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Flattening of conic reflectors via a transformation method

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This paper presents a general and rigorous transformation method to tailor planar conic reflectors. The proposed method enables the designed reflectors to scatter or reflect incident light in the same manner as a conic reflector while the whole device as well as the reflector would maintain planar profiles. In order to reduce the overall size, especially the aperture of the reflector, we further apply a set of compressed and folded spatial mapping to the planar reflectors. Planar reflectors with reduced sizes are finally obtained which may be useful in several optical and electromagnetic applications.

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I. INTRODUCTION

Traditionally geometrical optics tells us that conic reflectors and planar reflectors have different optical properties. For example, any light radiated from the focus would propagate parallel to the main axis when it is reflected by a parabolic reflector. However, when the light is reflected by a planar reflector, it would propagate in the manner as if it is radiated from a virtual image source behind the reflector. So it may seem impossible to obtain a planar reflector with the same reflection property as a conic reflector from the conventional optics point of view. Currently, transformation optics [1–3] has been proved as an effective tool to tailor unique artificial media to manipulate light in any desired manner. Although the initial application of this theory is in designing invisibility cloaks [2,4], a wide range of unconventional optical and electromagnetic (EM) devices have also been proposed based on this concept. For example, Chen *et al.* designed field rotators in which the incident EM wave shall rotate a certain angle to appear as coming from a different direction [5,6]; Luo *et al.* designed field concentrators which can make the power flow of the incident wave concentrated within a given region [6–9]; besides, several kinds of superlenses [10–14] have also been proposed by researchers. Based on the transformation optics, this paper proposes a general and rigorous method to tailor a class of reflectors which will have the same optical properties as a conic reflector—that is, parabolic, hyperbolic, or elliptic reflector—while maintaining planar profiles.

II. THEORETICAL DESCRIPTION

Figures 1(a) and 1(b) show the optical properties of a conventional parabolic reflector and planar reflector, respectively. Any light reflected by a reflector would obey reflection law. We will demonstrate that, by applying a transformation method, one can flatten a conic reflector to a planar reflector without changing its original optical property, as shown in Fig. 1(c). Note that when the reflector (bold full blue line) is not inserted, the device would be a perfect transparent lens in which the incident light will deviate from its original propagation direction but when it leave the lens it will return to its trajectory, as shown in the orange arrows. The schematic diagram of the proposed spatial transformation is depicted in Fig. 1(d). The original right-hand space AEOFD is compressed along the $+x$ axis into a rectangular region of AEFD; at the same time, the original left-hand space BEOFC is correspondingly expanded into a rectangular region of BEFC. Thus the conic surface EOF will be flattened into a planar surface EF after the transformation.

Taking two-dimensional (2D) space as an example, for simplicity and without loss of generality, we could choose the coordinate transformation in a linear form as

$$x = \frac{x_3 - x_1}{x_3 - x_2}(x' - x_2) + x_1, \quad y = y', \quad z = z'. \tag{1}$$

For a parabola defined by $x = py^2$, there is

$$x_1 = py'^2, \quad x_2 = x_0, \quad x_3 = a. \tag{2}$$

Substituting Eq. (2) into Eq. (1), we can obtain the detailed coordinate transformation as

$$x = \frac{a - py'^2}{a - x_0}(x' - x_0) + py'^2, \quad y = y', \quad z = z'. \tag{3}$$

According to the theory of transformation optics [2,3], the spatial transformation from an original space to a distorted space is equivalent to the constitutive parameter variations in the space. The permittivity and permeability of the transformation media are calculated by $\boldsymbol{\varepsilon}' = \mathbf{A}\boldsymbol{\varepsilon}\mathbf{A}^T/\det(\mathbf{A})$, $\boldsymbol{\mu}' = \mathbf{A}\boldsymbol{\mu}\mathbf{A}^T/\det(\mathbf{A})$ where $\mathbf{A}$ is the Jacobian transformation matrix, and $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ are the permittivity and permeability in the original space, respectively. So $\boldsymbol{\varepsilon}'$ and $\boldsymbol{\mu}'$ for the planar parabolic reflectors are calculated as

$$\varepsilon'_{xx} = \mu'_{xx} = \frac{a - x_0}{a - py'^2} + \frac{4p^2y'^2(a - x'^2)}{(a - py'^2)(a - x_0)}, \tag{4a}$$

$$\varepsilon'_{xy} = \mu'_{xy} = \frac{-2py'(a - x')}{a - x_0}, \tag{4b}$$

$$\varepsilon'_{yy} = \mu'_{yy} = \frac{a - py'^2}{a - x_0}, \tag{4c}$$

$$\varepsilon'_{zz} = \mu'_{zz} = \frac{a - py'^2}{a - x_0}. \tag{4d}$$

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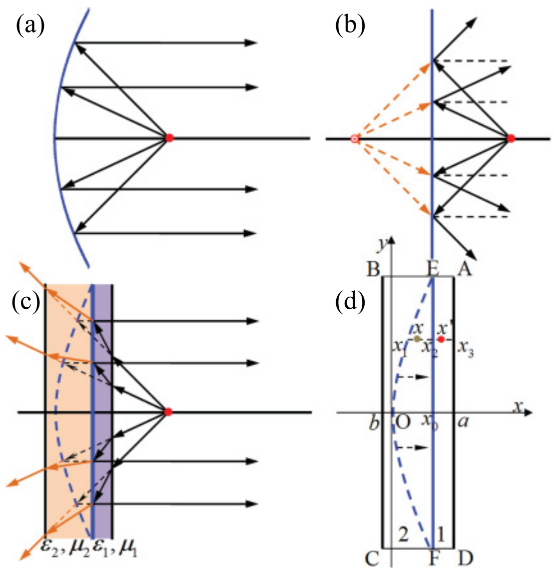

FIG. 1. (Color online) Optical properties of (a) conventional parabolic reflector, (b) planar reflector, and (c) proposed unconventional planar parabolic reflector. (d) The schematic diagram of spatial mapping used in the proposal.

Similarly, for an ellipse defined by $x^2/m^2 + y^2/n^2 = 1$, there is

$$x_1 = -\frac{m}{n}\sqrt{n^2 - y'^2}, \;\; x_2 = x_0, \;\; x_3 = a. \tag{5}$$

Substituting it into Eq. (1), we can obtain the detailed coordinate transformation as

$$\begin{aligned} x = {} & \frac{a + \frac{m}{n}\sqrt{n^2 - y'^2}}{a - x_0}(x' - x_0) \\ & - \frac{m}{n}\sqrt{n^2 - y'^2}, \;\; y = y', \;\; z = z'. \end{aligned} \tag{6}$$

$\boldsymbol{\varepsilon}'$ and $\boldsymbol{\mu}'$ are calculated as

$$\begin{aligned} \varepsilon'_{xx} = \mu'_{xx} = {} & \frac{m\sqrt{n^2 - y'^2} + na}{n(a - x_0)}\left(\frac{n^2(a - x_0)^2}{(m\sqrt{n^2 - y'^2} + na)^2}\right. \\ & \left. + \frac{m^2 y'^2 (a - x')^2}{(m(n^2 - y'^2) + na\sqrt{n^2 - y'^2})^2}\right), \end{aligned} \tag{7a}$$

$$\varepsilon'_{xy} = \mu'_{xy} = \frac{my'(a - x')}{n(x_0 - a)\sqrt{n^2 - y'^2}}, \tag{7b}$$

$$\varepsilon'_{yy} = \mu'_{yy} = \frac{m\sqrt{n^2 - y'^2} + na}{n(a - x_0)}, \tag{7c}$$

$$\varepsilon'_{zz} = \mu'_{zz} = \frac{m\sqrt{n^2 - y'^2} + na}{n(a - x_0)}. \tag{7d}$$

For a hyperbola defined by $x^2/m^2 - y^2/n^2 = 1$, there is

$$x_1 = \frac{m}{n}\sqrt{n^2 + y'^2}, \;\; x_2 = x_0, \;\; x_3 = a. \tag{8}$$

Substituting it into Eq. (1), we can obtain the detailed coordinate transformation as

$$\begin{aligned} x = {} & \frac{a - \frac{m}{n}\sqrt{n^2 + y'^2}}{a - x_0}(x' - x_0) + \frac{m}{n}\sqrt{n^2 + y'^2}, \\ & y = y', \;\; z = z'. \end{aligned} \tag{9}$$

$\boldsymbol{\varepsilon}'$ and $\boldsymbol{\mu}'$ are calculated as

$$\begin{aligned} \varepsilon'_{xx} = \mu'_{xx} = {} & \frac{m\sqrt{n^2 + y'^2} - na}{n(x_0 - a)}\left(\frac{n^2(a - x_0)^2}{(m\sqrt{n^2 + y'^2} - na)^2}\right. \\ & \left. + \frac{m^2 y'^2 (a - x')^2}{(m(n^2 + y'^2) - na\sqrt{n^2 + y'^2})^2}\right), \end{aligned} \tag{10a}$$

$$\varepsilon'_{xy} = \mu'_{xy} = \frac{my'(a - x')}{n(x_0 - a)\sqrt{n^2 + y'^2}}, \tag{10b}$$

$$\varepsilon'_{yy} = \mu'_{yy} = \frac{m\sqrt{n^2 + y'^2} - na}{n(x_0 - a)}, \tag{10c}$$

$$\varepsilon'_{zz} = \mu'_{zz} = \frac{m\sqrt{n^2 + y'^2} - na}{n(x_0 - a)}. \tag{10d}$$

Although the above calculations are specific to the right-hand space, in fact they are the same in the left-hand space as one only needs to replace the coordinate a by b in Eqs. (2)–(10).

One of the most important applications of a planar reflector is in building planar reflector antennas such as satellite antennas. Kong *et al.* discussed this issue to a certain extent [15]. In their work, they studied the parabolic case and calculated the transformation media for the concave side [$\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\mu}_1$ in Fig. 1(c)]. They also built an imperfect planar parabolic antenna, and demonstrated that when the wave normal incidents from the concave side, the reflection property of their designed planar reflector is nearly equivalent to a real parabolic reflector. However, they did not consider the spatial mapping in the convex side. Therefore, the distorted space did not equal to the original space during the transformation so that the tailored planar reflector could not be perfectly equivalent to the parabolic reflector especially from any other incident directions. In many cases the reflection or scattering properties of a convex surface of a reflector are required. For example, in a typical Cassegrain antenna (a kind of dual-reflector antenna) the concave surface of a parabolic reflector is used as the main reflector while the convex surface of a hyperbolic reflector is used as the subreflector. So a complete design for both convex and concave surfaces is necessary for potential applications of the flattened reflectors.

III. NUMERICAL VALIDATION

To validate the proposed method, full-wave simulations on several detailed examples are carried out in the following. The first example is a parabolic reflector with an equation of $x = 5y^2$ and geometrical parameters of $a = 0.08$ m, $b = -0.02$ m, and $x_0 = 0.05$ m. Figure 2(a) shows the total electric field distribution when the tailored planar device (without inserting a reflector) under a transverse electric (TE-mode) plane-wave

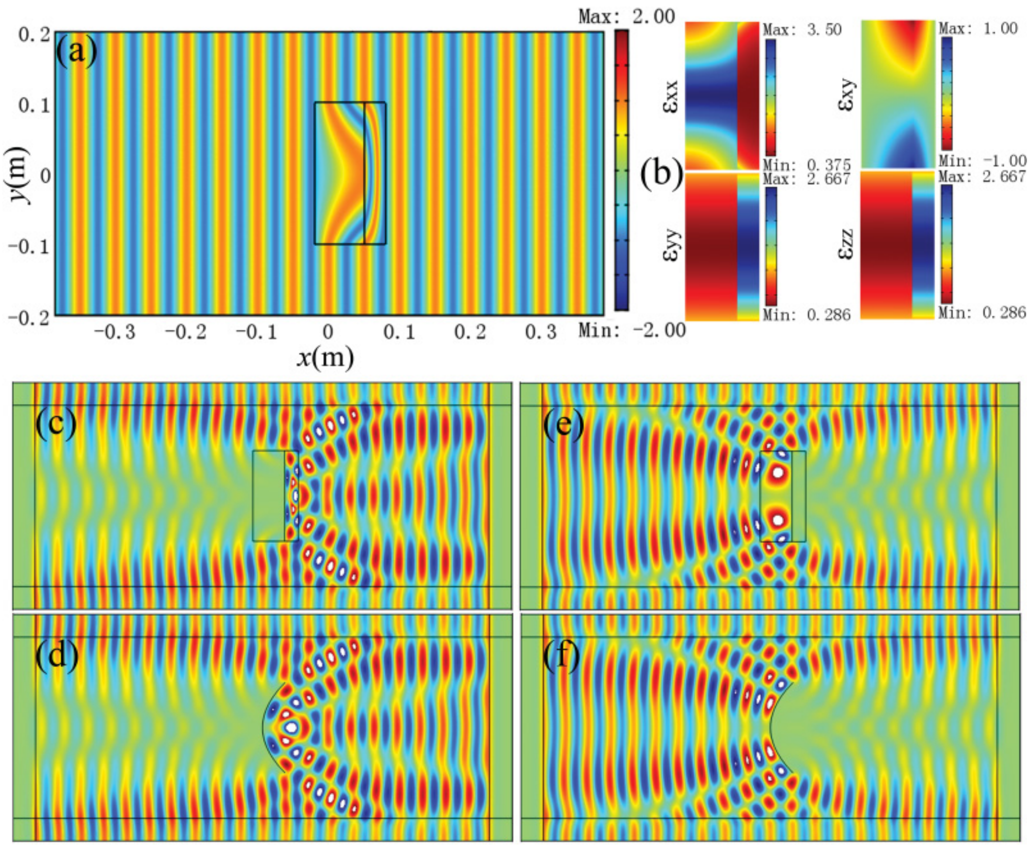

FIG. 2. (Color online) (a) Without the inserted reflector, the device is a perfect transparent lens in the incident wave. (b) The distribution of material parameters in the device. Scattering patterns of (c), (e) the designed planar reflector and (d), (f) the original parabolic reflector in TE-mode plane-wave incidents (c), (d) from the right-hand side to the left-hand side and (e), (f) from the left-hand side to the right-hand side, respectively.

incident from the left-hand side to the right-hand side. One can find the wave bent smoothly inside the media while it remained undisturbed outside the media. The whole device does not bring any scattering (invisible). To get an intuitive description of the tailored media we plot the distributions of $(\boldsymbol{\varepsilon}_1,\boldsymbol{\mu}_1)$ and $(\boldsymbol{\varepsilon}_2,\boldsymbol{\mu}_2)$ in Fig. 2(b). None of the parameters is singular and the changing ranges of them are relatively small. However, they are spatially gradient media, so when practical realization is concerned, the parameters may need to be simplified.

Now we insert a planar perfect electric conducting (PEC) reflector into the media. When the TE-mode plane wave is incident from the right-hand side to the left-hand side, the scattering pattern of the system is depicted in Fig. 2(c), which is identical to the case of the real parabolic PEC reflector as shown in Fig. 2(d). Obviously, Figs. 2(c) and 2(d) have validated media $(\boldsymbol{\varepsilon}_1,\boldsymbol{\mu}_1)$. To demonstrate the efficiency of media $(\boldsymbol{\varepsilon}_2,\boldsymbol{\mu}_2)$ we reset the wave incident from the left-hand side to the right-hand side, and the results are shown in Figs. 2(e) and 2(f), respectively. The scattering pattern of the planar reflector shown in Fig. 2(e) still remains the same as that of the real parabolic reflector shown in Fig. 2(f). So, from the results shown in Fig. 2, we may conclude that the flattened reflector is indeed equivalent to the real parabolic reflector.

In order to obtain a clearer demonstration of the reflection property, we further apply a Gaussian beam incident at a flattened reflector (with $x = 2.5y^2, a = 0.04$ m, $b = -0.02$ m, $x_0 = 0.025$) from the right-hand side, as shown in Fig. 3(a). One can find the beam is reflected by the reflector and then focused at the focus ($F = 0.1$ m). When the beam is reflected by the real parabolic reflector, the result is depicted in Fig. 3(b).

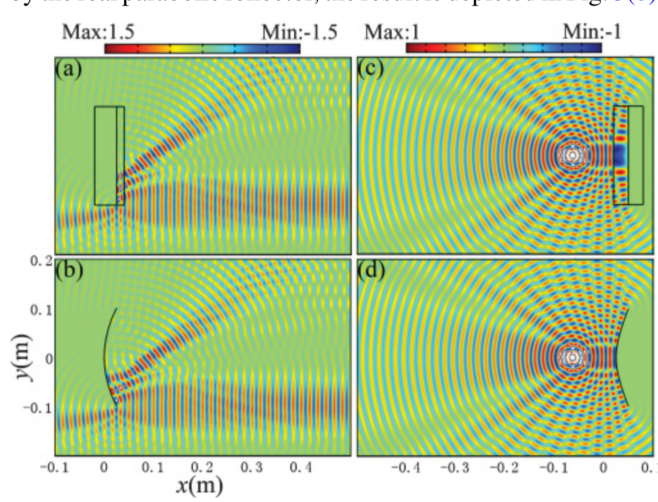

FIG. 3. (Color online) The total electric field distributions when a Gaussian beam incidents on the concave surface of (a) a designed planar parabolic reflector and (b) the original parabolic reflector. The total electric field distributions when a cylindrical wave incidents on the convex surface of (c) a designed planar hyperbolic reflector and (d) the original hyperbolic reflector.

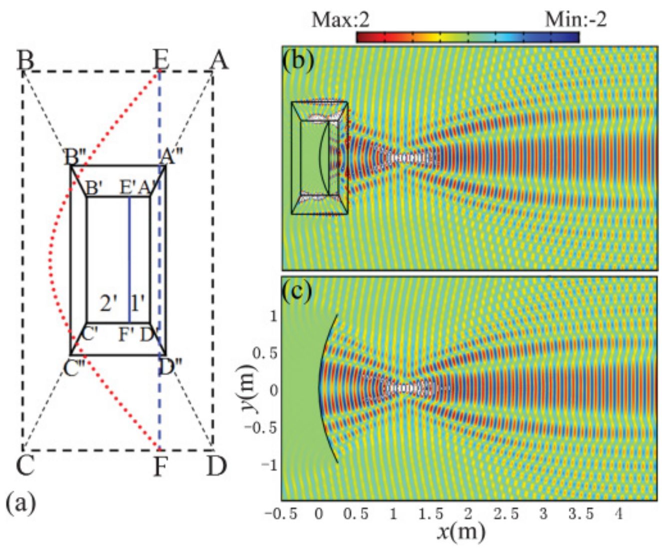

FIG. 4. (Color online) The schematic diagram of shrunken planar reflector. (a) A planar reflector ABCD is shrunken into A″B″C″D'' and the shrunken reflector E′F′ will still be equivalent to the conventional conic reflector (dotted red line). The total electric field distributions when a Gaussian beam incidents on (b) a designed shrunken planar reflector and (c) a conventional parabolic reflector.

From a simple comparison between Figs. 3(a) and 3(b), one can find the flattened planar reflector reflects light in the same manner as the real parabolic reflector.

As mentioned above, in some cases we use the concave surface to reflect light while in several applications we should use the convex surface. Figures 3(a) and 3(b) have provided a detailed example to show the former case. We also give an example to depict the latter case, although it has been partially embodied in Figs. 2(e) and 2(f). In this example we use a hyperbolic reflector with an equation of $x^2/(0.025)^2 - 3y^2/(0.1)^2 = 1$ and geometrical parameters of $a = 0.08$ m, $b = 0.02$ m, and $x_0 = 0.05$ m. When a point source radiates the wave toward the convex surface of the flattened hyperbolic reflector, the distribution of the total electric field is shown in Fig. 3(c), which still maintains the same shape as the case of the real hyperbolic reflector shown in Fig. 3(d). Therefore, examples shown in Fig. 3 clearly demonstrate that the designed planar reflectors have the same reflection properties as a real conic reflector in both convex and concave surfaces.

As shown above, we have tailored planar reflectors which are exactly equivalent to conic reflectors. It is doubtless that if we can further dramatically reduce the overall size—especially the aperture—of the planar reflector, then the proposed method may be more attractive in practical applications. Fortunately, folded transformation [16] and negative refracting media can provide us with an effective approach to tackle this issue [17]. A few small-sized antennas have been designed using this method [18,19]. Based on the structure shown in Fig. 1(d), we can shrink the device through two steps as shown in Fig. 4(a). First, we uniformly compress the flattened reflector ABCD into A′B′C′D′ by a transformation $r = kr'$, where $k > 1$. The media are correspondingly calculated as

$$\boldsymbol{\varepsilon}' = \boldsymbol{\mu}' = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & k^2\varepsilon_{zz} \end{bmatrix}, \tag{11}$$

where ε_{ij} is component of $\boldsymbol{\varepsilon}_1$ or $\boldsymbol{\varepsilon}_2$.

Second, we fold the surrounded four trapezoidal empty spaces (AA″B″B, BB″C″C, CC″D″D, DD″A″A) into four smaller trapezoidal sectors (A′A″B″B′, B′B″C″C′, C′C″D″D′, D′D″A″A′). Detailed descriptions of the required folded transformation and parameters calculation for these trapezoidal sectors are available in Ref. [20].

After the compressed and folded transformations, the shrunken device A″B″C″D″ will be equivalent to the original device ABCD, and the shrunken planar reflector E′F′ will map to the uncompressed planar reflector EF, which would further map to the conventional conic reflector (dotted red curve). The schematic diagram clearly indicates that the shrunken device can be designed to be thinner than the original conic reflector, and the aperture of the reflector can be dramatically reduced after compression.

A detailed example will be presented to validate the design. Figure 4(b) shows the total electric field distribution when a TE-mode Gaussian beam is incident on a shrunken planar reflector, while Fig. 4(c) shows the case when the beam is incident on a real parabolic reflector. The two figures demonstrate that the shrunken planar reflector is equivalent to the original parabolic reflector in the incident wave. The aperture of the parabolic reflector (EF) and the shrunken planar reflector (E′F′) is 1 and 0.5 m, respectively, while the overall radius of the planar device is 0.75 m.

IV. CONCLUSION

In conclusion, this paper presents a general and rigorous method to flatten conic reflectors in transformation optics. It ensures that the designed planar reflectors scatter or reflect any incident light or wave in the same manner as a desired conic reflector. In order to reduce the effective aperture of the reflectors, compressed as well as folded spatial mappings are further adopted. Finally, reduced profile and planar structure reflectors are achieved in the proposed method which may be useful in certain optical and EM applications, e.g., reflector antennas, etc.

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