PHYSICAL REVIEW B VOLUME 62, NUMBER 3 15 JULY 2000-I

Landauer-Büttiker formula for time-dependent transport through resonant-tunneling structures: A nonequilibrium Green's function approach

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(Received 18 January 2000; revised manuscript received 6 March 2000)

A nonequilibrium Green's-function formalism is employed to study the time-dependent transport through resonant-tunneling structures. With this formalism, we derive a time-dependent Landauer-Büttiker formula that guarantees current conservation and gauge invariance. Furthermore, we apply the formula to calculate the response behaviors of the resonant-tunneling structures in the presence of rectangular-pulse and harmonic-modulation fields. The results show that the displacement current plays the role of retarding the tunneling current.

The study of semiconductor nanostructures is one of the most active areas in condensed-matter physics. Many phenomena of electrical conduction in the dc situation have been successfully explained for mesoscopic systems[1] since the pioneering works of Landauer[2] and Büttiker *et al.*[3] Recently, time-dependent transport through mesoscopic systems has aroused much interest and the Keldysh nonequilibrium Green's functions are widely applied to analyze such phenomenon.[4] However, in many studies, only the tunneling current is calculated, while the displacement current induced by a time-dependent field is ignored. In recent years, this incompleteness has begun to be recognized.[5–7]

As a matter of fact, the importance of the displacement current was first noted by Büttiker and co-workers.[8–10] To analyze the electron transport in the presence of ac fields, they developed a linear, low-frequency theory[8,9] based on the scattering matrix formulation, in which current conservation and gauge invariance are guaranteed. Very recently, Wang *et al.*[7] further studied the ac problem by means of nonequilibrium Green's functions and also derived an expression for the linear dynamic conductance that conserves electric current and satisfies the gauge invariance. In the present paper, we investigate the time-dependent transport through resonant-tunneling structures. We also adopt the nonequilibrium Green's-function formalism such that far-from-equilibrium situations can be analyzed conveniently. In contrast to previous works by Büttiker *et al.*[8,9] and Wang *et al.*,[7] we conduct our calculations directly in the time domain and a time-dependent Landauer-Büttiker formula that guarantees current conservation and gauge invariance is derived. In our approach, the derived time-dependent current formula is applicable to both small and large bias-voltage situations. Finally, the Landauer-Büttiker formula is applied to the rectangular-pulse and harmonic-modulation cases to reveal the response behaviors of the resonant-tunneling structures.

We first consider the system described by the Hamiltonian

$$H=\sum_{\sigma,k\in L,R}\epsilon_{k\sigma}c_{k\sigma}^{\dagger}c_{k\sigma}+\epsilon_{d\sigma}(t)c_{d\sigma}^{\dagger}c_{d\sigma}+\sum_{\sigma,k\in L,R}(V_{k\sigma}c_{k\sigma}^{\dagger}c_{d\sigma}+\text{H.c.}). \tag{1}$$

The first term describes the leads. The operator $c_{k\sigma}^{\dagger}$ creates an electron with momentum k and spin σ in either the left (L) or right (R) lead. The second term models the scattering region and $c_{d\sigma}^{\dagger}$ creates an electron with a time dependent energy level $\epsilon_{d\sigma}(t)=\epsilon_{0\sigma}+\Delta(t)$. The third term characterizes the tunneling transfer of electrons between the leads and the scattering region. This Hamiltonian can be used to model either a quantum dot with one energy level (in which the intradot repulsion is not taken into account) or a double-barrier quantum-well structure.

The time dependence of the energy level $\epsilon_{d\sigma}(t)$ in the scattering region can be introduced by imposing a time-dependent field. For instance, in a lateral quantum-dot structure, it can be realized by applying a time-dependent voltage to the gate above the dot. As a result of the time dependence, charge accumulation occurs in the scattering region and tunneling current is not conserved. Nevertheless, conservation of the total current is preserved and one has[5,7,11]

$$J_{\sigma L}^{c}(t)+J_{\sigma R}^{c}(t)-e\frac{dN_{\sigma}(t)}{dt}=0, \tag{2}$$

where $J_{\sigma L(R)}^{c}(t)$ is the tunneling current with spin σ flowing from the left (right) lead into the scattering region, and

$J^d_\sigma(t)=-edN_\sigma(t)/dt$ is the displacement current. Here, $N_\sigma(t)=\langle c^\dagger_{d\sigma}(t)c_{d\sigma}(t)\rangle$ represents the occupation of the level $\epsilon_{d\sigma}(t)$. We will explain later that the displacement current can be partitioned into two terms, i.e., $J^d_\sigma(t)=J^d_{\sigma L}(t)+J^d_{\sigma R}(t)$. Equation (2) can then be rewritten as

$$J_{\sigma L}(t)+J_{\sigma R}(t)=0, \tag{3}$$

where $J_{\sigma\alpha}(t)=J^c_{\sigma\alpha}(t)+J^d_{\sigma\alpha}(t)$ is the total current with spin σ flowing from the lead α into the scattering region.

According to the theory of Jauho and co-workers,[11,12] the tunneling current with spin σ, flowing from the left (right) lead into the scattering region, is given by

$$J^c_{\sigma L(R)}(t)=-\frac{2e}{\hbar}\int_{-\infty}^{t}dt_1\int\frac{d\epsilon}{2\pi}\,\mathrm{Im}\{e^{-i\epsilon(t_1-t)}\Gamma^{L(R)}_\sigma(\epsilon)\times[G^<_\sigma(t,t_1)+f_{L(R)}(\epsilon)G^r_\sigma(t,t_1)]\}, \tag{4}$$

where

$$\Gamma^{L(R)}_\sigma(\epsilon)=2\pi\sum_{k\in L(R)}|V_{k\sigma}|^2\delta(\epsilon-\epsilon_{k\sigma})$$

and $f_{L(R)}(\epsilon)$ is the Fermi distribution function of the isolated left (right) lead. The retarded and Keldysh Green's functions, $G^r_\sigma(t,t_1)$ and $G^<_\sigma(t,t_1)$, are given, respectively, by

$$G^r_\sigma(t,t_1)=-i\theta(t-t_1)\langle\{c^\dagger_{d\sigma}(t_1),c_{d\sigma}(t)\}\rangle \tag{5}$$

and

$$G^<_\sigma(t,t_1)=i\langle c^\dagger_{d\sigma}(t_1)c_{d\sigma}(t)\rangle. \tag{6}$$

Here, we consider the wideband limit[11] where the elastic couplings to the leads Γ^α_σ, are independent of energy. In this limit, the total current with spin σ, $J_\sigma(t)=J_{\sigma L}(t)=-J_{\sigma R}(t)$, can be written as

$$\begin{aligned}J_\sigma(t)&=\frac{1}{\Gamma^L_\sigma+\Gamma^R_\sigma}[\Gamma^R_\sigma J_{\sigma L}(t)-\Gamma^L_\sigma J_{\sigma R}(t)]\\&=\frac{e}{\hbar}\int d\epsilon\frac{\Gamma^L_\sigma\Gamma^R_\sigma}{\Gamma^L_\sigma+\Gamma^R_\sigma}[f_L(\epsilon)-f_R(\epsilon)]\rho^{(0)}_\sigma(\epsilon,t)\\&\quad+\frac{1}{\Gamma^L_\sigma+\Gamma^R_\sigma}[\Gamma^R_\sigma J^d_{\sigma L}(t)-\Gamma^L_\sigma J^d_{\sigma R}(t)],\end{aligned} \tag{7}$$

with $\rho^{(0)}_\sigma(\epsilon,t)$ given by

$$\rho^{(0)}_\sigma(\epsilon,t)=-\frac{1}{\pi}\,\mathrm{Im}\{a_\sigma(\epsilon,t)\}, \tag{8}$$

where

$$a_\sigma(\epsilon,t)=\int dt_1 e^{i\epsilon(t-t_1)}G^r_\sigma(t,t_1). \tag{9}$$

The retarded Green's function is given by

$$G^r_\sigma(t,t_1)=-i\theta(t-t_1)\exp\left\{-i\int_{t_1}^{t}dt_2\left[\epsilon_{0\sigma}+\Delta(t_2)-\frac{i}{2}\Gamma_\sigma\right]\right\}, \tag{10}$$

with $\Gamma_\sigma=\Gamma^L_\sigma+\Gamma^R_\sigma$.

The occupation of the level $\epsilon_{d\sigma}(t)$ can be written as $N_\sigma(t)=\mathrm{Im}\{G^<_\sigma(t,t)\}$ according to the definition of the Keldysh Green's function. Following Jauho *et al.*,[11] it can be shown that

$$G^<_\sigma(t,t)=\sum_{L,R}\Gamma^{L/R}_\sigma\int\frac{d\epsilon}{\pi}f_{L/R}(\epsilon)A_\sigma(\epsilon,t), \tag{11}$$

where

$$A_\sigma(\epsilon,t)=\frac{i}{2}|a_\sigma(\epsilon,t)|^2. \tag{12}$$

Therefore, the displacement current $J^d_\sigma(t)=-edN_\sigma(t)/dt$ may naturally be cast into the partitioned form

$$J^d_\sigma(t)=J^d_{\sigma L}(t)+J^d_{\sigma R}(t), \tag{13}$$

where

$$J^d_{\sigma L(R)}(t)=-\frac{e}{\hbar}\int\frac{d\epsilon}{\pi}\Gamma^{L(R)}_\sigma f_{L(R)}(\epsilon)\mathrm{Im}\left\{\frac{dA_\sigma(\epsilon,t)}{dt}\right\}. \tag{14}$$

In the present situation, the displacement currents vanish in the absence of tunneling, i.e., $J^d_{\sigma L(R)}(t)=0$ for $\Gamma^{L(R)}_\sigma=0$. This is because the charge accumulation, only due to the tunneling, is taken into account. Clearly, it can be seen from Eq. (11) that $G^<_\sigma(t,t)=0$ and $dG^<_\sigma(t,t)/dt=0$ for both $\Gamma^L_\sigma=0$ and $\Gamma^R_\sigma=0$, leading to $N_\sigma(t)=0$ and $dN_\sigma(t)/dt=0$ in the absence of tunneling. If the charge accumulation due to a pure capacitor coupled to a time-varying voltage source[8,9] is also taken into account, the total charge number accumulated in the scattering region is then given by $N_\sigma(t)+N_{\sigma c}(t)$, where $N_{\sigma c}(t)$ is induced by the pure capacitor coupled to a time-varying voltage source and its derivative with respect to time, $dN_{\sigma c}(t)/dt$ can be nonzero in the absence of tunneling. In this case, one needs to further partition the accumulated charge number $N_{\sigma c}(t)$ in order that the total displacement current may be partitioned. However, the present approach does not concern this case, but focuses on the situation[11] that the charge accumulation in the scattering region results from tunneling.

Substituting Eq. (14) into Eq. (7), we finally obtain the total current through the resonant-tunneling system

$$J(t)=\sum_\sigma J_\sigma(t), \tag{15}$$

where

$$J_\sigma(t)=\frac{e}{\hbar}\int d\epsilon\frac{\Gamma^L_\sigma\Gamma^R_\sigma}{\Gamma^L_\sigma+\Gamma^R_\sigma}[f_L(\epsilon)-f_R(\epsilon)]\rho_\sigma(\epsilon,t), \tag{16}$$

with

$$\rho_\sigma(\epsilon,t)=\rho^{(0)}_\sigma(\epsilon,t)+\rho^{(1)}_\sigma(\epsilon,t). \tag{17}$$

Here $\rho^{(0)}_\sigma(\epsilon,t)$ is given by Eq. (8) and $\rho^{(1)}_\sigma(\epsilon,t)$ by

$$\rho^{(1)}_\sigma(\epsilon,t)=-\frac{1}{\pi}\,\mathrm{Im}\left\{\frac{dA_\sigma(\epsilon,t)}{dt}\right\}. \tag{18}$$

This is a Landauer-Büttiker formula[2,3] generalized to the time-dependent case. It satisfies total current conservation as is evident from the above derivation. The gauge invariance can also be easily verified by demonstrating that shifting the resonant level and the chemical potentials of the left and right leads by the same constant, leads to no change in our formula. Obviously, in the equilibrium situation, the displacement currents have the relation:[7] $J^d_{\sigma\alpha}=(\Gamma^\alpha_\sigma/\Gamma_\sigma)J^d_\sigma$.

When the interaction term $Un_{d\uparrow}n_{d\downarrow}$, with $n_{d\sigma}=c^\dagger_{d\sigma}c_{d\sigma}$, is included in Eq. (1), the Hamiltonian is the Anderson model. This new Hamiltonian is appropriate to model a quantum dot with intradot Coulomb repulsion.[13] If the Kondo effect is not taken into account, one can derive that the retarded Green's function is given by

$$\mathcal{G}^r_\sigma(t,t_1)=G^r_\sigma(\epsilon_{0\sigma},t,t_1)[1-N_{\bar\sigma}(t_1)] + G^r_\sigma(\epsilon_{0\sigma}+U,t,t_1)N_{\bar\sigma}(t_1), \tag{19}$$

where $G^r_\sigma(\epsilon_{0\sigma},t,t_1)$ has the same form as the retarded Green's function given in Eq. (10). In the steady case where $\epsilon_{d\sigma}(t)$ and $N_{\bar\sigma}(t)=\langle n_{d\bar\sigma}(t)\rangle$ are independent of time, the Fourier transform of $\mathcal{G}^r_\sigma(t,t_1)$ is

$$\mathcal{G}^r_\sigma(\epsilon)=\frac{1-\langle n_{d\bar\sigma}\rangle}{Z-\epsilon_{0\sigma}+i\Gamma_\sigma/2}+\frac{\langle n_{d\bar\sigma}\rangle}{Z-(\epsilon_{0\sigma}+U)+i\Gamma_\sigma/2}, \tag{20}$$

where $Z=\epsilon+i\eta$, with $\eta\to 0$. It is identical to the Green's function derived by Pals and MacKinnon[14] in their scheme two. The Keldysh Green's function $\mathcal{G}^<_\sigma(t,t)$ has the same form as in Eq. (11), i.e.,

$$\mathcal{G}^<_\sigma(t,t)=\sum_{L,R}\Gamma^{L/R}_\sigma\int\frac{d\epsilon}{\pi}f_{L/R}(\epsilon)A_\sigma(\epsilon,t), \tag{21}$$

but $A_\sigma(\epsilon,t)$ is replaced by

$$A_\sigma(\epsilon,t)=\frac{i}{2}\{|\alpha_\sigma(\epsilon_{0\sigma},\epsilon,t)|^2-\alpha_\sigma(\epsilon_{0\sigma},\epsilon,t)[\beta_\sigma(\epsilon_{0\sigma},\epsilon,t)]^* + \alpha_\sigma(\epsilon_{0\sigma}+U,\epsilon,t)[\beta_\sigma(\epsilon_{0\sigma}+U,\epsilon,t)]^*\}, \tag{22}$$

with

$$\alpha_\sigma(\epsilon_{0\sigma},\epsilon,t)=\int dt_1 e^{i\epsilon(t-t_1)}G^r_\sigma(\epsilon_{0\sigma},t,t_1) \tag{23}$$

and

$$\beta_\sigma(\epsilon_{0\sigma},\epsilon,t)=\int dt_1 e^{i\epsilon(t-t_1)}N_{\bar\sigma}(t_1)G^r_\sigma(\epsilon_{0\sigma},t,t_1), \tag{24}$$

where $N_\sigma(t)$ is determined by $N_\sigma(t)=\mathrm{Im}\{\mathcal{G}^<_\sigma(t,t)\}$. In terms of the Green's functions $\mathcal{G}^r_\sigma(t,t_1)$ and $\mathcal{G}^<_\sigma(t,t)$, it can be derived that in the interacting case, the displacement and total currents are also given by the same formulas as in Eqs. (13)–(17). Here, $\rho^{(0)}_\sigma(\epsilon,t)$ has the same form as in Eq. (8) and $\rho^{(1)}_\sigma(\epsilon,t)$ as in Eq. (18), but $a_\sigma(\epsilon,t)$ is given by

$$a_\sigma(\epsilon,t)=\int dt_1 e^{i\epsilon(t-t_1)}\mathcal{G}^r_\sigma(t,t_1) \tag{25}$$

and $A_\sigma(\epsilon,t)$ by Eq. (22).

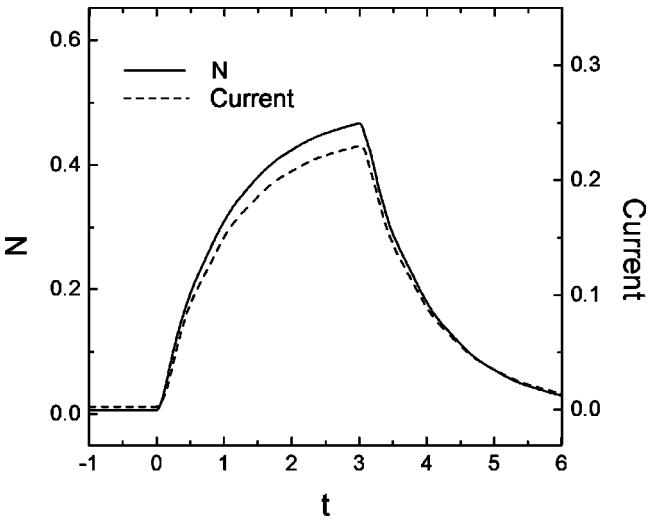

FIG. 1. Occupation of resonant level (solid curve) and total current through a resonant-tunneling structure (dashed curve) as a function of time, where the chemical potentials of the left and right leads are kept at $\mu_L=20$ and $\mu_R=0$. For $t<0$ and $t>3$, the resonant level is at $\epsilon_{0\sigma}=30$, while it is shifted down to $\epsilon_{0\sigma}-\Delta=10$ by a rectangular pulse during the period of $0<t<3$. In this figure and the following ones, the temperature is $k_BT=0.1$, the elastic couplings to the leads are $\Gamma^L_\sigma=\Gamma^R_\sigma=\frac{1}{2}\Gamma$, and the energy, current, and time are in units of Γ, $2e\Gamma/\hbar$, and $\hbar/\Gamma$, respectively.

The total current, $J_\alpha(t)=\Sigma_\sigma J_{\sigma\alpha}(t)$, flowing from lead α into the scattering region can be written as

$$J_\alpha(t)=\int d\omega J_\alpha(\omega)e^{-i\omega t}, \tag{26}$$

where the total current in the frequency domain $J_\alpha(\omega)$ can be expanded in a series form in terms of bias voltages applied to the leads,

$$J_\alpha(\omega)=\sum_\beta G_{\alpha\beta}(\omega)V_\beta+\sum_{\beta\gamma}G_{\alpha\beta\gamma}(\omega)V_\beta V_\gamma + \sum_{\beta\gamma\delta}G_{\alpha\beta\gamma\delta}(\omega)V_\beta V_\gamma V_\delta+\cdots. \tag{27}$$

When the bias voltages are very small, the linear expression $J_\alpha(\omega)=\Sigma_\beta G_{\alpha\beta}(\omega)V_\beta$ can be used approximately. As for large bias voltages, nonlinear dynamic conductances, $G_{\alpha\beta\gamma}(\omega), G_{\alpha\beta\gamma\delta}(\omega),\ldots$, have to be derived in order to obtain accurate $J_\alpha(\omega)$ and $J_\alpha(t)$. In Refs. 7 and 8, techniques are developed to derive the linear dynamic conductance $G_{\alpha\beta}(\omega)$, but no nonlinear dynamic conductance is obtained. Actually, it is a complicated and onerous task to derive the nonlinear dynamic conductances. Nevertheless, the current formulas derived in our approach are applicable to both small and large bias-voltage situations and the response behavior of the structures in the presence of time-varying fields can be studied directly in the time domain.

In the following, we employ the time-dependent Landauer-Büttiker formula to study the response of the resonant-tunneling structure to a rectangular pulse and a harmonic modulation, respectively. Here, only the noninteracting system is considered. In Fig. 1, we show the occupation $N(t)=N_\sigma(t)$ of the resonant level (solid curve) and the total

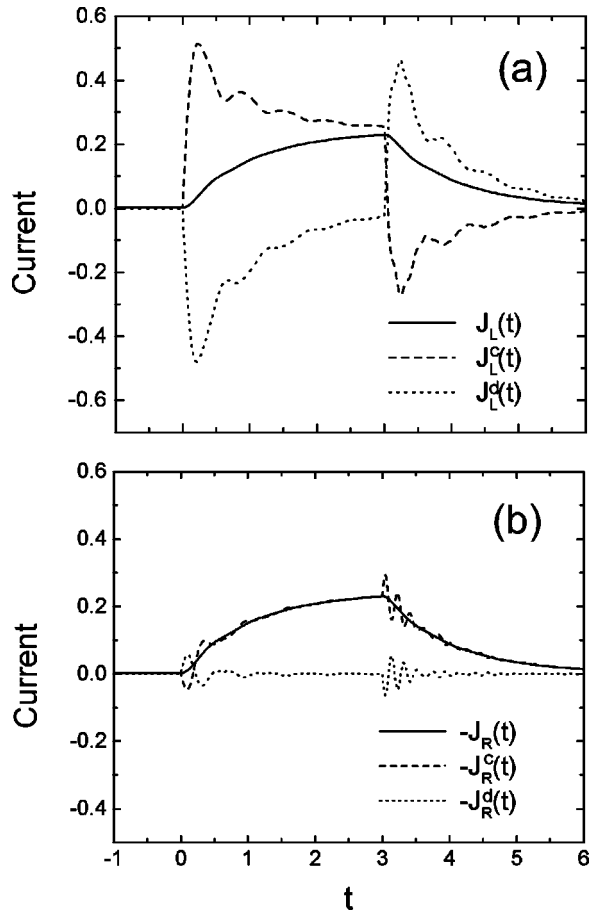

FIG. 2. (a) Time-dependent total (solid curve), tunneling (dashed curve), and displacement (dotted curve) currents flowing from the left lead into the scattering region and (b) those flowing from the scattering region into the right lead in the rectangular-pulse case.

current, $J(t)=\Sigma_\sigma J_\sigma(t)$, through the resonant-tunneling structure (dashed curve) in response to a rectangular-pulse potential. The temperature is chosen to be $k_BT=0.1$ and the chemical potentials of the left and right leads are maintained to be $\mu_L=20$ and $\mu_R=0$. The rectangular pulse begins at $t=0$ by shifting abruptly the resonant level from $\epsilon_{0\sigma}=30$ to $\epsilon_{0\sigma}-\Delta=10$ and lasts for a duration of $s=3$. At $t=3$, the pulse ends and the resonant level shifts back to $\epsilon_{0\sigma}=30$. Figure 1 reveals that the occupation of the resonant level and the total current have the same behavior in response to the rectangular-pulse potential. When $t<0$, the energy level $\epsilon_{d\sigma}(t)$ is above the chemical potential of the left lead as well as that of the right lead. As expected, the occupation of the level and the total current are constant and close to zero in this time region. During the period of $0<t<s$, the energy of the resonant level is located in between the chemical potentials of the leads. Both the occupation of the resonant level and the total current increase monotonously. Finally, with the ending of the pulse at $t=3$, the occupation of the energy level and the total current decay back toward zero.

The time-dependent tunneling (dashed curves) and displacement (dotted curves) currents are shown in Fig. 2 for the rectangular-pulse case. In Fig. 2(a), we present the tunneling and displacement currents,

$$J_L^c(t)=\sum_\sigma J_{\sigma L}^c(t)$$

and

$$J_L^d(t)=\sum_\sigma J_{\sigma L}^d(t),$$

flowing from the left lead into the scattering region. It is clear that the tunneling current $J_L^c(t)$ displays ringing structures[12] for both $0<t<s$ and $t>s$, but becomes negative for $t>s$, revealing that the tunneling current turns to flow from the scattering region into the left lead as the pulse ends. The displacement current $J_L^d(t)$ exhibits similar oscillatory behaviors and its flowing direction is, however, opposite to that of the tunneling current. This feature implies that the displacement current tends to retard the tunneling current. The tunneling and displacement currents,

$$-J_R^c(t)=-\sum_\sigma J_{\sigma R}^c(t)$$

and

$$-J_R^d(t)=-\sum_\sigma J_{\sigma R}^d(t),$$

flowing from the scattering region into the right lead are given in Fig. 2(b). Interestingly, both of them have oscillations of much smaller amplitude and their oscillatory behaviors are drastically different from those presented in Fig. 2(a). Nevertheless, the total currents, which are obtained by summing the corresponding tunneling and displacement currents, are the same [see the solid curves in Figs. 2(a) and 2(b), which are identical to the dashed curve in Fig. 1]. Also, it can be seen that the displacement current $-J_R^d(t)$ plays the retarding role as well.

The harmonic-modulation case is studied below, where the energy of the resonant level varies as $\epsilon_{d\sigma}(t)=\epsilon_{0\sigma}+\Delta_0\cos(\omega t)$. Here, we choose $\epsilon_{0\sigma}=5$ and $\omega=2$, and the chemical potentials of the left and right leads are kept at $\mu_L=10$ and $\mu_R=0$. Figure 3(a) presents the occupations of the resonant level for $\Delta_0=5$ (solid curve) and 10 (dashed curve) and the corresponding total currents are given in Fig. 3(b). The results show that the occupation of the resonant level and the total current are periodic but not harmonic; the occupation of the resonant level has the same frequency as the harmonic potential applied to the level, while the frequency of the total current doubles. Furthermore, one can see that the shoulders on the curves of both the occupation of the resonant level and the total current become dim as the amplitude of the applied harmonic potential is strengthened. In sharp contrast to the rectangular-pulse case, the total current does not exhibit the same response behavior as the occupation of the resonant level in the harmonic-modulation case.

In Fig. 4, we show the time-dependent tunneling (dashed curves) and displacement (dotted curves) currents for $\Delta_0=10$. The tunneling current flowing from the left lead into the scattering region and that from the scattering region into the right lead have analogous ringing structures as the tunneling current shown in Fig. 2 of Ref. 12, but there exists a phase difference between them. The displacement current os-

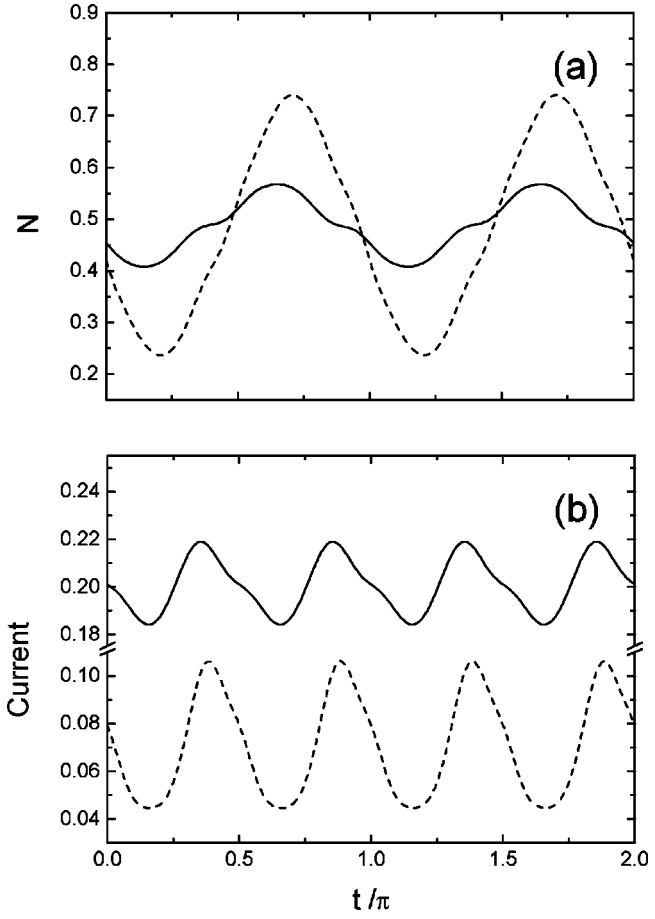

FIG. 3. (a) Occupation of resonant level versus time for $\Delta_0 = 5$ (solid curve) and 10 (dashed curve); (b) the corresponding time-dependent total current. The energy of the resonant level varies harmonically as $\epsilon_{d\sigma}(t) = \epsilon_{0\sigma} + \Delta_0 \cos(\omega t)$ and the chemical potentials of the left and right leads are kept at $\mu_L = 10$ and $\mu_R = 0$. Here, we choose $\epsilon_{0\sigma} = 5$ and $\omega = 2$, and the frequency ω is in units of $\Gamma/\hbar$.

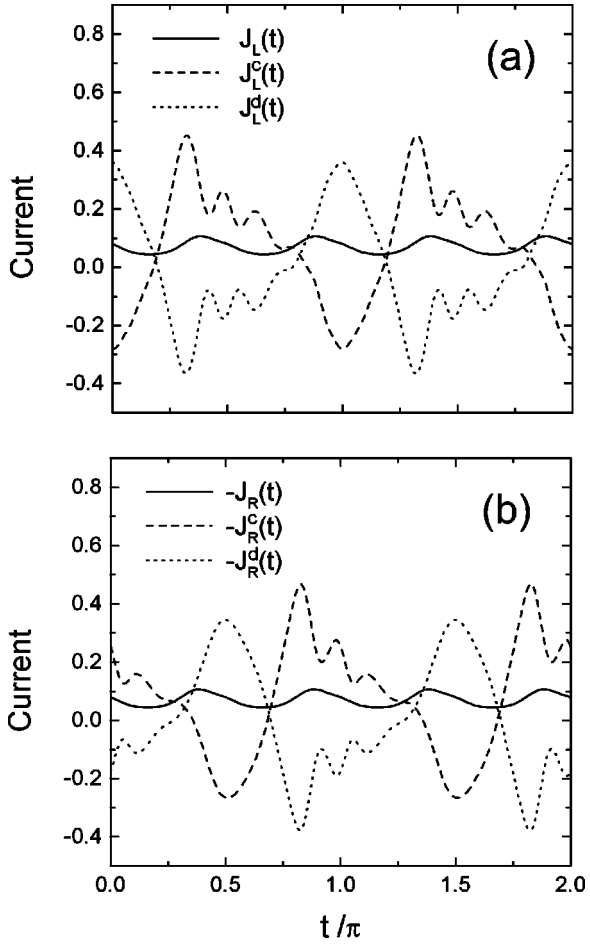

FIG. 4. (a) Time-dependent total (solid curve), tunneling (dashed curve), and displacement (dotted curve) currents flowing from the left lead into the scattering region and (b) those flowing from the scattering region into the right lead for $\Delta_0 = 10$ in the harmonic-modulation case.

cillates similarly like the tunneling current and flows in an opposite direction, indicating that it plays the role of retarding the tunneling current. Furthermore, it can be seen that the tunneling (displacement) current becomes negative (positive) in a certain part of each period. The solid curves in Figs. 4(a) and 4(b) are obtained by summing the corresponding tunneling and displacement currents. These solid curves are identical to the total current denoted by the dashed curve in Fig. 3(b). It is interesting to note that the frequency of the total current becomes doubled, though the tunneling and displacement currents have the same frequency as the applied harmonic potential $\Delta_0 \cos(\omega t)$.

In conclusion, we have studied the time-dependent transport through resonant-tunneling structures. The Keldysh nonequilibrium Green's-function approach is employed and the transport problem is treated directly in the time domain. A time-dependent Landauer-Büttiker formula is derived in the wideband limit, which guarantees current conservation and gauge invariance. As typical examples, this formula is applied to the rectangular-pulse and harmonic-modulation cases to demonstrate the response behaviors of the resonant-tunneling structures. It is found that the displacement current plays the role of retarding the tunneling current.

In addition to Eq. (14), there is another natural way of partitioning the total displacement current by choosing

$$J^d_{\sigma L(R)}(t) = -\frac{e}{\hbar}\int \frac{d\epsilon}{\pi}\,\Gamma^{R(L)}_{\sigma} f_{R(L)}(\epsilon)\,\mathrm{Im}\left\{\frac{dA_\sigma(\epsilon,t)}{dt}\right\}.$$

For these two natural partitions, which is superior needs to be determined by experiments.

ACKNOWLEDGMENTS

J.Q.Y. and H.Z.Z. were supported by the National Science Foundation of China and the National Climbing Program of China. J.Q.Y. also acknowledges the Applied Physics Department of The Hong Kong Polytechnic University for hospitality.

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[1] For a recent review, see S. Datta, *Electronic Transport in Mesoscopic Conductors* (Cambridge University Press, Cambridge, 1995).
[2] R. Landauer, Philos. Mag. **21**, 863 (1970).
[3] M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
[4] For a recent review, see A. P. Jauho, cond-mat/9911282 (unpublished).
[5] M. P. Anantram and S. Datta, Phys. Rev. B **51**, 7632 (1995).
[6] M. H. Hettler and H. Schoeller, Phys. Rev. Lett. **74**, 4907 (1995); A. P. Jauho, cond-mat/9711141 (unpublished).
[7] B. G. Wang, J. Wang, and H. Guo, Phys. Rev. Lett. **82**, 398 (1999).
[8] M. Büttiker, A. Prêtre, and H. Thomas, Phys. Rev. Lett. **70**, 4114 (1993).
[9] M. Büttiker and T. Christen, in *Quantum Transport in Semiconductor Submicron Structures*, edited by B. Kramer (Kluwer, Dordrecht, 1996), p. 263.
[10] M. Büttiker, Nuovo Cimento Soc. Ital. Fis. **110B**, 509 (1995).
[11] A. P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B **50**, 5528 (1994).
[12] N. S. Wingreen, A. P. Jauho, and Y. Meir, Phys. Rev. B **48**, 8487 (1993).
[13] A. L. Yeyati, A. Martin-Rodero, and F. Flores, Phys. Rev. Lett. **71**, 2991 (1993), and references therein.
[14] P. Pals and A. MacKinnon, J. Phys.: Condens. Matter **8**, 5401 (1996).