 |
| [$B$4] | ≥4 | ≥3 | ≥3 | ≥II | 0.9231 | 0.7692 | S2; S14 |
| [$B$5] | ≥5 | ≥3 | ≥3 | ≥II | 0.8750 | 0.8750 | S7 |
| [$B$6] | ≥1 | ≥3 | ≥3 | ≥II | 0.8636 | 0.7727 | S8; S12 |
| [$B$7] | ≥4 | ≥2 | ≥3 | ≥II | 0.8947 | 0.7895 | S21 |
| [$B$8] | ≥2 | ≥1 | ≥3 | ≥II | 0.7353 | 0.8529 | S30 |
| [$B$9] | ≤2 | ≤3 | ≤3 | ≤III | 0.7857 | 0.7857 | S35; S37 |
| [$B$10] | ≤2 | ≤2 | ≤3 | ≤III | 0.7500 | 0.7500 | S38 |
| [$B$11] | ≤3 | ≤2 | ≤3 | ≤III | 0.8750 | 0.3750 | S45; S47; S49; S50 |
| [$B$12] | ≤5 | ≤3 | ≤3 | ≤III | 0.7083 | 0.6250 | S48 |
| [$B$13] | ≤2 | ≤4 | ≤3 | ≤II | 0.8947 | 0.8421 | S26 |
| [$B$14] | ≤2 | ≤3 | ≤3 | ≤II | 0.8571 | 0.8571 | S35; S37 |
| [$B$15] | ≤5 | ≤3 | ≤3 | ≤II | 0.7917 | 0.8750 | S48 |

[8], VC-DRSA model [7], and the extended scheme [2] of DRSA models. Hereinto, C-DRSA can be regarded as consistency level L=1.0. VC-DRSA can be denoted as L<1.0, i.e. L=0.9, L=0.8, L=0.7, etc. The extended scheme is with the symbol ˆ, i.e. $\hat{L} = 1.0, \hat{L} = 0.9, \hat{L} = 0.8$, etc.

Table 4 illustrates the statistical results of assignments in multicriteria sorting. Through generated certain rules, sorting rates are given in the first six rows of this table. The proposed believable rules together with the induced certain rules (via C-DRSA) are also tested for sorting. The comparison result indicates that our proposal provides the highest *correct sorting rate* (the ratio of the number of correctly classified objects over the total number of testing objects), which equals to 0.94.

**Table 4.** A comparison of correct sorting rate

| Alternative Proposals | Correctly Sorted objects | | Incorrectly Sorted objects | | Unknown objects | |
|---|---|---|---|---|---|---|
| 1. L=1.0 | 42 | 84% | 0 | 0% | 8 | 16% |
| 2. L=0.9 | 45 | 90% | 1 | 2% | 4 | 8% |
| 3. L=0.8 | 41 | 82% | 2 | 4% | 7 | 14% |
| 4. $\hat{L}$=1.0 | 36 | 72% | 6 | 12% | 8 | 16% |
| 5. $\hat{L}$=0.9 | 43 | 86% | 5 | 10% | 2 | 4% |
| 6. $\hat{L}$=0.8 | 42 | 84% | 8 | 16% | 0 | 0% |
| 7. Our proposal | 47 | 94% | 3 | 6% | 0 | 0% |

## 5 Conclusion

This paper provided a new idea for extracting uncertain information within rough boundary regions. In terms of intuitionistic fuzzy values, we proposed a new coefficient, called believe factor, together with three measuring degrees. Aided by these measurements, we further provided the method for inducing believable rule as a new kind of uncertain rule for sorting problems. In the experimental testing, we illustrated the process of believable rule induction, and verified its sorting capability via comparison with other representative proposals. In the future work, we shall develop this approach for multicriteria ranking and expand its ability for prediction. We would have been investigating some real-world applications using the proposed approach like supplier chain management (i.e. supplier selection).

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