Estimation and Characterization of Nonlinear Behavior of Nonlinear Spatio-Temporal Systems

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1. Motivation

Spatio-temporal systems (STSs) described by partial differential equations (PDEs) are widely used in physical and engineering systems [1]. Traditional methods for the analysis of a PDE system rely on an analytical solution of the system, which is difficult to obtain for most nonlinear PDEs. Alternative methods such as qualitative analysis of solutions or numerical methods for an approximation are studied using functional analysis and generalized function theory [2], or finite element methods and difference methods [3]. Identification of STSs has also been studied recently using finite dimensional parametric MIMO models to approximate infinite dimensional systems [4]. The estimation of spatio-temporal systems is formulated into a traditional identification problem of an MIMO system. Because multiple input/output variables are involved including their nonlinear combinations, existing methods are usually computationally intensive and not applicable for online problems. Therefore, an effective and powerful method for the estimation of nonlinear behavior of a STS is deserved to be further investigated. Importantly, this study will also aim to estimate physical characteristics characterized by some important parameters in the system PDE model. This will provide an important insight into the analysis and design of physical and structural properties of the dynamical system under study.

2. The method

Nonlinear STSs are firstly transformed into a class of MIMO partially linear systems (PLSs), and a new online identification algorithm for this class of PLSs is proposed using a pruning error minimization principle and least squares support vector machines (LS-SVM). Many benchmark physical and engineering systems can be transformed into an MIMO-PLS which keeps important physical spatio-temporal relationships that are very helpful in system identification and also in analysis of the underlying system. Compared with existing methods, the proposed algorithm can make full use of prior structural information on system physical models, can realize online estimation of system dynamics and achieve online characterization of some important nonlinear physical characteristics of the system. Given a nonlinear STS with appropriate boundary conditions

$$\frac{\partial^2 u}{\partial t^2} = b(x,y)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g(u), \quad 0 < t \leq T, \tag{1}$$

using some discretization methods, it can always be transformed into a discrete lattice difference model [5], and further formulated into a more general MIMO partially linear system model as

$$y_m(t) = \beta_m^T x^{m_1}(t) + f_m(x^{m_2}(t)), \quad m = 1, \ldots, M. \tag{2}$$

where x(t) is a vector including all the possible regressors, the superscript m1 represents those appearing linearly in the regressive model and the superscript m2 those appearing nonlinearly, m denotes the m-th channel located in space, $y_m(t)$ represents the states of the m-th channel at time t. Noticeably, β_m is an explicit function of b(x,y). Estimation of (2) will simultaneously predict the dynamic behavior $y_m(t)$ and identify the physical characteristic parameter b(x,y). Once (2) is obtained, system nonlinear behavior at any time and position can thus be predicted. Model (2) can be expressed with support vector machines as

$$y_m(t) = \beta_m^T x^{m_1}(t) + W_m^T \varphi_m(x^{m_2}(t)) + c_m \tag{3}$$

Here c_m is a bias term, φ is nonlinear support vector. The LS-SVM [6] is used to find weights β_m and W_m subject to

$$J_m = \min_{W_m, c_m, e_m(t), \beta_m} \frac{1}{2} W_m^T W_m + \frac{1}{2\gamma} \sum_{t=1}^{N} e_m(t)^2 \tag{4}$$

which results in the solution of the following dual problem

$$\begin{pmatrix} 0 & 0 & X_{m_1}^T \\ 0 & 0 & 1^T \\ X_{m_1} & 1 & \Omega_m+\gamma I \end{pmatrix} \cdot \begin{pmatrix} \beta_m \\ c_m \\ \alpha_m \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ y_m \end{pmatrix} \tag{5}$$

where $X_{m_1} = [x^{m_1}(1)^T, x^{m_1}(2)^T, \ldots, x^{m_1}(N)^T] \in N \times R^{N_{m_1}}$ and $y_m = [y_m(1), y_m(2), \ldots, y_m(N)]$. The LS-SVM uses the inverse of the square matrix in (5), denoted by $A_{m,N}$ at N. At N+1, it is given by

$$A^{-1}_{m,N+1} = \begin{pmatrix} A^{-1}_{m,N} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} h_m - a_m^T A_N^{-1} a_m \end{pmatrix}^{-1} \begin{pmatrix} A^{-1}_{m,N} a_m \\ -1 \end{pmatrix} \begin{pmatrix} a_m^T A^{-1}_{m,N} & -1 \end{pmatrix} \tag{6}$$

To achieve the parsimony of model (3), a pruning technique can be adopted such that the support vector machine with the least contribution is pruned from the model, i.e.,

$$\arg\min_j \sum_{m=1}^{M} D_m(x(j)) = \arg\min_j \sum_{m=1}^{M} \frac{\alpha_m^{N+1}(j)}{[A^{-1}_{m,N+1}]_{jj}} \tag{7}$$

In case that the sample j is to be pruned, $A_{m;N}$ is reconstructed by deleting the $m1+1+j$-*th* row and $m1+1+j$-*th* column of $A_{m;N+1}$, whose inverse can be computed by

$$A^{-1}_{m,N} = \overline{A}^{-1}_{m,N+1} - \frac{1}{a_m^\star} H_{m,N+1} H^T_{m,N+1}. \tag{8}$$

where

$$\overline{A}^{-1}_{m,N+1} = \begin{pmatrix} A^{-1}_{m,N+1}(1,1) & A^{-1}_{m,N+1}(1,2) \\ A^{-1}_{m,N+1}(2,1) & A^{-1}_{m,N+1}(2,2) \end{pmatrix}, \quad H_{m,N+1} = \begin{pmatrix} a_m(1,j) \\ a_m(2,j) \end{pmatrix}, \quad A^{-1}_{m,N+1} = \begin{pmatrix} A^{-1}_{m,N+1}(1,1) & a_m(1,j) & A^{-1}_{m,N+1}(1,2) \\ a_m^T(1,j) & a_m^\star & a_m^T(2,j) \\ A^{-1}_{m,N+1}(2,1) & a_m(2,j) & A^{-1}_{m,N+1}(2,2) \end{pmatrix}$$

3. Simulations and conclusions

Considering system (1) with $b(x,y)=x^2+y^2$ and $g(u)=0.2u^2(x,y,t)$ and appropriate boundary conditions. The predict output and the identified b(x,y) are given in Figures.

The proposed method can online predict accurately the complex spatio- temporal behaviors and identify the nonlinear characteristics of the physical or structural properties of the underlying system.

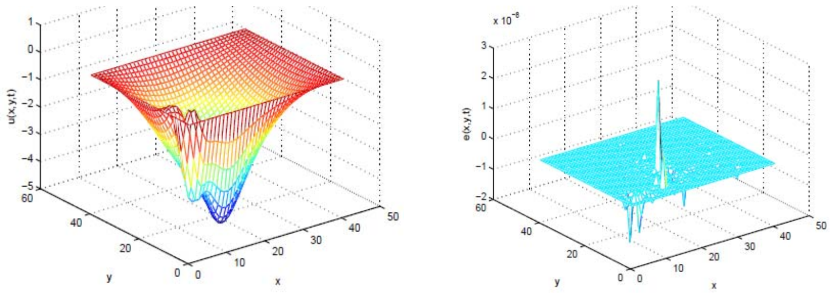

Figure 1 Prediction output at time t=1 and its error

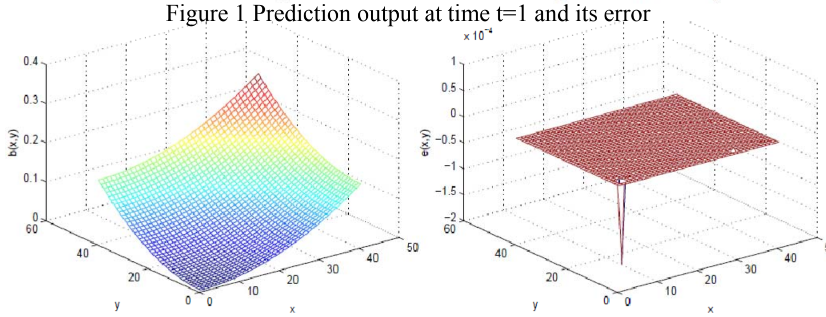

Figure 2. The identified b(x,y) and its error

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