Superconducting charge qubits: The roles of self and mutual inductances

J. Q. You
National Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, P.O. Box 912, Beijing 100083, China
and Department of Applied Physics, Hong Kong Polytechnic University, Hung Hom, Hong Kong

Chi-Hang Lam
Department of Applied Physics, Hong Kong Polytechnic University, Hung Hom, Hong Kong

H. Z. Zheng
National Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, P.O. Box 912, Beijing 100083, China
(Received 11 December 2000; published 2 April 2001)

We study the essential roles of self and mutual inductances in superconducting charge qubits and propose a scheme to couple charge qubits by means of mutual inductance. We also show that the Hamiltonians can be exactly formulated in compact forms in the spin-$\frac{1}{2}$ representation for both single- and double-qubit structures.

DOI: 10.1103/PhysRevB.63.180501 PACS number(s): 74.50.+r, 73.23.−b, 85.25.Cp

Based on the principles of quantum mechanics such as quantum parallelism and entanglement, quantum computers are expected to be capable of performing certain tasks which no classical computers can do in practical time scales. Early physical realizations of quantum computation were performed in optical cavities,[1] trapped ions,[2] and nuclear spins in bulk solutions.[3] These systems have the advantage of high quantum coherence, but cannot be integrated easily to form large-scale circuits. On the contrary, there exists better potential to realize large-scale quantum computers by implementations of qubits in solid-state systems based on electron spins in quantum dots[4,5] or nuclear spins of donor atoms in silicon.[6]

Recently, superconducting charge[7,8] and phase qubits[9,10] have attracted much attention because of possible large-scale integration and relatively high quantum coherence. In particular, recent experimental realizations of a single charge qubit[11] prove that it is promising to construct quantum computers by means of superconducting charge qubits. The next immediate challenge involved is to realize two-bit gates by coupling single charge qubits in a feasible way, since two-bit gates in combination of one-bit operations provide a complete set of gates required for quantum computation.[12] Interbit coupling can be implemented by coupling the single charge qubits to a common inductor to form an LC circuit,[7] but the interbit coupling terms calculated in Ref. 7 only applies to the case when the phase conjugate to the total charge on the external capacitors of both single qubits fluctuates weakly. Here, motivated by the work of Mooij *et al.*,[9] we propose a scheme to couple charge qubits by mutual inductance. In our calculations, self and mutual inductances will be taken into account and their essential roles are demonstrated. Even for charge qubits separated far apart, mutual inductive coupling can still be implemented by means of the superconducting flux transporter as originally designed by Mooij *et al.*[9] for superconducting phase qubits. Furthermore, we show that the Hamiltonians can be exactly formulated in compact forms in the spin-$\frac{1}{2}$ representation for both single and double qubits. From the quantum computing point of view, this is important since using these compact forms, the evolution of the systems can be conveniently described.

Single-qubit structure. For the single-qubit structure[7,11] shown in Fig. 1(a), the Hamiltonian of the system without including self-inductance energy is given by

$$H=4E_c(n-CV_X/2e)^2-E_J(\Phi)\cos\varphi, \tag{1}$$

where $E_c=e^2/2C_\Sigma$, with $C_\Sigma=C+2C_J$, is the single-particle charging energy and $E_J(\Phi)=2E_J^0\cos(\pi\Phi/\Phi_0)$ is the effective Josephson coupling. The number n of the extra Cooper pairs on the island and the average phase drop $\varphi=(\phi_1+\phi_2)/2$ are canonically conjugate. The gauge-

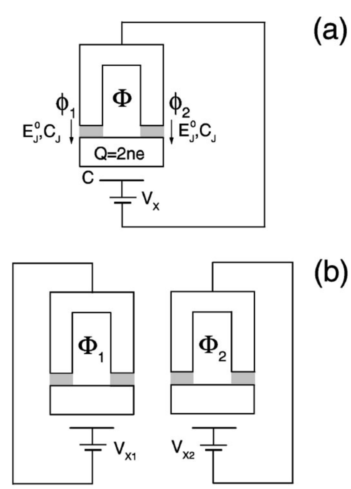

FIG. 1. Superconducting charge qubits. (a) Single-qubit structure, where a superconducting island is coupled by two Josephson tunnel barriers (each with capacitance C_J and Josephson coupling energy E_J^0) to a segment of a superconducting ring and through a gate capacitance C to a voltage source. (b) Double-qubit structure, where the interbit coupling is realized by mutual inductance between left and right SQUID loops.

invariant phase drops ϕ_1 and ϕ_2 across the junctions are related to the total flux $\Phi=\Phi_X+LI$ through the superconducting-quantum-interference-device (SQUID) loop by the constraint $\phi_2-\phi_1=2\pi\Phi/\Phi_0$, where Φ_X is the external flux, L the self-inductance of the loop, and $\Phi_0=h/2e$ the flux quantum. The circulating current in the SQUID loop is given by $I=2I_c\sin(\pi\Phi/\Phi_0)\cos\varphi$, where $I_c=\pi E_J^0/\Phi_0$. Here we consider the charging regime with E_c much larger than the Josephson coupling E_J^0, so large fluctuations in n are suppressed and the charge states play a dominative role. Also, we assume that the superconducting gap Δ is larger than E_c and $E_c\gg k_BT$ so that quasiparticle tunneling is prohibited. As the external gate voltage V_X approaches $(2n+1)e/C$, charge states $|n\rangle$ and $|n+1\rangle$ become degenerate. The Josephson coupling strongly mixes them. However, in the vicinity of $V_X=(2n+1)e/C$, other charge states have much higher energies since the energy difference between $|n-k\rangle$ ($|n+k+1\rangle$) and $|n\rangle$ ($|n+1\rangle$) is $\sim 4E_ck(k+1)$ for $k=1,2,3,\ldots$. In the charging regime with $E_c\gg E_J^0$, transition to higher charge states is hence negligible.

For the special case of extremely small self-inductance, Φ reduces to the classical variable Φ_X. In the spin-$\frac{1}{2}$ representation based on the charge states $|\uparrow\rangle=|n\rangle$ and $|\downarrow\rangle=|n+1\rangle$, the reduced two-state Hamiltonian can be cast into[7]

$$H=\varepsilon(V_X)\sigma_z-\frac{1}{2}E_J(\Phi_X)\sigma_x, \tag{2}$$

where $\varepsilon(V_X)=2E_c[CV_X/e-(2n+1)]$. Hamiltonian (2) has two eigenvalues $E_\pm=\pm\frac{1}{2}[4\varepsilon^2(V_X)+E_J^2(\Phi_X)]^{1/2}$ and the corresponding eigenstates are

$$|\Psi_\pm\rangle=\frac{2}{\sqrt{4[\varepsilon(V_X)-E_\pm]^2+E_J^2(\Phi_X)}}\begin{pmatrix}\frac{1}{2}E_J(\Phi_X)\\ \varepsilon(V_X)-E_\pm\end{pmatrix}.$$

In the spin-$\frac{1}{2}$ representation, we have $\cos\varphi=\frac{1}{2}\sigma_x$ and the circulating current in the superconducting loop is $I=I_c\sin(\pi\Phi_X/\Phi_0)\sigma_x$. The expectation values of the circulating current for these two eigenstates are given by

$$I_\pm=\langle\Psi_\pm|I|\Psi_\pm\rangle=\mp\frac{I_cE_J(\Phi_X)\sin(\pi\Phi_X/\Phi_0)}{[4\varepsilon^2(V_X)+E_J^2(\Phi_X)]^{1/2}},$$

which are identical to the results derived directly from the thermodynamic relation $I_\pm=-\partial E_\pm/\partial\Phi_X$.

For the general case, self-inductance needs to be included and the circulating current in the SQUID loop obeys the relation

$$I=2I_c\cos\varphi\sin(\pi\Phi_X/\Phi_0+\pi LI/\Phi_0). \tag{3}$$

This equation implies that I is a continuous function of $\cos\varphi$ and it can be expanded as the Taylor series

$$I=2I_c\cos\varphi\sum_{n=1}^{\infty}a_n\cos^{n-1}\varphi. \tag{4}$$

Equation (3) can also be rewritten as follows:

$$\begin{aligned}I&=2I_c\cos\varphi[\sin(\pi\Phi_X/\Phi_0)\cos(\pi LI/\Phi_0)\\&\quad+\cos(\pi\Phi_X/\Phi_0)\sin(\pi LI/\Phi_0)]\\&=2I_c\cos\varphi\left\{\sin\left(\frac{\pi\Phi_X}{\Phi_0}\right)\left[1-\frac{1}{2!}\left(\frac{\pi LI}{\Phi_0}\right)^2+\frac{1}{4!}\left(\frac{\pi LI}{\Phi_0}\right)^4\right.\right.\\&\quad\left.-\ldots\right]+\cos\left(\frac{\pi\Phi_X}{\Phi_0}\right)\left.\left[\left(\frac{\pi LI}{\Phi_0}\right)-\frac{1}{3!}\left(\frac{\pi LI}{\Phi_0}\right)^3+\ldots\right]\right\}.\end{aligned} \tag{5}$$

Substituting Eq. (4) into Eq. (5) and comparing coefficients of $\cos^n\varphi$ on both sides, one can obtain the coefficients a_n:

$$a_1=\sin(\pi\Phi_X/\Phi_0),$$

$$a_2=(2\pi LI_c/\Phi_0)\sin(\pi\Phi_X/\Phi_0)\cos(\pi\Phi_X/\Phi_0),$$

$$a_3=(2\pi LI_c/\Phi_0)^2\sin(\pi\Phi_X/\Phi_0)[1-\tfrac{3}{2}\sin^2(\pi\Phi_X/\Phi_0)],$$

$$\ldots\ldots\ldots\ldots\ldots$$

Similarly, the effective Josephson coupling $E_J(\Phi)=2E_J^0\cos(\pi\Phi_X/\Phi_0+\pi LI/\Phi_0)$ can also be expanded as the Taylor series

$$E_J(\Phi)=2E_J^0\sum_{n=0}^{\infty}b_n\cos^n\varphi. \tag{6}$$

Rewriting $E_J(\Phi)$ as

$$\begin{aligned}E_J(\Phi)&=2E_J^0[\cos(\pi\Phi_X/\Phi_0)\cos(\pi LI/\Phi_0)\\&\quad-\sin(\pi\Phi_X/\Phi_0)\sin(\pi LI/\Phi_0)]\\&=2E_J^0\left\{\cos\left(\frac{\pi\Phi_X}{\Phi_0}\right)\left[1-\frac{1}{2!}\left(\frac{\pi LI}{\Phi_0}\right)^2+\frac{1}{4!}\left(\frac{\pi LI}{\Phi_0}\right)^4\right.\right.\\&\quad\left.-\ldots\right]-\sin\left(\frac{\pi\Phi_X}{\Phi_0}\right)\left.\left[\left(\frac{\pi LI}{\Phi_0}\right)-\frac{1}{3!}\left(\frac{\pi LI}{\Phi_0}\right)^3+\ldots\right]\right\},\end{aligned} \tag{7}$$

and substituting Eq. (7) into Eq. (6), one can obtain the coefficients b_n:

$$b_0=\cos(\pi\Phi_X/\Phi_0),$$

$$b_1=-(2\pi LI_c/\Phi_0)\sin^2(\pi\Phi_X/\Phi_0),$$

$$b_2=-\tfrac{3}{2}(2\pi LI_c/\Phi_0)^2\sin^2(\pi\Phi_X/\Phi_0)\cos(\pi\Phi_X/\Phi_0),$$

$$\ldots\ldots\ldots\ldots\ldots$$

From Eqs. (4)–(7) it follows that $a_n\sim(2\pi LI_c/\Phi_0)^{n-1}$ and $b_n\sim(2\pi LI_c/\Phi_0)^n$. The series (4) and (6) converge rapidly since the flux generated by the circulating current is typically much smaller than the flux quantum,[13] i.e., $L\langle I\rangle\sim LI_c\ll\Phi_0$.

In the spin-$\frac{1}{2}$ representation, $\cos^{2n}\varphi=1/2^{2n}$ and $\cos^{2n+1}\varphi=\sigma_x/2^{2n+1}$. The circulating current given in Eq. (4) thus simplifies to

$$I=I_c(\alpha\sigma_x+\beta), \tag{8}$$

and the Hamiltonian of the single-qubit structure in Eq. (1), after including self-inductance energy $\frac{1}{2}LI^2$, is reduced to

$$H=\varepsilon(V_x)\sigma_z-E_J^0[\gamma-(\pi LI_c/\Phi_0)\alpha\beta]\sigma_x, \tag{9}$$

where

$$\alpha=\sum_{n=0}^{\infty}\frac{a_{2n+1}}{2^{2n}},\quad \beta=\sum_{n=0}^{\infty}\frac{a_{2n+2}}{2^{2n+1}},\quad \gamma=\sum_{n=0}^{\infty}\frac{b_{2n}}{2^{2n}}. \tag{10}$$

In Eq. (9), we have dropped the term which only leads to a shift of the energy. Retained up to terms second order in the expansion parameter $\pi LI_c/\Phi_0$, Hamiltonian (9) becomes

$$H=\varepsilon(V_x)\sigma_z-E_J^0\cos\left(\frac{\pi\Phi_X}{\Phi_0}\right)\times\left[1-\frac{5}{2}\left(\frac{\pi LI_c}{\Phi_0}\right)^2\sin^2\left(\frac{\pi\Phi_X}{\Phi_0}\right)\right]\sigma_x. \tag{11}$$

This is in general a good approximation since typically $LI_c/\Phi_0\sim 10^{-3}$. If the applied gate voltage V_X is set to $V_X=(2n+1)e/C$, then $\varepsilon(V_X)=0$. The Hamiltonian is reduced to $H=-E_J^0[\gamma-(\pi LI_c/\Phi_0)\alpha\beta]\sigma_x$ and the system evolves according to the unitary transformation

$$U_{1b}(\tau)=e^{iE_J^0[\gamma-(\pi LI_c/\Phi_0)\alpha\beta]\tau\sigma_x/\hbar}. \tag{12}$$

When $\Phi_X=0$, $\gamma-(\pi LI_c/\Phi_0)\alpha\beta\sim 1$. The typical switching time during a one-bit operation is of the order $\hbar/E_J^0$. For typical experimental value of $E_J^0\sim 1$ K, one arrives at a very short operation time $\tau\sim 0.1$ ns.[7]

Double-qubit structure. When two single-qubit structures are put together [Fig. 1(b)], mutual inductance becomes relevant. The Hamiltonian of the system with self and mutual inductances included is given by

$$H=\sum_{i=1,2}[4E_{ci}(n_i-C_iV_{Xi}/2e)^2-E_{Ji}(\Phi_i)\cos\varphi_i], +\frac{1}{2}\sum_{i=1,2}L_iI_i^2+M_{12}I_1I_2, \tag{13}$$

where $i=1,2$ denote, respectively, the left and right single-qubit structures and

$$\Phi_i=\Phi_{Xi}+L_iI_i+M_{ij}I_j. \tag{14}$$

In Eq. (14) and below, $i,j=1,2$ with $i\neq j$, and $M_{12}=M_{21}$ is the mutual inductance between left and right SQUID loops shown in Fig. 1(b).

The circulating currents in the left and right superconducting loops

$$I_i=2I_{ci}\cos\varphi_i\sin[\pi\Phi_{Xi}/\Phi_0+\pi(L_iI_i+M_{ij}I_j)/\Phi_0], \tag{15}$$

are continuous functions of $\cos\varphi_1$ and $\cos\varphi_2$, and they can be expanded as

$$I_i=2I_{ci}\cos\varphi_i\sum_{m,n=0}^{\infty}a_{m+1,n}^{(i)}\cos^m\varphi_i\cos^n\varphi_j. \tag{16}$$

Similar to the single-qubit case, expanding I_1 and I_2 in Eq. (15) as series and substituting Eq. (16) into them, one can obtain the coefficients $a_{mn}^{(i)}$:

$$a_{10}^{(i)}=\sin(\pi\Phi_{Xi}/\Phi_0),$$

$$a_{11}^{(i)}=(2\pi M_{ij}I_{cj}/\Phi_0)\cos(\pi\Phi_{Xi}/\Phi_0)\sin(\pi\Phi_{Xj}/\Phi_0),$$

$$a_{20}^{(i)}=(2\pi L_iI_{ci}/\Phi_0)\sin(\pi\Phi_{Xi}/\Phi_0)\cos(\pi\Phi_{Xi}/\Phi_0),$$

$$\cdots\cdots\cdots\cdots$$

Also, the effective Josephson couplings $E_{Ji}(\Phi_i)=2E_{Ji}^0\cos(\pi\Phi_i/\Phi_0)$, $i=1,2$, can be expanded as Taylor series

$$E_{Ji}(\Phi_i)=2E_{Ji}^0\sum_{m,n=0}^{\infty}b_{mn}^{(i)}\cos^m\varphi_i\cos^n\varphi_j, \tag{17}$$

and the coefficients $b_{mn}^{(i)}$ can be obtained similarly:

$$b_{00}^{(i)}=\cos(\pi\Phi_{Xi}/\Phi_0),$$

$$b_{10}^{(i)}=-(2\pi L_iI_{ci}/\Phi_0)\sin^2(\pi\Phi_{Xi}/\Phi_0),$$

$$b_{01}^{(i)}=-(2\pi M_{ij}I_{cj}/\Phi_0)\sin(\pi\Phi_{Xi}/\Phi_0)\sin(\pi\Phi_{Xj}/\Phi_0),$$

$$\cdots\cdots\cdots\cdots$$

In the spin-$\frac{1}{2}$ representation, the circulating currents in the left and right superconducting loops are cast to

$$I_i=I_{ci}(\alpha_i\sigma_x^{(i)}+\beta_i\sigma_x^{(j)}+\gamma_i\sigma_x^{(i)}\sigma_x^{(j)}+\delta_i), \tag{18}$$

where

$$\alpha_i=\sum_{k,l=0}^{\infty}\frac{a_{2k+1,2l}^{(i)}}{2^{2(k+l)}},\quad \beta_i=\sum_{k,l=0}^{\infty}\frac{a_{2k+2,2l+1}^{(i)}}{2^{2(k+l)+2}},$$

$$\gamma_i=\sum_{k,l=0}^{\infty}\frac{a_{2k+1,2l+1}^{(i)}}{2^{2(k+l)+1}},\quad \delta_i=\sum_{k,l=0}^{\infty}\frac{a_{2k+2,2l}^{(i)}}{2^{2(k+l)+1}}, \tag{19}$$

while the Hamiltonian of the double-qubit structure is given by

$$H=\sum_{i=1,2}[\varepsilon_i(V_{Xi})\sigma_z^{(i)}-\overline{E}_{Ji}\sigma_x^{(i)}]+\Pi_{12}\sigma_x^{(1)}\sigma_x^{(2)}, \tag{20}$$

where

$$\overline{E}_{J1}=(E_{J1}^0A_1+E_{J2}^0B_2)-L_1I_{c1}^2(\alpha_1\delta_1+\beta_1\gamma_1)-L_2I_{c2}^2(\alpha_2\gamma_2+\beta_2\delta_2)-M_{12}I_{c1}I_{c2}(\alpha_1\delta_2+\beta_1\gamma_2+\gamma_1\alpha_2+\delta_1\beta_2), \tag{21}$$

$$\overline{E}_{J2}=(E_{J1}^0B_1+E_{J2}^0A_2)-L_1I_{c1}^2(\alpha_1\gamma_1+\beta_1\delta_1)-L_2I_{c2}^2(\alpha_2\delta_2+\beta_2\gamma_2)-M_{12}I_{c1}I_{c2}(\alpha_1\gamma_2+\beta_1\delta_2+\gamma_1\beta_2+\delta_1\alpha_2), \tag{22}$$

and

$$\Pi_{12}=\sum_{i=1,2}[L_iI_{ci}^2(\alpha_i\beta_i+\gamma_i\delta_i)-E_{Ji}^0C_i]+M_{12}I_{c1}I_{c2}(\alpha_1\alpha_2+\beta_1\beta_2+\gamma_1\delta_2+\delta_1\gamma_2). \tag{23}$$

In Eqs. (21)–(23), A_i, B_i, and C_i are given by

$$A_i=\sum_{k,l=0}^{\infty}\frac{b_{2k,2l}^{(i)}}{2^{2(k+l)}},\quad B_i=\sum_{k,l=0}^{\infty}\frac{b_{2k+1,2l+1}^{(i)}}{2^{2(k+l)+2}},$$

$$C_i=\sum_{k,l=0}^{\infty}\frac{b_{2k,2l+1}^{(i)}}{2^{2(k+l)+1}},\quad i=1,2. \tag{24}$$

Retaining the Hamiltonian up to the second order in expansion parameters, we have

$$\overline{E}_{Ji}=E_{Ji}^0\cos\left(\frac{\pi\Phi_{Xi}}{\Phi_0}\right)\left\{1-\frac{5}{2}\left[\left(\frac{\pi L_iI_{ci}}{\Phi_0}\right)^2\sin^2\left(\frac{\pi\Phi_{Xi}}{\Phi_0}\right)+\left(\frac{\pi M_{ij}I_{cj}}{\Phi_0}\right)^2\sin^2\left(\frac{\pi\Phi_{Xj}}{\Phi_0}\right)+\left(\frac{\pi M_{ij}I_{cj}}{\Phi_0}\right)\times\left(\frac{\pi L_jI_{cj}}{\Phi_0}\right)\tan\left(\frac{\pi\Phi_{Xi}}{\Phi_0}\right)\sin\left(\frac{2\pi\Phi_{Xj}}{\Phi_0}\right)\right]\right\}, \tag{25}$$

and $\Pi_{12}=3M_{12}I_{c1}I_{c2}\sin(\pi\Phi_{X1}/\Phi_0)\sin(\pi\Phi_{X2}/\Phi_0)$.

Choosing the applied gate voltages to be $V_{Xi}=(2n+1)e/C_i$, $i=1,2$, one has $\varepsilon_i(V_{Xi})=0$ and the Hamiltonian is reduced to

$$H=-\overline{E}_{J1}\sigma_x^{(1)}-\overline{E}_{J2}\sigma_x^{(2)}+\Pi_{12}\sigma_x^{(1)}\sigma_x^{(2)}. \tag{26}$$

The two-bit operations are realized by evolving the system according to the unitary transformation

$$U_{2b}(\tau)=e^{-iH\tau/\hbar}. \tag{27}$$

When the external fluxes are set to $\Phi_{Xi}=\Phi_0/2$, $i=1,2$, $\overline{E}_{Ji}\sim0$ and $\Pi_{12}\sim3M_{12}I_{c1}I_{c2}$. The typical switching time in a two-bit operation is of the order $\hbar/3M_{12}I_{c1}I_{c2}$. Since $M_{12}I_{c1}\sim10^{-3}\Phi_0$ and $I_{c2}=\pi E_{J2}^0/\Phi_0$, one has $3M_{12}I_{c1}I_{c2}\sim10^{-2}E_{J2}^0$. For the typical experimental value of $E_{J2}^0\sim1$K, the two-bit operation time is also very short and is about $\tau\sim10$ns.

In Ref. 7, interbit coupling is realized by connecting the single charge qubits to a common inductor to form an LC circuit, but it is assumed in the derivation of the interbit coupling term that the phase conjugate to the total charge on the external capacitors of both single qubits must fluctuate weakly. In contrast, an interbit coupling term $\Pi_{12}\sigma_x^{(1)}\sigma_x^{(2)}$ is exactly derived in our approach. As for the case when the two charge qubits are far apart, mutual inductive coupling of the two qubits can still be effectively realized by means of the superconducting flux transporter as designed by Mooij *et al.*[9] for phase qubits.

In conclusion, the essential roles of self and mutual inductances in SQUID charge qubits are studied and a scheme to couple charge qubits is proposed by means of mutual inductance. It is shown that the Hamiltonians can be exactly formulated in compact forms in the spin-$\frac{1}{2}$ representation for both single- and double-qubit structures. Using these compact forms, one can conveniently describe the evolution of the systems in the quantum computing language.

Discussions with H. F. Chau and C. Y. Hu are gratefully acknowledged. J.Q.Y. also acknowledges the National Natural Science Foundation of China and the Chinese Academy of Sciences for partial financial support.

[1] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).

[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995); C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, *ibid.* **75**, 4714 (1995).

[3] N. A. Gershenfeld and I. L. Chuang, Science **275**, 350 (1997); D. G. Cory, A. F. Fahmy, and T. F. Havel, Proc. Natl. Acad. Sci. U.S.A. **94**, 1634 (1997).

[4] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B **59**, 2070 (1999).

[5] X. Hu and S. Das Sarma, Phys. Rev. A **61**, 062301 (2000).

[6] B. Kane, Nature (London) **393**, 133 (1998).

[7] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997); Y. Makhlin, G. Schön, and A. Shnirman, Nature (London) **398**, 305 (1999).

[8] D. V. Averin, Solid State Commun. **105**, 659 (1998).

[9] J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999); T. P. Orlando, J. E. Mooij, L. Tian, C. H. van der Wal, L. S. Levitov, S. Lloyd, and J. J. Mazo, Phys. Rev. B **60**, 15 398 (1999).

[10] L. B. Ioffe, V. B. Geshkenbein, M. V. Feigel'man, A. L. Fauchère, and G. Blatter, Nature (London) **398**, 679 (1999).

[11] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).

[12] S. Lloyd, Phys. Rev. Lett. **75**, 346 (1995); D. Deutsch, A. Barenco, and A. Ekert, Proc. R. Soc. London, Ser. A **449**, 669 (1995).

[13] The flux generated by the circulating current in the SQUID loop is typically about $10^{-3}\Phi_0$ for a qubit whose size is of order $1\,\mu$m (see, e.g., Ref. 9).