Scaling property and multi-resonance of PCF-based long period gratings

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Abstract: In a long period grating (LPG) made on a silica-based single material photonic crystal fibre (PCF), the effect of material dispersion on the resonance wavelength of the LPG is negligible. The resonance wavelength, the period and length of the LPG, and the diameter and pitch of the air-hole lattice of the PCF are found to obey a scaling law that is derived from the scaling property of the Maxwell's Equations. Simulations show that the resonance wavelength has a non-monotonic dependence on the grating period and, for a particular grating period, there could exist multiple resonance wavelengths and hence multiple transmission dips due to phase matching between the fundamental core mode and a cladding mode simultaneously at multiple wavelengths.

OCIS codes: (060.2270) Fiber characterization; (350.2770) Gratings; (999.999) Photonic crystal fiber

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1. Introduction

Photonic Crystal Fibers (PCFs) have attracted significant attentions recently because of their unusual properties [1, 2]. PCFs are usually single material fibers that incorporated numerous air holes in the cladding that run along the length of the fiber. The shape, size and distribution of holes can be controlled or designed, which allows for PCFs to have unusual properties that cannot be achieved with conventional fibers such as controllable dispersion and mode-field confinement, very broad single-mode range and novel polarization characteristics.

Long period gratings (LPGs) are periodic refractive index/geometry perturbation along the length of an optical fiber. The period of the perturbation matches the beat length between a core mode and a cladding mode and hence facilitate coupling between the two modes. Previous research on LPGs mainly focused on conventional Ge-doped single mode optical fibers [3]. Recently considerable research has been carried out on LPGs written on PCFs [4-10]. LPGs were produced in pure silica PCFs by glass structure change [5], by physical deformation of the air holes [6], and by flexural acoustic waves [7]. LPGs in PCFs were investigated experimentally with their guiding properties reported in a number of literatures [5-11]. It has been found that the resonance wavelength of a PCF-based LPG is blue-shifted when the grating period is increased [5-9], which is contrary to that of a conventional LPG.

In this paper, we report the results of our recent investigation on LPGs based on index-guiding PCFs with a triangular lattice of air-holes in the cladding. The effect of material dispersion on the resonance wavelength is discussed in section 2, followed by a discussion on the scaling property of the LPGs on PCFs in section 3. The non-monotonic relationship between the grating period and the resonance wavelength of the LPGs, which is believed to be reported for the first time, is presented in section 4 and a brief summary of the given in section 5.

2. The effect of material dispersion

It is well known that mode coupling occurs when the period of the LPG (Λ_g) is matched to the beat length (L_B) between the core mode and a cladding mode, i.e., [3]

$$\Lambda_g = L_B = \lambda/(n_{co} - n_{cl}), \tag{1}$$

where n_{co} and n_{cl} are respectively the effective refractive indices of the fundamental core mode and an individual cladding mode of the fiber.

In general, L_B depends on material dispersion because of the wavelength-dependent refractive index of the material. For a silica-based fiber, the chromatic dispersion D, which is defined as D=dτ/dλ=d(dβ/dω)/dλ, may be decomposed into the sum of the material dispersion D_m and the waveguide dispersion D_w, i.e., $D=D_m+D_w$. Similar to this decomposition, the group delay τ, the propagation constant β, and hence the effective refractive index n_{eff} of each of the modes, may be virtually split into a material component and a waveguide component, i.e.,

$$\tau=\tau_m+\tau_w,\ \beta=\beta_m+\beta_w,\ n_{eff}=n_{effm}+n_{effw}, \tag{2}$$

where n_{effm} is the characteristic of the fiber material, and is determined by the well-known Sellmeier's equation and independent of the particular mode [12]. n_{effw} can be regarded as a virtual waveguide index. By using Eq.(2), Eq. (1) may be rewritten as

$$\Lambda_g=L_B=\lambda/(n_{co,w} - n_{cl,w}). \tag{3}$$

Equation (3) means that the material dispersion has no effect on the beat length and hence the resonance wavelength of an LPG made on a silica-based PCF. Extensive calculations using Eq.(1) and Eq.(3) show that the results with and without considering the material dispersion only differ by less than one percent. This finding is useful in that it significantly reduces the numerical computations while keep a reasonably good accuracy in determining the beat length between the modes and hence the resonance wavelength of the LPG.

3. Scaling property of PCF-based LPG

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3.1 Scaling property of the resonance wavelength

Consider a silica-based PCF with a cross-section as shown in Fig.1. This PCF has a triangular lattice of circular air-holes in the cladding and is characterized by two parameters: the spacing between the air-holes or pitch Λ and the relative hole-size $f=d/\Lambda$.

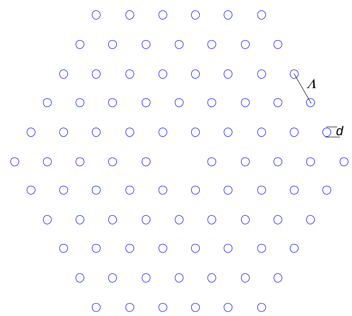

Fig.1 PCF with a triangular lattice of air-holes in a silica base. The definition of pitch Λ and hole diameter d are shown in the figure.

Under the condition that the material dispersion is neglected, the scaling properties [13] of the Maxwell's Equations says, when a PCF with pitch Λ is scaled by a factor M while keeping the relative diameter f unchanged, the field profile of the newer one can be obtained from the older one by the same scaling factor, i.e.

$$\mathbf{e}(M\Lambda, \mathbf{r}, f, \lambda) = \mathbf{e}(\Lambda, \mathbf{r}/M, f, \lambda/M). \tag{4}$$

The mode effective index also obeys the scaling law, which can be expressed as

$$n_{eff}(M\Lambda, f, \lambda) = n_{eff}(\Lambda, f, \lambda/M). \tag{5}$$

Substituting Eq. (5) into Eq.(3), we obtain

$$\Lambda_g(M\Lambda, f, \lambda) = \frac{\lambda}{n_{co}(M\Lambda, f, \lambda) - n_{cl}(M\Lambda, f, \lambda)} = M\Lambda_g(\Lambda, f, \lambda/M),$$

which can be rewritten as

$$\frac{\Lambda_g(M\Lambda, f, \lambda)}{M\Lambda} = \frac{\Lambda_g(\Lambda, f, \lambda/M)}{\Lambda}. \tag{6}$$

Eq.(6) shows that the resonance wavelength λ will be scaled by the same factor M as the grating period Λ_g and the hole-pitch Λ.

3.2 Scaling property of the coupling strength

At the resonance wavelength, the strength of the mode coupling from the core mode to the cladding mode, i.e., the depth of the transmission dip, oscillates as $\cos(K_{co,cl}L)$, where L is the length of the LPG, $K_{co,cl}$ is the coupling coefficient between the core mode and the cladding mode and can be evaluated from [3]

$$K_{co,cl} = \frac{2\pi n_1 \delta n_{\mathrm{mod}}}{\lambda\sqrt{n_{co}n_{cl}}} \frac{\iint_{grating-area} dxdy\, \mathbf{e}_{co,t}(x,y)\cdot \mathbf{e}_{cl,t}(x,y)}{\sqrt{\iint_{\infty} dxdy \left|\mathbf{e}_{co,t}(x,y)\right|^2} \cdot \sqrt{\iint_{\infty} dxdy \left|\mathbf{e}_{cl,t}(x,y)\right|^2}}, \tag{7}$$

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where δn_{mod} is the index modulation in the fiber grating area, n_1 is the core index, $\mathbf{e}_{co,t}$ and $\mathbf{e}_{cl,t}$ are the transverse electric field of the core mode and the cladding mode, respectively.

By using Eqs.(4), (5) and (7), it can be easily shown that

$$K_{co,cl}(M\Lambda, f, \lambda) = K_{co,cl}(\Lambda, f, \lambda/M)/M\,. \tag{8}$$

The confinement loss (L_{conf}) of the cladding mode will affect the LPG spectrum. Fortunately, it can be deduced that $L_{conf}(M\Lambda, f, \lambda) = L_{conf}(\Lambda, f, \lambda/M)$, which is also called the scaling law. The scaling properties, as expressed in Eqs. (6) and (8), are useful for the design of PCF-based LPGs by scaling the structural parameters of the PCFs and LPGs. It says, if an LPG with period Λ_g and length L inscribed in a PCF with structural parameters Λ and f has a transmission dip at wavelength λ, then an LPG with period $M\Lambda_g$ and length ML inscribed in a PCF with structural parameters $M\Lambda$ and f will have a transmission dip at wavelength $M\lambda$.

4. Relationship between beat length and wavelength

It has been shown that the resonance wavelength of the PCF-based LPG decreases monotonically with an increase in the beat length or the grating period [5-9]. This is contrary to that of LPGs based on conventional single mode fibers and is believed to be originated from the highly dispersive characteristics of the cladding mode due to the existence of the air-holes [9]. In this section, the relationship between the beat length and the wavelength is investigated further and compared with that of the LPGs on conventional step index fibers.

4.1 LPGs on conventional optical fibers

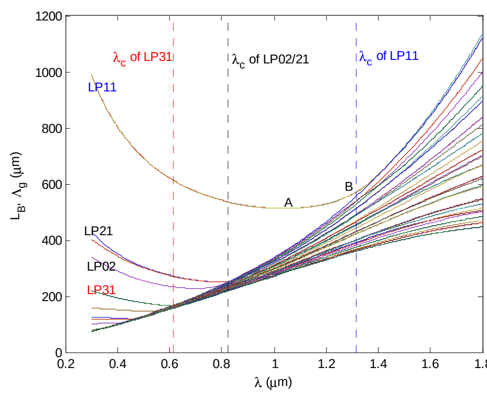

Fig. 2 Wavelength dependent beat lengths between the fundamental LP_{01} mode and a few HOMs of a conventional silica-based step index fiber with core radius of 4.5μm and relative index difference of 0.3%. λ_c of LP_{11}, LP_{21}, LP_{02} and LP_{31} are plotted as vertical lines.

The beat length between the fundamental LP_{01} mode and higher-order modes (HOMs) of a conventional optical fiber is shown in Fig. 2. The cutoff wavelengths (λ_c) of the first few HOMs are also plotted as three vertical dotted lines. It is clear that the beat length does not always increase with λ over the whole wavelength range. However, the beat length between the fundamental LP_{01} mode and a particular HOM does increase with wavelength when operating above the cut-off wavelength λ_c of the HOM where it is not bounded to the core region and becomes a cladding mode. When operating below the cut-off wavelength, the HOM is a bounded high order core mode and the beat length between the LP_{01} and the HOM is actually decreasing with λ at wavelength well-below cut-off and become increasing with λ at wavelength close to cut-off, as indicated in the region from A to B for the LP_{11} mode. As most research on LPGs uses single mode fibers with operating wavelength above the cut-off wavelength of the LP_{11} mode, the beat length and hence the grating period has a monotonic relation with the resonance wavelength. If the operating wavelength range were broad enough to cover the regions both below and above the cut-off wavelength of a HOM, the beat length

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between the fundamental mode and the HOM would have a non-monotonic dependence on the wavelength. This means that, for a particular grating period Λ_g, there could be more than one resonance wavelengths that couples power between the fundamental mode and the HOM. The LPG, which is inscribed in few-mode fiber, is particularly useful as a broadband mode-converter that can route light into other modes, as would be desirable for band selection filters, switches, variable attenuators [14]. However, the coupling at shorter wavelength is not reflected in the LPG transmission dips because the HOM is still bounded and not contributing to the transmission loss.

4.2 LPGs on PCFs

Figure 3 shows the normalized beat length (L_B/Λ) between the fundamental LP_{01} mode and the LP_{02} cladding mode of the PCF shown in Fig.1 as a function of the normalized wavelength (λ/Λ), for various relative hole size of f from 0.2 to 0.6. The results were obtained using a super-cell method [15]. The results obtained from a commercial FEM software are also shown as '+' or 'o' which agree well with the results from the super-cell method.

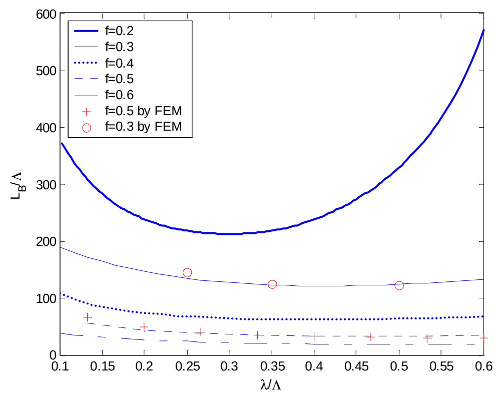


Fig.3 Normalized beat length between LP_{01} and LP_{02} modes as a function of the normalized wavelength for the PCF shown in Fig.1. 'o' and '+' are obtained by FEM method and others are obtained by the super cell method.

It is clear that, for small value of the normalized wavelength λ/Λ, the normalized beat length decreases with an increase in the normalized wavelength, which agrees with the results reported [5-9]. However, for larger values of λ/Λ, the beat length actually increase with an increase of the wavelength. This non-monotonic dependence of the beat length on wavelength is believed to be reported for the first time and shows that, for the same grating period Λ_g, there could be multiple wavelengths that stratifying the resonance condition and hence multiple dips in the transmission spectrum of the LPG. This phenomenon is more easily observed for smaller values of f.

Figure 4 shows field intensity profile of LP_{01} and LP_{02} modes for a PCF with Λ=3μm and f=0.2 at wavelength of 1550nm. As is well known that PCF with f less than 0.406 supports 'endlessly-single mode' [16], the LP_{02} mode shown in Fig.4 is definitely a cladding mode. This means that the beat length between the fundamental core mode and the cladding mode in the PCF has non-monotonic dependence on wavelength, and, for a particular grating period Λ_g, there would be multiple resonance wavelength and hence multiple dips in the transmission spectrum of the PCF-based LPG.

For the particular PCF with Λ=3μm and f=0.2, the beat length L_B as function of wavelength is re-plotted in Fig. 5(a). The transmission spectra of three PCF-based LPGs with grating period of 800μm, 700μm and 650μm were calculated by using the conventional mode coupling theory [3] and shown in Fig.5 (b). Double dips appear in the spectrum for each of the grating periods with the locations of dips approximately the same as predicted in Fig.5 (a).

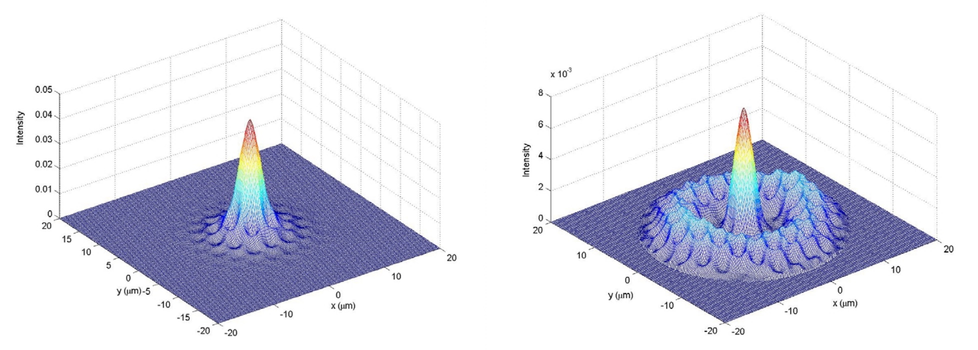
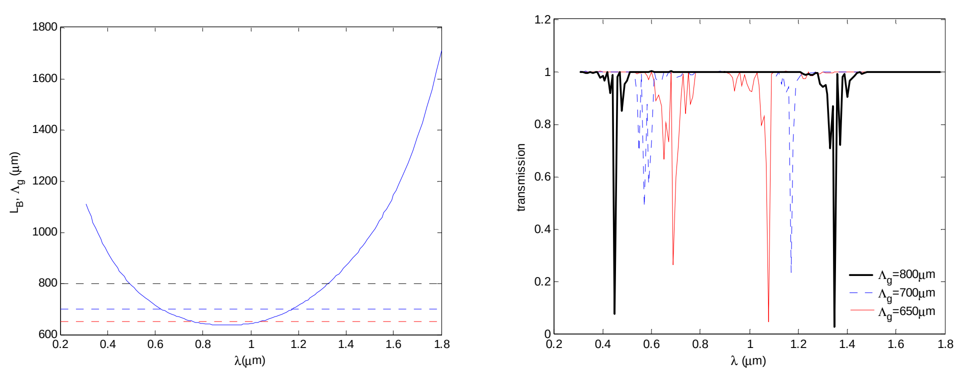


Fig. 4 The electric field intensity profile of the core mode LP_{01} (left panel) and the cladding mode LP_{02} (right panel) at wavelength 1.55μm. The PCF parameters are: Λ=3μm and f=0.2.



(a) (b)

Fig. 5 (a) Beat length between modes LP_{01} and LP_{02} as a function of wavelength for a PCF with Λ=3μm and f=0.2. (b) Transmission spectra of three LPGs inscribed in the PCF with Λ=3μm and f=0.2. There are two resonance transmission dips for each of the LPGs.

5. Conclusion

In conclusion, the scaling property of LPG inscribed in silica based PCF is obtained from the scaling properties of the Maxwell's Equations. The effect of the material dispersion on the resonance wavelength of the LPGs was found negligible. According the scaling property, if a LPG with period Λ_g and length L inscribed in a PCF with structural parameters Λ and f has transmission dip at wavelength λ, then a LPG with period $M\Lambda_g$ and length ML inscribed in a PCF with structure parameters $M\Lambda$ and f will have a transmission dip at wavelength $M\lambda$.

The beat length of the fundamental core mode and the higher order cladding mode is found to have a non-monotonic relation with wavelength. This means that there can be multiple resonance wavelengths for the same LPG inscribed in a PCF. This will result multiple dips in the transmission spectrum of a PCF-based LPG. This property is different from the LPG inscribed in a conventional single mode fiber and could have potential for new device applications. Further experimental investigations should be conducted to further understand the unusual transmission properties of the PCF-based LPGs and to develop novel fiber devices.

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