

Changing the scattering of sheltered targets

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(Received 20 September 2010; published 11 April 2011)

In this paper, we propose a kind of illusion cloak that does not provide invisibility but instead changes the scattering of a coated target to that of a totally different one. Different from other illusion cloaks such as those based on “anti-object” or active sources, the proposed one is independent of the information of concealed targets or incident waves and can reshape the scattering of any targets. In addition, we also provide a general method to imitate arbitrary conductor line segments, as a special case of conductor reshapers. Electromagnetic (EM) simulations by a finite-element solver on detailed examples have been carried to validate the design.

DOI: [10.1103/PhysRevA.83.043809](https://doi.org/10.1103/PhysRevA.83.043809)

PACS number(s): 41.20.Jb, 42.25.Bs, 42.25.Fx

I. INTRODUCTION

Recent research has shown that waves and fields can be controlled and guided in transformation media in almost any manner [1–3]. Under the guidance of this method of transformation optics, both theoretical [4–12] and experimental [13–19] improvements on invisibility cloaks have been moving steadily from the microwave regime [13–16] into the optical frequency range [17–19]. In essence, the waves or lights are smoothly guided around the sheltered region by a properly designed media to emerge traveling in the same direction as if they had passed through an empty space, in which invisibility is thus achieved.

Inspired by the success in invisibility cloaks, one may ponder if it would be possible to design another kind of cloak that does not cause the sheltered target to become invisible but enables it to appear as a completely different object under lights. In other words, the waves or lights in such a cloak should not only be guided around the sheltered target but also be scattered as if they have encountered a different object that one desired.

Before approaching this issue formally, previous work concerning illusion optics will be described. Based on the cloaking strategy [2], Chen *et al.* proposed a perfectly electric conductor (PEC) reshaper [20] that can change the scattering of one perfect conductor to that of another one. In their design, the boundary of illusion PEC cylinder was mapped onto the inner boundary of the designed cloak. As a result, the sheltered PEC cylinder appeared as the illusion PEC cylinder. Because electromagnetic (EM) waves cannot penetrate into the conductor (in which the skin depth is very thin) yet are almost completely reflected on the surface, only a spatial mapping would be required to rebuild a proper optical path without material mapping. If the sheltered PEC cylinder is virtually extended to be larger than the cloaking device, then the cloaking material must be negative refractive indexed, as demonstrated in [21]. Currently, PEC reshapers have been applied extensively as superscatterers [22–24], which can enhance the scattering cross section (SCS) of an object, or on the contrary constructed as reduced scatterers [24,25], which would have a smaller SCS than the original scatterer. Liu

provided proof for these reshapers, and also demonstrated that this kind of reshaper can be designed not only in the EM domain but also in an acoustic regime [24]. However, whether in the EM domain or in an acoustic regime, the above-mentioned reshapers are only applicable for impenetrable targets, but not for penetrable objects.

Later on, a different illusion strategy based on folded transformation [26], complementary media [27], and an “anti-object” was proposed by Lai *et al.* [28]. They designed an EM illusion device that can transform the scattered light of an object into another object to achieve illusion. The basic principle employed in this approach is to eliminate the scattering of the original object by an “anti-object” before reconstructing an optical path identical to that of the illusion object. One key feature of this design is that it works at a distance away from the object. Unfortunately, this illusion system would demand specific knowledge of the shape and the material properties of both the original object and the illusion object. More importantly, this strategy cannot satisfactorily cancel a metallic (or perfect conductor) object by using an “anti-object” since it would be difficult to define an “anti-object” for a metallic object. Such an approach is also unsuitable for objects that absorb light [29]. Although designing external cloaks without embedded “anti-objects” [30] may help to solve the first problem, the other two inherent defects remain a challenge to overcome. Another interesting exterior optical illusion [31] can be achieved by using active sources [32] instead of transformation media. Zheng *et al.* demonstrated that active sources, if properly placed, would generate fields so that any object inside a certain domain would become invisible and the external observer would see an illusion of another object inside. However, the employed active sources are dependent on the property of the incident waves, therefore the information of the incident wave must be captured in advance.

In this paper, we continue to employ the cloaking strategy to address the so-called “illusion” issue as it enables us to create a tailor-made conformal cloak for any target, which would be very appealing for practical applications. Compared with the previous PEC reshapers, our work has made progress in at least two aspects. First a general description of scattering reshaping can be obtained. The mimicked (illusion) object would no longer be limited to an impenetrable one. We shall

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demonstrate that material mapping along with spatial mapping could help to render the designed cloak to reshape scattering of any dielectric target. Our second area of progress is that the proposed cloak would probably provide the best solution for reshaping an arbitrary conducting line in a near-perfect manner, an area that remains problematic. On the other hand, different from the illusion strategy based on an “anti-object” or active sources, our proposal would not require information from the concealed targets or the incident waves, which may facilitate and generalize the design. While the current study is limited to the EM domain, this idea may be extended to an acoustic regime as well [24].

II. THEORETICAL DESCRIPTION

This section will describe how an illusion cloak is designed in detail. Since the metallic case (PEC reshaper) has been fully discussed [20,22–25], the following analyses are mainly based on the dielectric case. One notable difference between the metallic case and the dielectric case is that material mapping is not required for a metallic object but is indispensable for a dielectric one. As shown in Fig. 1, to design such an illusion cloak, we would need to compress the desired illusion object **B** [with contour Γ_M and material parameters ϵ_M and μ_M in Fig. 1(b)] into a transformation media shell [the inner shell of the cloak with boundaries of Γ'_{IN} and Γ'_M in Fig. 1(a)] through proper spatial transformation, generally written as

$$r' = f_{\text{inner}}(r, \theta, \varphi), \quad \theta' = \theta, \quad \varphi' = \varphi \quad (1)$$

combined with boundary conditions of

$$r'|_{\Gamma'_{IN}} = f_{\text{inner}}(r, \theta, \varphi)|_{r=0}, \quad r'|_{\Gamma'_M} = f_{\text{inner}}(r, \theta, \varphi)|_{\Gamma_M}. \quad (2)$$

In addition, a surrounded space, assuming $\Gamma_M < r < \Gamma_{\text{OUT}}$, should also be compressed into another shell [the outer shell of the cloak with boundaries of Γ'_M and $\Gamma'_{\text{OUT}} = \Gamma_{\text{OUT}}$ in Fig. 1(a)] through spatial transformation of

$$r' = f_{\text{outer}}(r, \theta, \varphi), \quad \theta' = \theta, \quad \varphi' = \varphi, \quad (3)$$

combined with boundary conditions of

$$r'|_{\Gamma'_M} = f_{\text{outer}}(r, \theta, \varphi)|_{\Gamma_M}, \quad r'|_{\Gamma'_{\text{OUT}}} = f_{\text{outer}}(r, \theta, \varphi)|_{\Gamma_{\text{OUT}}}. \quad (4)$$

In the case of $\Gamma_{\text{OUT}} < r < \Gamma_M$, which indicates that the mimicked object **B** is bigger than the designed cloak, the spatial

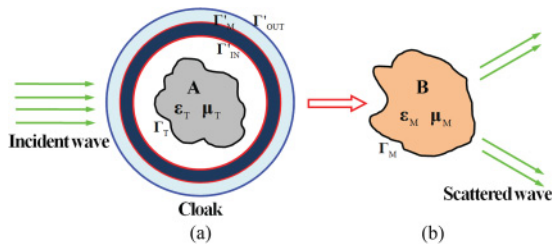


FIG. 1. (Color online) A given target **A** (with contour Γ_T and material parameters ϵ_T and μ_T) coated with a specially tailored cloak shown in (a) has the same “appearance” as a different desired object **B** (with contour Γ_M and material parameters ϵ_M and μ_M) shown in (b) under EM waves.

transformation is a folded one and the above analyses are still applicable.

According to the theory of transformation optics, once the spatial mapping from original space to distorted space is obtained, the permittivity ϵ' and permeability μ' of the transformation media can be calculated by the formulas of $\epsilon' = \mathbf{A}\epsilon\mathbf{A}^T/\det(\mathbf{A})$ and $\mu' = \mathbf{A}\mu\mathbf{A}^T/\det(\mathbf{A})$, where \mathbf{A} is the Jacobian transformation matrix from an original coordinate system \mathbf{x} to a transformed coordinate system $\mathbf{x}'(\mathbf{x})$ with an element of $A_{ij} = \partial x'_i/\partial x_j$, and ϵ and μ are the permittivity and permeability tensors in the original space, respectively. Therefore, with respect to the corresponding coordinate transformation of $(r, \theta, \varphi) \rightarrow (r', \theta', \varphi')$, the transformation media in the inner shell of the cloak can be calculated as

$$\epsilon' = \frac{\mathbf{A}\epsilon_M\mathbf{A}^T}{\det(\mathbf{A})}, \quad \mu' = \frac{\mathbf{A}\mu_M\mathbf{A}^T}{\det(\mathbf{A})}, \quad (5)$$

and in the outer shell, the transformation media would have a simplified form of

$$\epsilon' = \mu' = \frac{\mathbf{A}\mathbf{A}^T}{\det(\mathbf{A})} \quad (6)$$

since ϵ and μ are equal to 1 in the free space.

One can find that the designed cloak will have two transformation media shells. The outer shell [see Eq.(6)] embodies the spatial mapping; while the inner shell [see Eq. (5)] embodies the material mapping, which is distinguished from the invisibility cloaks and PEC reshapers. Undoubtedly, the analyses are suitable for any dielectric mimicked objects. In fact, they also contain the solution for the metallic case. In other words, the PEC or perfectly magnetic conductor (PMC) reshaper can also be designed with the proposal. In this case, the material mapping described by Eqs. (1), (2), and (5) is not needed any more. One only needs a thin metallic shell (usually replaced by the PEC/PMC boundary for simplicity in analyses) to reflect the lights and a properly tailored transformation media shell [the outer shell, tailored by Eqs. (3), (4), and (6)] to reconstruct the correct optical path.

So the recipe proposed here enables the tailored cloak to imitate any object (either dielectric or metallic). And apparently the design does not rely on the material property of the sheltered target **A**, nor does it require the information of the incident waves at all. Besides, the shape Γ_T of the sheltered target **A** is also not required unless one wants to tailor a conformal cloak for **A**, which means $\Gamma'_{IN} = \Gamma_T$.

To illustrate the recipe more clearly, a two-dimensional (2D) cloak with cylindrical structure, for simplicity and without loss of generality, is analyzed in detail as an example.

We denote the boundaries of the cloaking shells as Γ'_{IN} : $r' = a$, Γ'_{OUT} : $r' = b$, and Γ'_M : $r' = c$, respectively, while the mimicked object is assumed to be a dielectric cylinder with boundary Γ_M : $r = d$ and material parameters of

$$\epsilon = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{xy} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}. \quad (7)$$

For the inner shell, the spatial transformation satisfying the boundary conditions $a = f_{\text{inner}}(r, \theta)|_{r=0}$ and $c = f_{\text{inner}}(r, \theta)|_{r=d}$ is chosen as $r' = (c - a)r/d + a$.

According to Eq. (5), the material parameters in the shell can be calculated as

$$\varepsilon'_{xx} = \frac{(r'^3 - ax'^2)^2 \varepsilon_{xx} - 2ax'y'(r'^3 - ax'^2)\varepsilon_{xy} + a^2x'^2y'^2\varepsilon_{yy}}{r'^5(r' - a)}, \quad (8a)$$

$$\varepsilon'_{xy} = \frac{ax'y'(ax'^2 - r'^3)\varepsilon_{xx} + (r'^6 + 2a^2x'^2y'^2 - ar'^5)\varepsilon_{xy} + ax'y'(ay'^2 - r'^3)\varepsilon_{yy}}{r'^5(r' - a)}, \quad (8b)$$

$$\varepsilon'_{yy} = \frac{(r'^3 - ay'^2)^2 \varepsilon_{yy} - 2ax'y'(r'^3 - ay'^2)\varepsilon_{xy} + a^2x'^2y'^2\varepsilon_{xx}}{r'^5(r' - a)}, \quad (8c)$$

$$\varepsilon'_{zz} = \frac{d^2(r' - a)}{(a - c)^2 r'} \varepsilon_{zz}, \quad (8d)$$

and

$$\mu'_{xx} = \frac{(r'^3 - ax'^2)^2 \mu_{xx} - 2ax'y'(r'^3 - ax'^2)\mu_{xy} + a^2x'^2y'^2\mu_{yy}}{r'^5(r' - a)}, \quad (9a)$$

$$\mu'_{xy} = \frac{ax'y'(ax'^2 - r'^3)\mu_{xx} + (r'^6 + 2a^2x'^2y'^2 - ar'^5)\mu_{xy} + ax'y'(ay'^2 - r'^3)\mu_{yy}}{r'^5(r' - a)}, \quad (9b)$$

$$\mu'_{yy} = \frac{(r'^3 - ay'^2)^2 \mu_{yy} - 2ax'y'(r'^3 - ay'^2)\mu_{xy} + a^2x'^2y'^2\mu_{xx}}{r'^5(r' - a)}, \quad (9c)$$

$$\mu'_{zz} = \frac{d^2(r' - a)}{(a - c)^2 r'} \mu_{zz}. \quad (9d)$$

Similarly, for the outer shell, the spatial transformation satisfying the boundary conditions $c = f_{\text{outer}}(r, \theta)|_{r=d}$ and $b = f_{\text{outer}}(r, \theta)|_{r=b}$ is chosen as $r' = (b - c)(r - d)/(b - d) + c$. According to Eq. (6), the material parameters in this shell are calculated as

$$\begin{aligned} \varepsilon'_{xx} &= \mu'_{xx} \\ &= \frac{(b - d)r' - b(c - d)}{(b - d)r'} \frac{x'^2}{r'^2} + \frac{(b - d)r'}{(b - d)r' - b(c - d)} \frac{y'^2}{r'^2}, \end{aligned} \quad (10a)$$

$$\begin{aligned} \varepsilon'_{xy} &= \mu'_{xy} \\ &= \left[\frac{(b - d)r' - b(c - d)}{(b - d)r'} - \frac{(b - d)r'}{(b - d)r' - b(c - d)} \right] \frac{x'y'}{r'^2}, \end{aligned} \quad (10b)$$

$$\begin{aligned} \varepsilon'_{yy} &= \mu'_{yy} \\ &= \frac{(b - d)r' - b(c - d)}{(b - d)r'} \frac{y'^2}{r'^2} + \frac{(b - d)r'}{(b - d)r' - b(c - d)} \frac{x'^2}{r'^2}, \end{aligned} \quad (10c)$$

$$\varepsilon'_{zz} = \mu'_{zz} = \frac{(b - d)^2 r' - b(c - d)(b - d)}{(b - c)^2 r'}. \quad (10d)$$

III. NUMERICAL VALIDATION

To validate the design, full-wave simulations on a few detailed cloaks are carried out in the following. We assume the transverse electric (TE mode) plane wave is incident from left to right with wavelength $\lambda = 1$ unit, and the geometrical sizes of the cloak and the mimicked object are $a = 1.4$, $b = 2$, $c = 1.7$, and $d = 0.8$. We would first study a simple case in which the mimicked object is homogeneous and isotropic with material parameters $\varepsilon = 2$ and $\mu = 1$. Figure 2(a) presents

the total electric-field distribution when the dielectric cylinder is under the incident wave. A cloak that is supposed to mimic the dielectric cylinder could be tailored directly by Eqs. (8)–(10). The “appearance” of the as-designed cloak under the incident wave is depicted in Fig. 2(b). One can see that the cloak scatters off the waves almost in the same manner as the dielectric cylinder. The result suggests that a homogeneous and isotropic object could be perfectly imitated by a properly tailored illusion cloak in the proposal. However, one may argue that the mimicked object people are interested in is usually more complex than a homogeneous and isotropic one. Therefore, further demonstration is necessary to verify our design. Figure 2(c) shows the “appearance” of an anisotropic cylinder with material parameters $\varepsilon = \mu = [1, 4, 0; 4, 6, 0; 0, 0, 2]$ under the incident waves. Apparently, the scattering pattern of this anisotropic object is different from the previous homogeneous and isotropic one. Nevertheless, it can still be imitated exactly by a specially tailored cloak, as shown in Fig. 2(d). An even more complex object, which is inhomogeneous and anisotropic with material parameters $\varepsilon = \mu = [3 + r, 2 + r, 0; 2 + r, 5 + r, 0; 0, 0, 4 + r]$, is shown in Fig. 2(e). Although the interaction between the wave and the material is quite complicated inside the cylinder, as shown in Fig. 2(e), the overall performance of the cylinder under the wave is still perfectly imitated by a specially tailored cloak, as shown in Fig. 2(f).

The above examples suggest that the proposal indeed enables the as-designed cloaks to change scattering of the sheltered target to any other dielectric objects. To demonstrate the generality of the proposal, a simple metallic example is also provided. A metallic cylinder under the incident wave is shown in Fig. 2(g), while a specially designed cloak is presented in Fig. 2(h). The inner shell of the cloak is a metallic shell and the outer shell is a transformation media shell tailored by Eq. (10). By comparing the scattering patterns

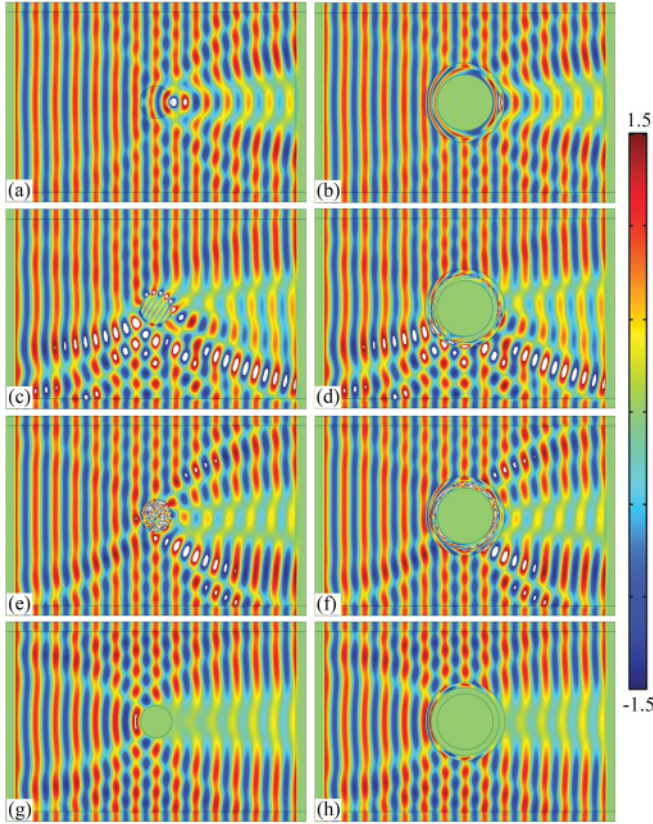


FIG. 2. (Color) Changing the scattering of sheltered target into another object. The TE-mode plane wave is incident from left to right with wavelength $\lambda = 1$. Homogeneous and isotropic dielectric cylinder with $\epsilon = 2$ and $\mu = 1$ shown in (a), anisotropic dielectric cylinder with $\epsilon = \mu = [1, 4, 0; 4, 6, 0; 0, 0, 2]$ shown in (c), inhomogeneous and anisotropic dielectric cylinder with $\epsilon = \mu = [3 + r, 2 + r, 0; 2 + r, 5 + r, 0; 0, 0, 4 + r]$, shown in (e), and metallic cylinder shown in (g) are imitated perfectly by the properly tailored cloaks as shown in (b), (d), (f), and (h), respectively.

in these two figures [Figs. 2(g) and 2(h)], one may conclude that the as-designed cloak indeed resembles the metallic cylinder.

The case in which the mimicked object is a metallic line segment has attracted particular attention. The issue of “line transformation,” as named by some researchers, has been raised recently [33–35]. However, this issue has not been solved satisfactorily apart from a special solution for elliptical structures with strict constrains in a 2D elliptic coordinate system. Although a line segment cannot be stretched to a contour by radial coordinate transformation, it is possible to achieve a general and near-perfect approximation through our proposed method, as an extremely thin structure can approach a line segment rather effectively in either 2D or 3D space. For example, to imitate a line in 3D space, we may choose a mimicked object as a very thin ellipsoid or cube. With the help of geometric approximation, we could design illusion cloaks to imitate any metallic line whose length and location are no longer as restricted as they were in previous works [33–35].

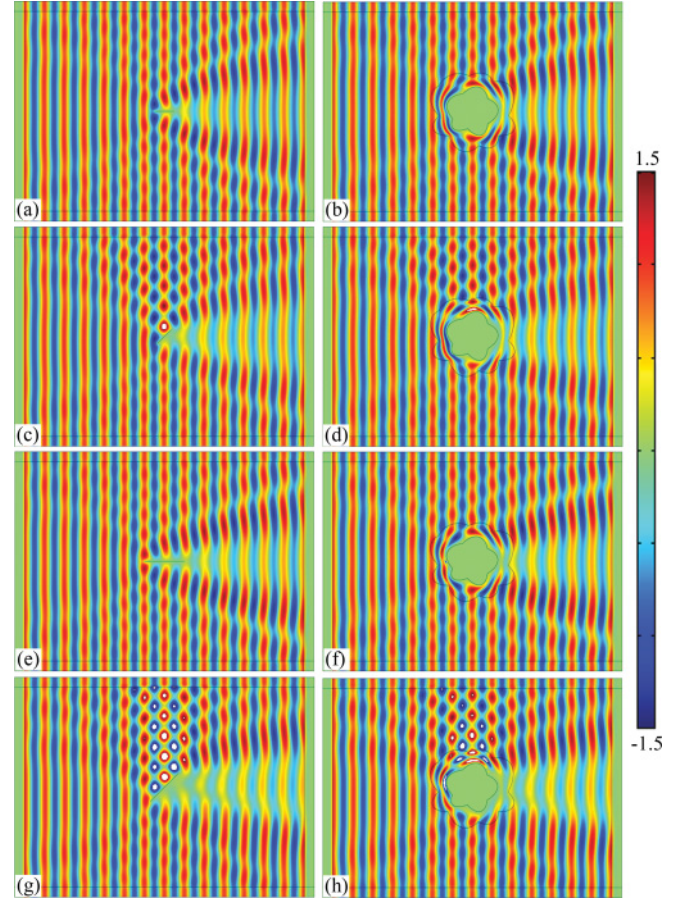


FIG. 3. (Color) Changing the scattering of the sheltered target with arbitrary cross section into a PEC line. The TE-mode plane wave is incident from left to right with wavelength $\lambda = 1$. A PEC line with length $l = 1$ located in the parallel direction (a) and in the oblique direction (c) (with an oblique angle of 45° from the x axis) is imitated by properly tailored cloaks shown in (b) and (d), respectively. When the length of the line increases to $l = 2$, as shown in (e) and (g), these targets can also be well imitated by their accordingly tailored cloaks, as shown in (f) and (h).

More detailed examples (still in 2D space) would further confirm our supposition. Here we choose the ellipse as the approximation geometry. One should note that when the axis ratio n (major axis versus minor axis) is very large, the ellipse would get quite close to the major axis l (also the mimicked line). To better exhibit the generality of the method, we would like to present a small number of cloaks with irregular cross section rather than regular cylindrical structure.

As far as the cloaks with arbitrary shapes are concerned, there are several approaches to obtain the required coordinate transformation, either analytical [7,8] or numerical [9,10]. Here we take the numerical solutions of the Laplace equation (a detailed introduction is available in [9]) to design the cloaks. For the outer shell, the spatial transformation $r' = f_{\text{outer}}(r, \theta)$ combined with boundary conditions of $r'|_{\Gamma'_M} = f_{\text{outer}}(r, \theta)|_{\Gamma_M}$ and $r'|_{\Gamma'_{\text{OUT}}} = f_{\text{outer}}(r, \theta)|_{\Gamma_{\text{OUT}}}$ is numerically solved by the Laplace equation in the partial differential equation (PDE) mode of commercial software COMSOL MULTIPHYSICS. Then

the parameters ϵ' and μ' can be calculated with the numerical solutions.

Figure 3 presents the simulation results of several PEC line segments and the correspondingly tailored cloaks (in which all demonstrated cloaks have the same cross section). The scattering of a parallel (along the x axis) PEC line segment with length $l = 1$ under the incident wave is depicted in Fig. 3(a). Accordingly, the ‘‘appearance’’ of a cloak which is designed with $l = 1$, $n = 20$ with the major axis parallel to the x axis is shown in Fig. 3(b). By comparing Figs. 3(a) and 3(b), it can be seen that the scattering of the tailored cloak approximates well that of the PEC line. To gain further insight into this phenomenon, an oblique located line is taken into account. The scattering of a PEC line with length $l = 1$ but an oblique angle of 45° from the x axis under the parallel incident wave is depicted in Fig. 3(c). The line is also well imitated by a cloak whose major axis is correspondingly designed with an angle of 45° from the x axis, as shown in Fig. 3(d). The above examples [Figs. 3(a)–3(d)] should verify that the cloak in the proposal can imitate line segments in any location. We have also tested our approach in terms of length. We increase the length of the line from 1 to 2 and redesign the cloaks. Consistent with previous results, the line located in either the parallel direction, as shown in Fig. 3(e), or the oblique direction, as shown in Fig. 3(g), could be well imitated by the specially tailored cloaks, as shown in Figs. 3(f) and 3(h), respectively.

To evaluate the illusion performance quantitatively, the SCS is employed. Here the scattering width $\sigma = 2\pi r |E^s(\varphi)|^2 / |E^i|^2$ (2D case) of an object is calculated based on Huygens’ principle. The simulated data of near field is used to calculate far field by near-field–far-field transformation. The electric field in the far-field region is thus calculated as

$$E(\varphi) = \frac{1}{2} \sqrt{\frac{jk}{2\pi r}} \vec{\tau} \times \oint_l [\eta \vec{\tau} \times (\vec{n} \times \vec{H}) - (\vec{n} \times \vec{E})] \times \exp(-j \vec{k} \cdot \vec{r}') dl', \quad (11)$$

where \vec{E} and \vec{H} are EM fields on the integration contour l , \vec{r}' is the position vector on the contour l , $\vec{\tau}$ is the unit vector of the scattering direction, \vec{n} is unit vector of the contour outward normal direction, and η is the wave impedance of free space. A more detailed description of the SCS of cloaks is also available in Ref. [16].

As shown in Fig. 4, the normalized scattering widths of the cloaks by calculation are close to those of PEC line segments. The deviation is partially due to the approximation method and partially due to the numerical calculation of the software. Based on the results shown in Figs. 3 and 4, it can be concluded that the proposed approximation method enables the tailored cloak to mimic arbitrary metallic lines.

Because of the numerical method, no analytical expressions of ϵ' and μ' for the cloaks (shown in Fig. 3) are available. To obtain a clear picture of the parametric requirements for the fabrication of such cloaks, the numerically calculated distributions of the ϵ' (μ') in these cloaks are presented in Fig. 5. It can be seen that the parameters are distributed still quite irregularly in the cloaking shells, although the singularity is

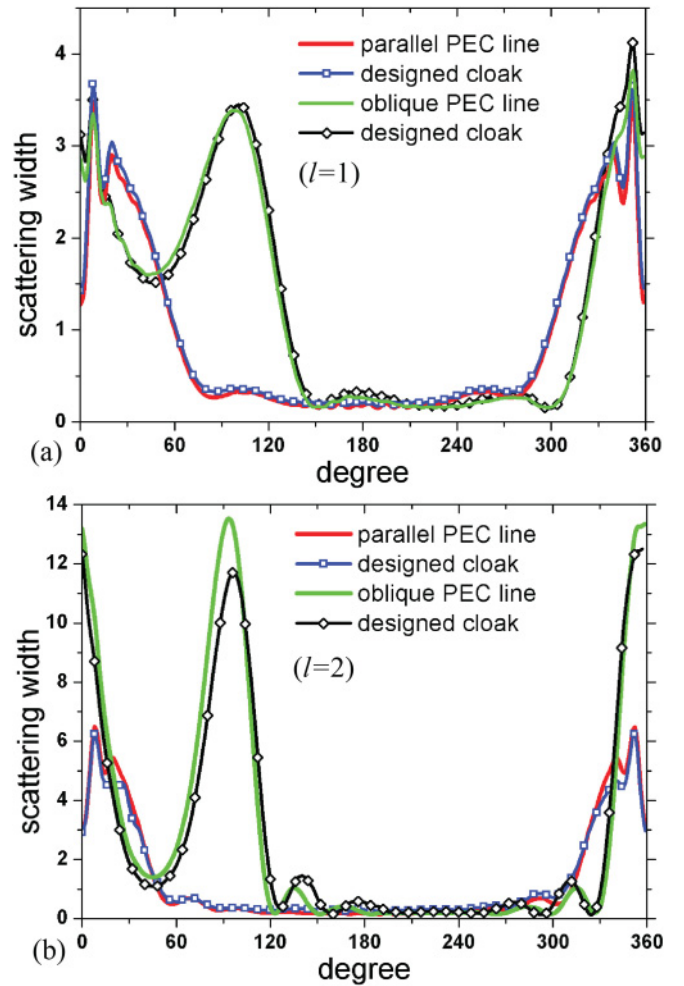


FIG. 4. (Color online) The calculated scattering width of the PEC line segments and cloaks in Fig. 3. The line segments are with length (a) $l = 1$ and (b) $l = 2$, respectively.

eliminated. Therefore, when it comes to a practical realization, it would be necessary to simplify the parameters, and certain existing methods applied to invisibility cloaks may help to tackle this problem [36–39]. A metamaterial element, split ring resonators (SRR’s) [13,15], or a nonresonant element [14], for example, may be used to construct the cloaks in a microwave frequency domain, and metallic cutwires [40] may be available in the optical range. Finish machining may also be required during fabrication.

IV. CONCLUSION

We have proposed a general recipe that enables the designed shell-like cloak to scatter waves similar to that of any other desired objects. Different from other illusion systems, the one presented here does not rely on the specific knowledge of sheltered targets or incident waves. Any mimicked objects, including dielectric objects, metallic objects, and even an arbitrary metallic line, could be well imitated by a properly designed cloak. Although we are fully aware that it would have a very high demand on fabrication technique

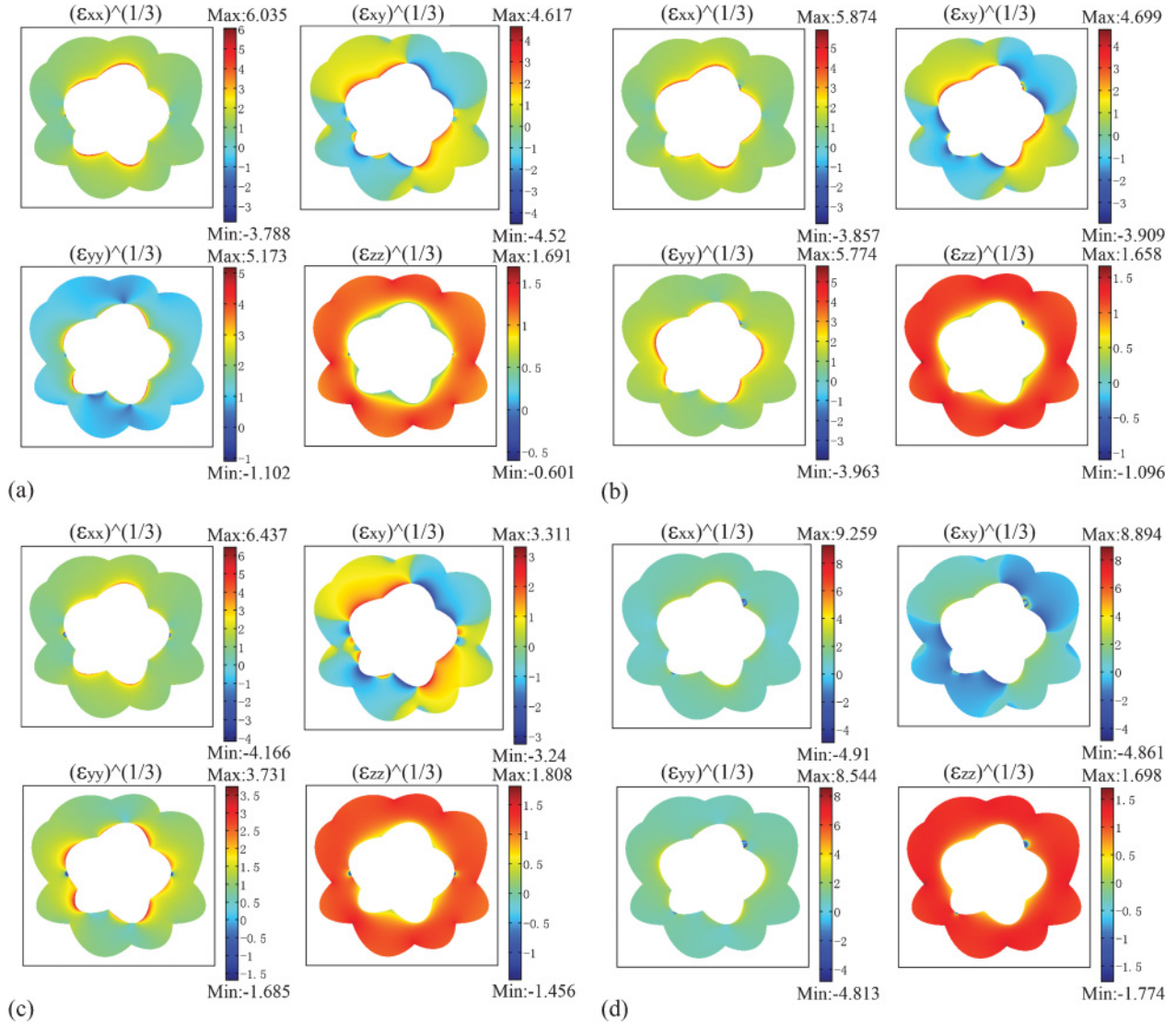


FIG. 5. (Color) The distributions of the ϵ' (μ') in the designed cloaks shown in Fig. 3. Parameters in (a), (b), (c), and (d) correspond to the cloak (b), (d), (f), and (h) in Fig. 3, respectively.

when practical realization is considered, given the current experimental progress in invisibility cloaks and metamaterials, we are confident about the potential applications of our proposal.

ACKNOWLEDGMENTS

This work is supported by the Hong Kong Innovation and Technology Fund (ITP/026/09NP); and the STCSM-Philips Joint R&D Fund (07SP07001).

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