

## Compound pulse solitons in a fiber ring laser

D. Y. Tang, B. Zhao, D. Y. Shen, and C. Lu

*School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798*

W. S. Man and H. Y. Tam

*Department of Electrical Engineering, Hong Kong Polytechnic University, Hung Hom, Hong Kong*

(Received 3 March 2003; published 24 July 2003)

We report on the existence of a different form of solitary waves in a passively mode-locked fiber ring laser. Studying the interaction between bound solitons observed in a passively mode-locked fiber laser revealed that the bound-soliton pair behaves as a unit, and the properties of their interaction have exactly the same features as those of the single-pulse soliton in the laser, which suggests that the observed bound solitons are in fact another form of solitary waves in the laser. Numerical simulation confirmed the existence of a different form of solitons in the laser.

DOI: 10.1103/PhysRevA.68.013816

PACS number(s): 42.81.Dp, 42.55.Wd, 42.60.Fc, 42.65.Re

Solitary wave generation is a generic property of many nonlinear dynamic systems and has been widely investigated [1–3]. Optical solitons due to their theoretical importance and potential practical applications in optical communication and signal processing systems have attracted special attention [4–7]. An optical soliton generally refers to an optical pulse that can propagate in media without changing its shape and pulse width even under weak perturbations. A typical example of optical soliton generation is the nonlinear pulse propagation in optical fiber, where due to the balanced interaction between the optical Kerr effect and the fiber chromatic dispersion on a pulse, whose shape and pulse width become constant with propagation. Optical pulse propagation in fiber is governed by the nonlinear Schrödinger equation (NLSE).

Optical solitons have also been observed in the passively mode-locked fiber lasers [8–10]. However, as in a laser, apart from the optical fiber, there exist other cavity components, such as the gain medium and the output coupler, whose existence affects the detailed dynamics of the formed solitons. It was found that solitons observed in fiber lasers exhibited special features. These include the soliton energy quantization [11], the soliton bunching [12], and the quasi-harmonic and harmonic mode locking [13]. Only under weak perturbations caused by the gain, loss, and saturable absorption, the average soliton dynamics of a laser could be described by the NLSE; generally, it is described by the complex Ginzburg-Landau equation or the coupled complex Ginzburg-Landau equations [14].

So far, all solitons observed in the optical fiber and/or the fiber laser systems are characterized as having a single peak, sech-form pulse profile. Although the higher-order NLSE solitons have more complicated pulse profiles, these solitons are intrinsically unstable [15]. They are difficult to be observed in a practical system. In this paper, we report on the experimental evidence of a different form of double-pulse solitary waves in a passively mode-locked fiber ring laser. In a previous paper, we have reported on the experimental observation of bound solitons in a passively mode-locked fiber laser [16]. Further studies on the interaction between the bound solitons revealed that the two bound pulses always behave as a unit. They could not break up. In particular, the

interaction properties of the bound solitons have exactly the same features as those of the single-pulse solitons in a laser. These properties of the bound solitons suggest strongly that they are not simply a bound state of solitons, predicted previously based on the soliton interaction [17,18], but rather another form of solitary wave in the laser system, namely, a compound pulse form of soliton. As bound states of solitons with different pulse separations have been observed in our laser, it shows that a family of such solitary wave exists.

The fiber laser used in our experiment is schematically shown in Fig. 1. To check if the bound states of solitons observed are a generic property of all passively mode-locked fiber lasers, whose appearance is independent of the laser setup, in the present experiment we have deliberately changed some of the laser parameters compared to those of our previous laser [16]. The laser cavity is now a 5.5-m-long loop, which comprises a 3.5-m-long, 2000-ppm erbium-doped fiber with a group-velocity dispersion of about  $-10$  ps/nm km, one piece of 1-m-long single-mode dispersion shifted fiber, whose group-velocity dispersion is  $-2$  ps/nm km, and one piece of 1-m-long standard single-mode fiber (SM-28). The nonlinear polarization rotation technique is still used to achieve the self-started mode locking. To this end, a polarization dependent isolator, together with two po-

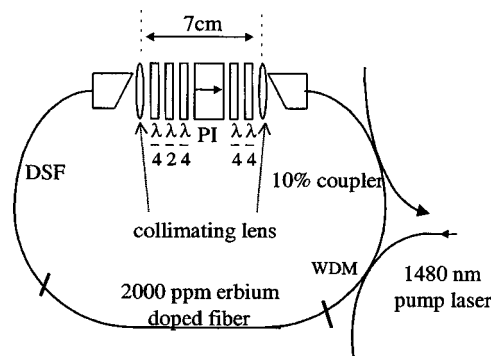


FIG. 1. A schematic of the fiber laser setup.  $\lambda/4$ —quarter-wave plate,  $\lambda/2$ —half-wave plate, and PI—polarization-dependent isolator. WDM—wavelength-dividing multiplexer. DSF—dispersion-shifted fiber.

larization controllers (one consisting of two quarter-wave plates and the other of two quarter-wave plates and one half-wave plate), is used to adjust the polarization of light in the cavity. The polarization dependent isolator and the polarization controllers are mounted on a 7-cm-long fiber bench to easily and accurately control the polarization of the light. The laser is pumped by a pigtailed In-Ga-As-P semiconductor diode of wavelength 1480 nm. A 10% output coupler is used to let the soliton pulses out of the laser cavity. The output of the laser is analyzed with an optical spectrum analyzer (Ando AQ-6315B) and a commercial optical autocorrelator (Inrad 5-14-LDA). A 50-GHz-wide-bandwidth sampling oscilloscope (Agilent 86100A) and a 25-GHz photodetector (New Focus 1414FC) are used to study soliton evolution in the laser cavity.

As mentioned above, the single-pulse, sech-form soliton emission is a typical feature of passively mode-locked ultrashort pulse fiber lasers, and the soliton dynamics could be well described by the extended nonlinear Schrödinger equation or the complex Ginzburg-Landau equation. Single-pulse solitons of the passively mode-locked fiber lasers have been intensively investigated previously. A typical feature of the solitons is that higher-order solitons are unstable as a result of the periodic perturbations caused by the laser gain and loss, etc. Solitons can only exist in their fundamental forms. Furthermore, due to the internal energy balance between the gain and loss, all fundamental solitons in a laser have exactly the same property: the same pulse width and the same pulse energy. This property of the solitons is known as the soliton energy quantization [11]. To visualize the feature we have shown in Fig. 2(a) an experimentally recorded oscilloscope trace of a typical single-pulse soliton operation state of our laser with multiple solitons coexisting in the cavity. The round-trip time of our laser cavity is about 26 ns. There are three single-pulse solitons in the cavity. From the oscilloscope trace it is clearly seen that all the solitons have exactly the same pulse height. Figures 2(b) and 2(c) show the corresponding optical spectrum and autocorrelation trace of the solitons, respectively. Except for the stronger spectral strength the soliton spectrum shown in Fig. 2(b) has exactly the same shape as that measured when only one soliton is in the cavity. A similar feature has also been observed in the measured autocorrelation trace. Despite the fact that there are three solitons in the cavity, the autocorrelation trace still gives the same pulse width as that measured when only one soliton is in the cavity, which under our present cavity configuration is about 340 fs. This feature of the optical spectrum and autocorrelation trace, together with the measured oscilloscope trace, demonstrates clearly that all the soliton pulses in the cavity have exactly the same property, e.g., the same pulse width and the same pulse energy. We have also experimentally checked cases where several tens of solitons coexist in the cavity. As far as the solitons are far apart ( $>800$  ps), the same feature of the soliton spectra and autocorrelation traces has always been observed. Soliton energy quantization is a generic property of soliton pulses in fiber lasers, and because of this property one can also precisely control the number of solitons in a laser cavity.

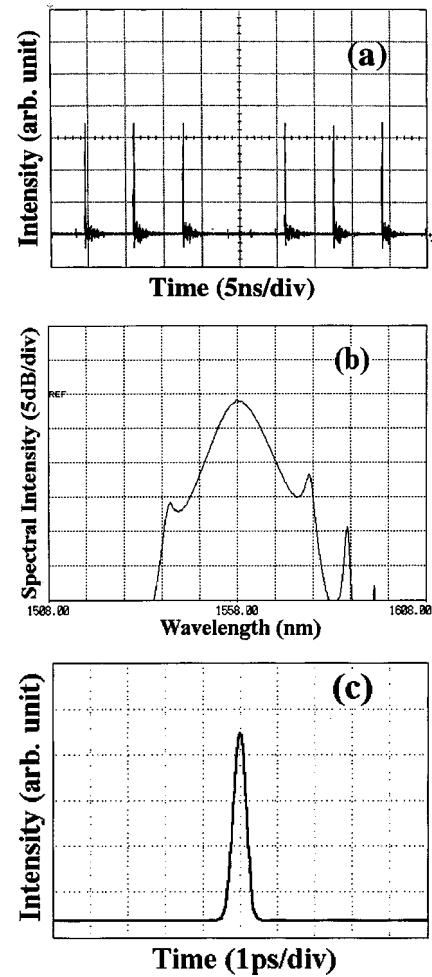


FIG. 2. A single-pulse soliton operation state of the laser with multiple solitons in the cavity. (a) Oscilloscope trace of the state. Note that there are three solitons in the cavity and they are far separated apart. (b) Soliton spectrum of the state. (c) Autocorrelation trace of the state.

Two forms of soliton interaction are well known to exist between the single-pulse solitons: direct soliton interaction and a long-range interaction mediated through the dispersive waves. Direct soliton interaction between the NLSE solitons had been intensively investigated previously [19,20], which was found to be the soliton phase dependent and to fall off exponentially with increasing soliton separation. The long-range soliton interaction mediated through dispersive waves between solitons in optical-fiber transmission systems and periodically amplified fiber links had also been investigated [21,22], however, its effect on the soliton dynamics in a passively mode-locked fiber laser was less emphasized. Apart from these two well-known forms of soliton interaction, it was also theoretically shown that in a fiber laser the gain depletion and the recovery of the erbium-doped fiber amplifier and the electrostriction caused by the solitons propagating in the fiber could induce extra interactions between solitons in a laser cavity and result in soliton bunching or quasiharmonic and harmonic mode locking [23,24]. To give an idea on how strong the various forms of soliton interaction could be in a laser, we have performed an experiment in

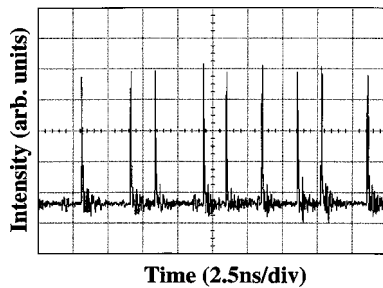


FIG. 3. Oscilloscope trace of a quasiharmonically mode-locked single-pulse soliton state.

order to investigate the interaction between two adjacent solitons in our laser through monitoring their relative movement. Details of the experimental results will be reported elsewhere. To summarize them, we have experimentally observed two types of soliton interactions in our laser. One is a global type of soliton interaction. This type of soliton interaction extends to the whole laser cavity and affects every soliton in the cavity. The physical origin of the soliton interaction could be due to the laser gain and electrostriction effects, as pointed out previously [23,24], the existence of a cw lasing component in the laser cavity as we found out, or their combined effect. Under the influence of the soliton interaction, solitons adjust their positions automatically relative to each other and eventually form an equilibrium state with a random soliton distribution pattern. Figure 2(a) is an example of the, thus, formed state. As another example we have shown in Fig. 3 a situation where more solitons are in the cavity. Such a soliton distribution was also called as quasi-harmonic mode locking, where solitons are unequally scattered over the whole cavity length. Occasionally, harmonic mode-locked patterns can also be obtained with solitons equally spaced in the cavity. However, there seem no effective ways to control the soliton separations, therefore, a harmonic mode-locked state can only be achieved by chance. Nevertheless, all the formed soliton distribution patterns are unstable in the sense that changing the laser operation condition such as the wave-plate orientation, pump strength could result in a rearrangement of the soliton positions. Also, whenever a soliton pulse is destroyed or a new soliton is generated, it will always cause all the solitons in the cavity to change their positions and reform a new equilibrium state with a different soliton distribution pattern. Another one is a local type of soliton interaction. This type of soliton interaction only extends to a maximum soliton separation of about 500 ps in our laser. The physical origin of interaction can be attributed to the dispersive-wave mediated soliton interaction [21,22]. When two solitons are spaced larger than the maximum value, they can remain stationary with respect to each other even under the influence of the global soliton interaction, as shown in Figs. 2(a) and 3. While if their separation is less than the maximum value, the local type of interaction takes place and consequently both solitons oscillate relative to one another. The local interaction does not affect the positions of other solitons in the cavity if they are far apart separated. Experimentally, we found that the closer the solitons are, the larger is the oscillation range. In cases where the

dispersive waves are strong and solitons are close to each other, two solitons can simply cross one another or make a collision. With a fixed number of solitons in the cavity, the strength of the dispersive waves can be controlled experimentally by carefully changing the pump power. Consequently, the actual range of the local soliton interaction also varies.

Apart from the single-pulse soliton, we have also observed bound states of solitons in our laser. In a previous paper, we reported on how to experimentally obtain the bound states of solitons and some of their features [16]. For the completeness of the paper, here we describe the experimental procedure again, briefly. Normally, we start from a single-pulse soliton operation state. Then, we carefully tune the orientations of the wave plates in our experimental setup to such a position where a strong cw lasing component could build up in the laser cavity. The cw emission coexists with the soliton pulses and must have a wavelength possibly close to or at the center of the soliton spectrum. At such a cavity condition the bound states of solitons would automatically appear. In our experimental setup, tuning the orientations of the wave plates physically corresponds to changing the linear phase bias of the cavity, which also alters the linear cavity transmission. Therefore, the main purpose of the procedure is to find the right linear cavity phase bias. If the linear cavity phase bias is correctly set, the bound states of solitons can also be directly obtained by simply increasing the pump beyond the laser mode-locking threshold. The observed bound solitons have the characteristic that they consist of two single-pulse solitons with fixed discrete pulse separations. Four bound states of solitons have been revealed in our experiments and their pulse separations roughly follow a relation of geometrical series of factor of 2. With a fixed fiber laser system, namely, the same cavity length and cavity components, it was found that the pulse separations of the observed bound solitons were independent of the laser operation condition. Whenever a bound state of solitons is observed, it will always have the same fixed pulse separations, no matter what the exact values of the pump strength, the frequency position of the cw component, and the orientations of each individual wave plate. With different fiber laser systems, although the concrete values of the pulse separations changed, the above-mentioned pulse separation relationship between the different bound-soliton states remained unchanged, showing that it is an invariant property of the bound solitons. Like the single-pulse soliton operation of the laser, the observed bound solitons are very stable. Once they are obtained, they can remain there for several hours, even under a noise experimental environment.

Bound states of solitons as a result of direct soliton interaction have been theoretically predicted in the coupled nonlinear Schrödinger equations [17] and the quintic complex Ginzburg-Landau equation [18,25]. Bound solitons thus formed were found to have discrete, fixed soliton separations, but they are weakly stable. Obviously, the bound-soliton pair observed in our laser is, in form, a kind of bound solitons. However, different from what theoretically predicted, they are ultrastable. To find out the possible reason of this difference, we have conducted experiments to further

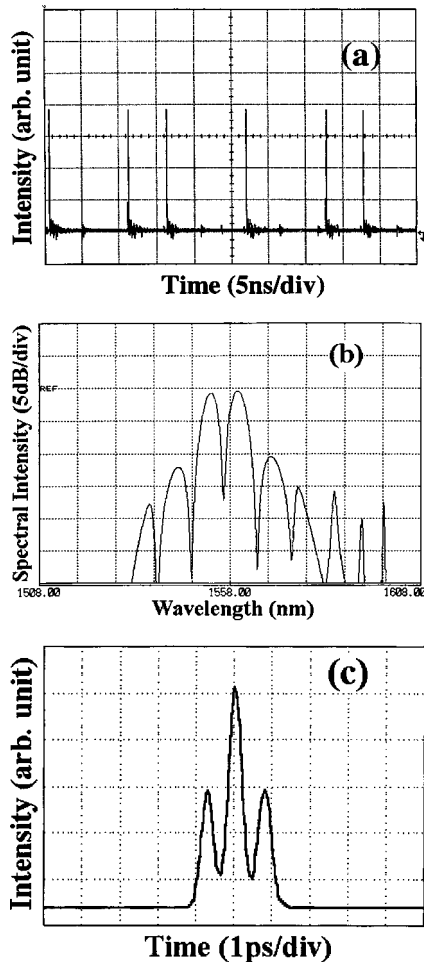


FIG. 4. A bound-soliton operation state of the laser with multiple bound solitons in the cavity. (a) Oscilloscope trace of the state. Note that there are three bound solitons in cavity. Separation between the two pulses of the bound solitons is about 930 fs, therefore, it cannot be resolved in the oscilloscope trace. (b) Soliton spectrum of the state. (c) Autocorrelation trace of the state.

study the interaction between the observed bound solitons in our laser. Surprisingly, we found that the bound-soliton pair always behaves as a unit. They are in fact not separable. Especially, the bound-soliton pair as a unit exhibits all the features of the single-pulse solitons observed in the laser, such as the soliton energy quantization [11], formation of soliton bunching [12] and harmonic mode locking [13], and the particlelike soliton collision. To illustrate the close similarity in property between the bound solitons and the single-pulse soliton, we show in Fig. 4(a) a typical oscilloscope trace of bound-soliton operation of our laser with multiple bound solitons in the cavity. For comparison with Fig. 2 we have selected a case where three bound solitons coexist. In our experiment, exactly the same detection system was used to record the two oscilloscope traces [shown in Fig. 2(a) and Fig. 4(a)]. Compared to Fig. 2(a), except that the bound solitons have higher pulse height the two oscilloscope traces show almost no difference. We emphasize that even all the bound solitons in the cavity have exactly the same pulse height. Figure 4(b) shows the corresponding soliton spec-

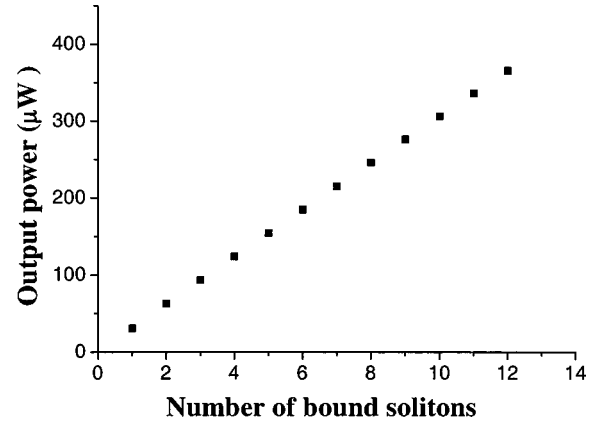


FIG. 5. Measured laser output power vs bound-soliton number in cavity relation.

trum and the autocorrelation trace of the bound solitons, which clearly show that the pulses are bound solitons rather than the single-pulse solitons. As different from the spectrum of the single-pulse solitons, the optical spectrum of the bound solitons exhibits strong spectral modulations. In a previous paper, we have already shown that the spectral modulation is a result of two solitons that are closely spaced in the time domain [16]. Mathematically, it can be easily proved that the Fourier transform of two closely spaced pulses in the time domain will give a modulated spectral distribution in the frequency domain. Based on this relationship one could directly calculate the pulse separation in the time domain by simply measuring a period of the spectral modulations. Like the cases of single-pulse solitons, the optical spectrum shown in Fig. 4(b) has exactly the same shape as that measured when only a single bound soliton is in the cavity. The measured autocorrelation trace shown in Fig. 4(c) also shows no difference in the pulse width and the soliton separation to that measured when only a single bound soliton is in cavity. Based on all these information we conclude that all the bound-soliton pulses in the cavity have identical properties, namely, they have the same soliton pulse separation and pulse energy. We have also checked cases where several tens of bound solitons coexist in the cavity, and also the cases of bound solitons with different pulse separations, the same result was obtained, showing that even the energy of the bound solitons in a laser is quantized. In addition, we have also experimentally measured the energy of each individual bound soliton by simultaneously measuring the laser output power and monitoring the number of bound solitons in the cavity, as shown in Fig. 5; changing the number of bound solitons in the cavity, the laser output does exhibit quantized jumps of equal amounts of energy.

With our present fiber laser configuration a total number of four different bound states of solitons were obtained. Different bound states of solitons are distinguished by their different soliton pulse separations. However, once a state of bound solitons is achieved, in a stable state neither the single-pulse solitons nor other forms of bound solitons are observed to coexist, showing that at a time only one bound state of solitons is stable. Like the single-pulse soliton case, the number of bound solitons in the cavity can be controlled

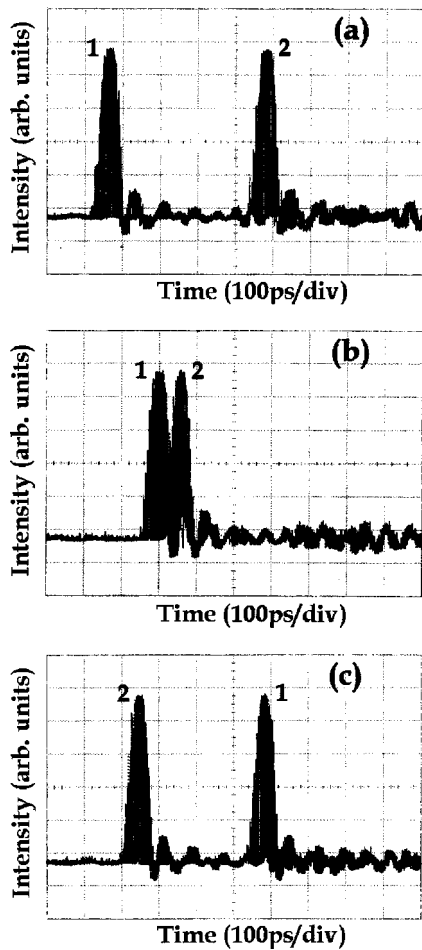


FIG. 6. Stroboscopic oscilloscope record of a bound-soliton collision process. (a) Two bound solitons moved toward each other. (b) Just before the collision. (c) After the collision they moved away from each other.

by carefully changing the pump power. The bound solitons have a structure of two bound pulses, however, changing the pump power can only generate or destroy the soliton pair simultaneously. In other words, the bound-soliton pair behaves always as a unit and cannot be separated. Actually, this property of the bound solitons is already reflected in the result shown in Fig. 5. Each time a bound soliton is destroyed a constant amount of output power drop is observed. To further illustrate the property of the bound solitons we show in Fig. 6 a stroboscopic record of a direct bound-soliton interaction process. Like the case of single-pulse solitons, there exists the dispersive-wave-mediated local bound-soliton interaction. As a result of the interaction, two nearby bound solitons oscillate relatively with respect to each other and make collisions. The relative oscillation and collisions have a very slow speed, so that one can monitor and follow them on the oscilloscope screen. Figure 6 shows that two bound-soliton pairs moved towards each other, collided, and then separated. As shown in the figures, after the collision each of the two bound solitons still remained at the same pulse height, indicating no change in their pulse energy during the collision or else their energy is quickly restored to the original value within one round-trip time due to the cavity

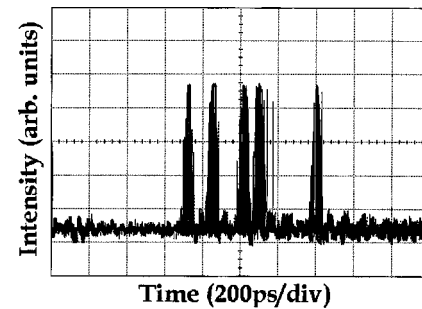


FIG. 7. Stroboscopic oscilloscope record of a mutual multiple bound-soliton interaction state.

feedback effect. Again, the bound soliton nature of the pulses shown is confirmed by their soliton spectrum which was simultaneously monitored by an optical spectral analyzer, and the soliton separations between the bound pulses are determined by both their soliton spectral modulation periods and the autocorrelation traces. No change in the soliton spectra and autocorrelation traces have been found between results measured before and after the collision, indicating that there is no breaking of the soliton binding and even no change in their pulse separations. Figure 7 shows again a case of random interaction among five bound solitons. The bound-soliton pairs either collided mutually with each other or simply passed through one another, no annihilation of solitons or breaking of the bound pulse pair was observed during the collisions.

Analogous to the features of the single-pulse solitons, experimentally we have also obtained bunching of bound solitons, quasiharmonic and harmonic bound-soliton mode locking. Figure 8 shows as an example a case of the quasiharmonic mode locking of the bound solitons. Exactly like the case of single-pulse solitons, under the influence of the global soliton interaction the bound solitons in the cavity can adjust automatically their positions to form various equilibrium states of random bound-soliton distribution patterns. Again, the bound-soliton nature of the pulses shown in Fig. 8 is determined by their optical spectra and autocorrelation traces. All the bound-soliton patterns thus formed are also unstable in the sense that changing the laser operation conditions causes bound solitons to rearrange their positions and form a new equilibrium pattern. These experimental evidences demonstrate that the observed “bound-soliton pair” as an entity is equivalent to the single-pulse soliton of the

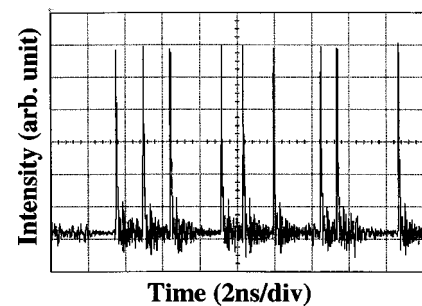


FIG. 8. Oscilloscope trace of a bound-soliton quasiharmonically mode-locked state.

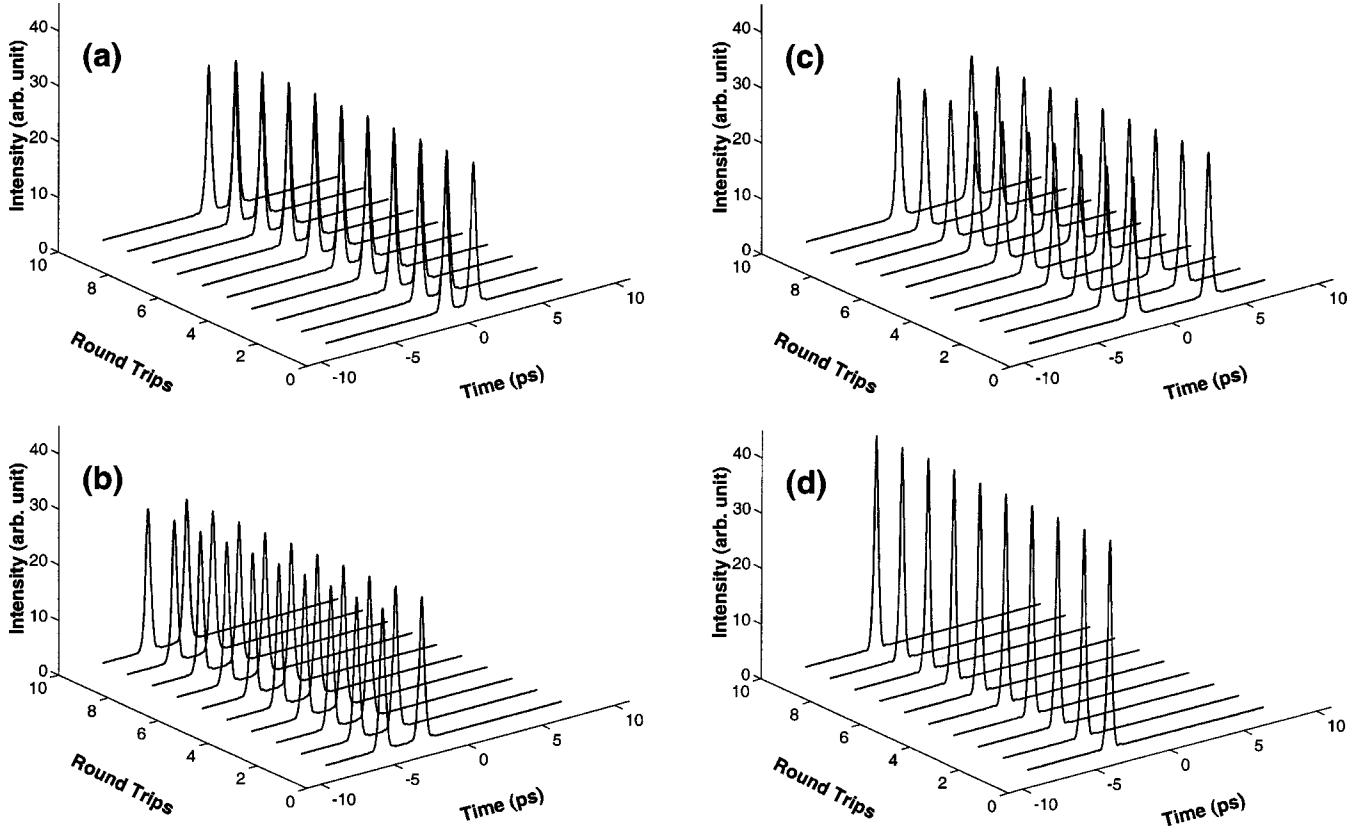


FIG. 9. States of solitons numerically calculated for the laser. (a–c) Evolution of bound states of solitons obtained.  $g_0=255$ ,  $E_s=105$ . Linear cavity phase bias is  $1.48\pi$ , and from (a–c) only the initial conditions are changed. (d) Evolution of single pulse soliton in the laser,  $g_0=265$ ,  $E_s=100$ , linear cavity phase bias is  $1.65\pi$ . Other parameters used are  $\gamma=3 \text{ W}^{-1} \text{ km}^{-1}$ ,  $\kappa''=-2 \text{ ps/nm km}$  (for dispersion shifted fiber),  $\kappa''=-10 \text{ ps/nm km}$  (for erbium-doped fiber),  $\Omega_g=2\pi\times 10 \text{ THz}$ , cavity length  $L=11 \text{ m}$ , and beat length  $L_b=L/2$ .

laser. It should be of another form of solitary wave in the laser.

To further support that the double-peak pulses are a different form of solitary wave in the laser, we have numerically simulated the operation of our laser. To this end we have built up a numerical model based on the coupled extended nonlinear Schrödinger equations that explicitly take into account the birefringence of the fiber and the gain and loss of the laser cavity:

$$\begin{aligned} \frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i}{2} \kappa'' \frac{\partial^2 u}{\partial t^2} + i\gamma \left( |u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* \\ &+ \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i}{2} \kappa'' \frac{\partial^2 v}{\partial t^2} + i\gamma \left( |v|^2 + \frac{2}{3} |u|^2 \right) v \\ &+ \frac{i\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}, \end{aligned} \quad (1)$$

where  $u$  and  $v$  are the two normalized slowly varying pulse envelopes along the slow and the fast axes,  $u^*$  and  $v^*$  are their conjugations, respectively.  $u$  and  $v$  are normalized in such a way that  $|u|^2$  and  $|v|^2$  represent their power.  $2\beta$

$=2\pi\Delta n/\lambda$  is the wave-number difference,  $2\delta=2\beta\lambda/2\pi c$  is the inverse group-velocity difference,  $\kappa''$  is the dispersion parameter,  $\gamma$  is the nonlinearity of the fiber,  $g$  is the laser gain coefficient, and  $\Omega_g$  is the bandwidth of the laser gain. The gain saturation of the laser is considered by taking

$$g = \frac{g_0}{1 + \frac{\int (|u|^2 + |v|^2) dt}{E_s}}, \quad (2)$$

where  $g_0$  is the small signal gain and  $E_s$  is the saturation energy. The laser gain is saturated by the total pulse energy inside the laser cavity. To precisely simulate the laser action and its influence on the solitons formed in the laser cavity, we have also considered the cavity feedback effect, linear cavity transmission, and the actions caused by various cavity components on the solitons in our model. Technically, this was done as the following: we first set an arbitrary input light pulse as the initial condition, and then let the light pulse circulate in the laser cavity exactly according to the laser configuration. When the light pulse propagates in the  $\text{Er}^{3+}$ -doped fiber we simulate its propagation with the coupled nonlinear Schrödinger equations shown above. If it is in the undoped fiber the gain will be set to zero. Whenever the light pulse meets an open-air cavity component we multiply the transformation matrix of the cavity component to

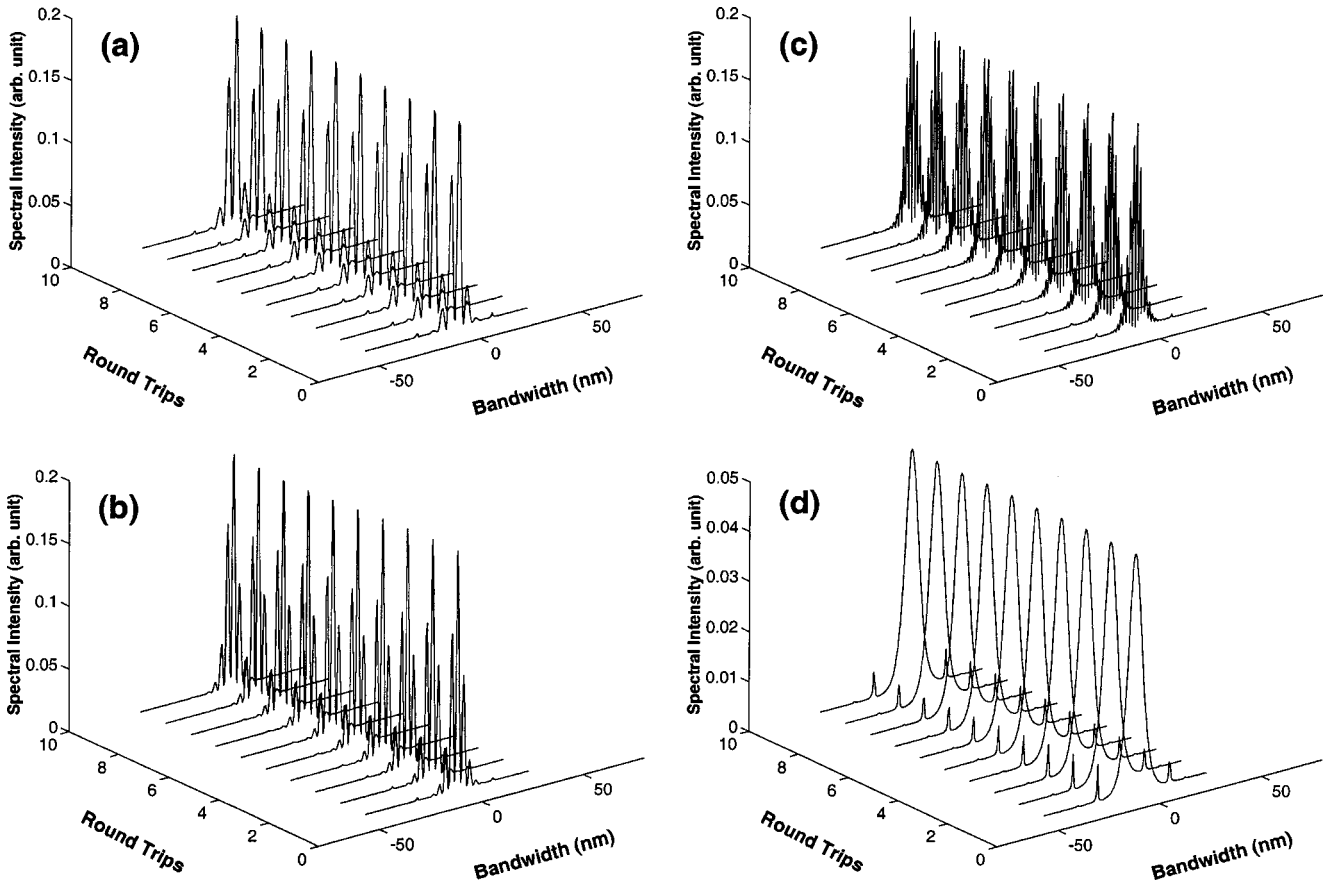


FIG. 10. Optical spectra corresponding to the soliton states shown in Fig. 9.

the light field. We circulate the light until a steady state is reached. Wherever possible we have used the actual fiber and laser parameters for our simulations. We found that by appropriately setting the linear cavity transmission and the polarization orientation of the isolator, we can actually obtain either the single-pulse soliton operation or the bound-soliton pair operation with discrete, fixed pulse separations in our model. Figure 9 shows, for example, the calculated single-pulse and bound-pulse solitons of the laser. In our simulations, three stable bound states of solitons have been numerically obtained. Exactly like the experimental observations, the bound states of solitons exhibit fixed, discrete pulse separations. The single-pulse soliton can be obtained in our model with a wide range of parameter settings. However, the bound-pulse solitons can only be achieved in a very narrow parameter range, which explains why the double-pulse solitons were difficult to observe experimentally. As observed experimentally, both the pulses in a bound state of solitons will be destroyed simultaneously if the gain of the laser is reduced below the value necessarily required to support them, which confirms numerically that the two pulses must exist together and cannot be destroyed one by one. In our simulations we have *tried with* different arbitrary initial pulse forms, as far as we have tested they all eventually stabilize at one of the bound soliton states shown in Fig. 9, showing that they are unique stable states of the laser system. We have also numerically tested the stability of the bound solitons obtained by slightly changing the simulation parameters such

as the saturation power and the small signal gain value, no change in the pulse separation has been observed. For the completeness we have also shown in Fig. 10 the corresponding soliton spectra of the bound-soliton states together with that of the single-pulse soliton. The calculated bound-soliton spectra show asymmetrical spectral modulations with a dip near the center, suggesting that the optical phase difference between the two bound pulses is close to  $\pi/2$ . For both the single-pulse and the bound-pulse solitons, the sidebands caused by the periodic perturbation of the laser gain and loss, etc., are clearly visible in their spectra. Our numerical simulations demonstrated an excellent agreement with the experimental observations and confirm the existence of the double-pulse solitary waves in the laser.

In summary, we have experimentally studied the interaction between the bound solitons observed in a passively mode-locked fiber laser and revealed that the bound-soliton pair always behaves as a unit and exhibit exactly the same properties as those of the single-pulse soliton in the laser. The close similarity in property between the bound solitons and the single-pulse soliton, together with that the observed bound solitons have discrete, fixed pulse separations, suggests strongly that the observed bound solitons are not results of a simple soliton binding through the soliton interaction, but rather another form of solitary waves in the laser. Numerical simulations of the operation of the laser have also confirmed the existence of such solitons in lasers. Although

this experimental result is observed as a special case in the fiber laser, as the dynamics of the laser is mainly governed by the coupled complex Ginzburg-Landau equations and the same equations describe a wide range of nonlinear dynamical systems, we believe it could be a general property of nonlinear dynamical systems such that, apart from the simple single-pulse soliton, there exists another stable compound pulse form of solitons. In fact, we noticed that recently a similar multihump spatial solitons have been observed in a

dispersive nonlinear medium and confirm it theoretically [26,27]. However, to the best of our knowledge, our result shows the first experimental evidence of stable multipulse solitons in the temporal domain.

W.S.M. and H.Y.T. acknowledge support from a university research grant of The Hong Kong Polytechnic University.

- 
- [1] Y. C. Newell, *Solitons in Mathematics and Physics* (SIAM, Philadelphia, 1985).
- [2] A. W. Snyder and D. J. Mitchell, *Science* **276**, 1538 (1997).
- [3] V. V. Afanasjev and N. Akhmediev, *Phys. Rev. E* **53**, 6471 (1996).
- [4] N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crepeo, *J. Opt. Soc. Am. B* **15**, 515 (1998).
- [5] Y. Kodama and A. Hasegawa, *Opt. Lett.* **17**, 31 (1992).
- [6] M. Nakazawa, H. Kubota, K. Kurokawa, and E. Yamada, *J. Opt. Soc. Am. B* **8**, 1811 (1991).
- [7] R. J. Essiambre and G. P. Agrawal, *Electron. Lett.* **31**, 1461 (1995).
- [8] D. U. Noske, N. Pandit, and J. R. Taylor, *Electron. Lett.* **28**, 2185 (1992).
- [9] D. J. Richardson, R. I. Laming, D. N. Payne, M. W. Phillips, and V. J. Matsas, *Electron. Lett.* **27**, 730 (1991).
- [10] M. J. Guy, D. U. Noske, and J. R. Taylor, *Opt. Lett.* **18**, 1447 (1993).
- [11] A. B. Grudinin, D. J. Richardson, and D. N. Payne, *Electron. Lett.* **28**, 67 (1992).
- [12] D. J. Richardson, R. I. Laming, D. N. Payne, V. I. Matsas, and M. W. Phillips, *Electron. Lett.* **27**, 1451 (1991).
- [13] A. B. Grudinin, D. J. Richardson, and D. N. Payne, *Electron. Lett.* **27**, 1860 (1993).
- [14] D. Y. Tang, W. S. Man, H. Y. Tam, and M. S. Demokan, *Phys. Rev. A* **61**, 023804 (2000).
- [15] S. R. Friberg and K. W. DeLong, *Opt. Lett.* **17**, 979 (1992).
- [16] D. Y. Tang, W. S. Man, H. Y. Tam, and P. D. Drummond, *Phys. Rev. A* **64**, 033814 (2001).
- [17] B. A. Malomed, *Phys. Rev. A* **45**, R8321 (1992).
- [18] B. A. Malomed, *Phys. Rev. A* **44**, 6954 (1991).
- [19] J. P. Gordon, *Opt. Lett.* **8**, 596 (1983).
- [20] P. L. Chu and C. Desem, *Electron. Lett.* **19**, 429 (1983).
- [21] K. Smith and L. F. Mollenauer, *Opt. Lett.* **14**, 1284 (1989).
- [22] L. Socci and M. Romagnoli, *J. Opt. Soc. Am. B* **16**, 12 (1999).
- [23] J. Nathan Kutz, B. C. Collings, K. Bergman, and W. H. Knox, *IEEE J. Quantum Electron.* **34**, 1749 (1998).
- [24] A. N. Pilipetskii, E. A. Golovchenko, and C. R. Menyuk, *Opt. Lett.* **20**, 907 (1995).
- [25] N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crepeo, *Phys. Rev. Lett.* **79**, 4047 (1997).
- [26] M. Mitchell, M. Segev, and D. N. Christodoulides, *Phys. Rev. Lett.* **80**, 4657 (1998).
- [27] E. A. Ostrovskaya, Y. S. Kivshar, D. V. Skryabin, and W. J. Firth, *Phys. Rev. Lett.* **83**, 296 (1999).