

**Title:** An Adaptive Squared-Distance-Weighted Interpolation for Volume Reconstruction in 3D Freehand Ultrasound

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**Abstract:** Volume reconstruction is a key procedure in 3D ultrasound imaging. An algorithm named as squared-distance-weighted (SDW) interpolation has been earlier proposed to reduce the blurring effect in the 3D ultrasonic images caused by the conventional distance weighted (DW) interpolation. However, the SDW parameter  $\alpha$ , which controls the weight distribution, is a constant assigned by operators so that the interpolation effect is invariant for both sharp edges and speckle noises. In this paper, we introduced a new adaptive algorithm based on SDW interpolation for volume reconstruction of 3D freehand ultrasound. In the algorithm, the local statistics of pixels surrounding each voxel grid were used to adaptively adjust the parameter  $\alpha$  in SDW. The voxel grids with a higher ratio of local variance and mean in their neighbourhoods would have a smaller  $\alpha$  to make the image details sharper, while the voxel grids locating in regions with a lower ratio of local variance and mean would have a larger  $\alpha$  to smooth image content in homogeneous regions, where speckle noise is usually observed and damages the image quality. By comparing the simulation results using the SDW and new adaptive algorithm, it was demonstrated that this new algorithm worked well in both edge preservation and speckle reduction.

**Keywords:** Adaptive SDW, volume reconstruction, 3D ultrasound.

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## 1 **1. Introduction**

2 In the past decades, 3D ultrasound (US) imaging has been recognized as a  
3 promising tool for a wide range of clinical applications. In comparison with conventional  
4 2D US, 3D US offers a whole volume data of tissues and organs for visualization and  
5 analysis. The 3D representation of anatomy greatly facilitates the diagnosis by offering a  
6 variety of useful display methods and more accurate quantitative measurements in  
7 comparison with 2D US. In recent years, many types of systems capable of obtaining 3D  
8 US imaging have been developed [1, 2, 3]. One of often-used types is freehand imaging  
9 with spatial tracking devices. During the scanning, the US probe is moved by the hand in  
10 an arbitrary manner. The irregularly located B-scans are collected and then used to  
11 reconstruct 3D data set. The positional information recorded from the tracking device for  
12 each B-scan is used to register the image plane to the regularly arranged voxel grids.  
13 Because of its advantages of being inexpensive and flexible in scanning, tracked freehand  
14 technique holds great promise in many research and clinical areas [4].

15 One of the key procedures in the freehand 3D US is volume reconstruction. As  
16 the collected B-scan images are arbitrarily located in space, there must be solutions for  
17 assigning pixel intensities from the original B-scans to the voxel grids. The algorithms  
18 used for volume reconstruction are important because the diagnostic information should  
19 be well preserved and the introduction of noises and artefacts should be avoided as much  
20 as possible. Many previous researchers [1, 5-8] have made contributions to this problem.  
21 According to a survey of the well-known reconstruction methods [6], voxel nearest-  
22 neighbour (VNN) interpolation, pixel nearest-neighbour (PNN) interpolation, and  
23 distance weighted (DW) interpolation are three main categories of reconstruction

1 technique in freehand 3D US systems. Particularly, DW interpolation performs well in  
 2 speckle and shadowing reduction in many practices. It computes each voxel value by  
 3 assigning the weighted average of a set of pixels falling into a predefined 3-D region  
 4 centred about each voxel. The pixel intensities in the region are weighed by the inverse  
 5 distances between the pixels and the voxel centre [5, 6]. However, averaging operation  
 6 can greatly blur image details, especially the boundaries of small tissues.

7 To reduce the blurring in conventional DW interpolation, Huang et al. [1]  
 8 proposed an improved method named as squared distance weighted (SDW) interpolation,  
 9 which used the square of inverse distance as the weight for each pixel. Fig. 1 depicts a 2D  
 10 representation of voxel calculation using SDW. The corresponding formula was  
 11 expressed as:

$$12 \quad I(\vec{V}_C) = \frac{\sum_{k=0}^n W_k I(\vec{V}_P^k)}{\sum_{k=0}^n W_k}, \quad W_k = \frac{1}{(d_k + \alpha)^2}, \quad (1)$$

13 where  $I(\vec{V}_C)$  is the intensity of the voxel at the volume coordinate  $\vec{V}_C$ ,  $n$  the number of  
 14 pixels falling within the predefined spherical region centred about voxel  $\vec{V}_C$ ,  $I(\vec{V}_P^k)$  the  
 15 intensity of the pixel at the  $k$ th image coordinate  $\vec{V}_P^k$ ,  $W_k$  the weight for the  $k$ th pixel,  
 16  $d_k$  the distance from the  $k$ th pixel ( $\vec{V}_P^k$ ) to the centre of the voxel ( $\vec{V}_C$ ), and  $\alpha$  a positive  
 17 constant for adjusting the effect of the interpolation. As SDW offers a non-linear  
 18 assignment for the weights, the reconstructed voxel array is less blurred in comparison  
 19 with conventional DW method.

20 With respect to Eq. (1), different values of  $\alpha$  result in different reconstruction  
 21 results. We tested the reconstruction results on an US resolution phantom (Model 44,

1 CIRS Inc, USA) using different values of  $\alpha$ . As illustrated in Fig. 2, SDW preserved more  
2 texture patterns in US images with a smaller  $\alpha$  ( $\alpha=0.1$ ), while smoothed the image content  
3 much more with a larger  $\alpha$  ( $\alpha = 10.0$ ). It is apparent that the larger  $\alpha$  is good for speckle  
4 suppression and the smaller  $\alpha$  works well for the preservation of details. However, there  
5 is a trade-off between noise reduction and detail preservation in the SDW method.

6 To provide a 3D US image with relatively good edge preservation as well as  
7 speckle reduction, we proposed an adaptive strategy to adjust  $\alpha$  in SDW in this paper.  
8 This new method made use of the local statistics of pixels in the spherical region to  
9 adaptively control the value of  $\alpha$ . The methods were described in the following section.  
10 The simulation results were presented in section 3 to evaluate the usefulness of the  
11 proposed algorithm. Discussions and conclusions were finally given in section 4.

12

## 13 **2. Methods**

### 14 *2.1 Adaptive algorithm for SDW*

15 There are three objectives, including speckle reduction, tissue boundary  
16 enhancement, and shadowing reduction, in the volume reconstruction of 3D US [9]. As  
17 illustrated in Fig. 2, the smaller  $\alpha$  in SDW could lead to well preserved edges, but many  
18 speckle noises were retained. Meanwhile, the larger  $\alpha$  blurred the edges and details  
19 though the speckle noises were much reduced. Therefore, we aimed to design an  
20 improved reconstruction method that was capable of both preserving tissue edges and  
21 reducing speckles in this study.

22 Loupas et al. [10] proposed the following signal-dependent noise model for  
23 speckle specifications in US images:

1 
$$y = s + n\sqrt{s}, \quad (2)$$

2 where,  $y$ ,  $s$ , and  $n$  represent the observed US signal, noise-free signal, and noise,  
 3 respectively. In a homogeneous region, it can be assumed that  $s$  is a constant. Therefore,  
 4 the variance ( $\sigma^2$ ) of the region has the following linear relationship to that of noise ( $\sigma_n^2$ ):

5 
$$\sigma^2 = s\sigma_n^2. \quad (3)$$

6 Assume arithmetic mean  $\mu$  of the region is the expectation of  $s$ , Eq. (3) can be rewritten  
 7 as  $\sigma^2 = \mu\sigma_n^2$ . If there are only speckle noises in a homogeneous region, Eq. (3) implies the  
 8 variance is proportional to the mean.

9 According to this model, it is obvious that a region containing tissue edges has  
 10 relatively larger ratio of  $\sigma^2/\mu$ . In order to preserve the edges, the parameter  $\alpha$  in SDW  
 11 should be as small as possible. Otherwise, the parameter  $\alpha$  should be as large as possible  
 12 if the spherical region is homogeneous. Thus, we designed an adaptive method to control  
 13 the value of  $\alpha$  in Eq. (1). This new adaptive squared distance weighted (ASDW)  
 14 interpolation was described as:

15 
$$I(\vec{V}_C) = \frac{\sum_{k=0}^n W_k I(\vec{V}_P^k)}{\sum_{k=0}^n W_k}, \quad W_k = \frac{1}{\left(d_k + ae^{-b\sigma^2/\mu}\right)^2}, \quad (4)$$

16 where,  $\sigma^2$  and  $\mu$  are the variance and mean of the pixels in a spherical region centred  
 17 about each voxel,  $a$  and  $b$  are two positive parameters empirically determined by the  
 18 operator.

19

20 *2.2 Simulation and comparison methods*

1 An amputee subject's forearm was scanned using a freehand 3D US system [1].  
2 The voxel array was defined as a 3D data set with a dimension of  $150 \times 151 \times 115$  voxels  
3 and a resolution of  $0.22 \times 0.22 \times 0.22$  mm<sup>3</sup>. The SDW with  $\alpha=0.1$  and 2.0, respectively, and  
4 the proposed ASDW with  $a=1000.0$  and  $b=2.0$  were employed for volume reconstruction,  
5 respectively. The spherical region for SDW and ASDW was defined as an ellipsoid with  
6 a radius of 0.54 mm. After the volume data was ready, three slices at the same location of  
7 the volumes were qualitatively compared. The intensities along column 124 in the three  
8 slices were presented to evaluate the performance of ASDW in boundary enhancement  
9 and speckle reduction. In addition, the averaged local contrast measure [11], where the  
10 measure for each voxel was carried out in a  $3 \times 3 \times 3$  neighbourhood, was applied to  
11 quantitatively assess the total contrast in the reconstructed 3D images using SDW and  
12 ASDW, respectively.

13

### 14 **3. Results**

15 Fig. 3 shows the three reconstructed slices of the amputee subject. It can be  
16 observed that SDW with  $\alpha=0.1$  generated the most texture patterns (Fig. 3(a)) in  
17 comparison with the other two methods. However, the speckles were also retained. The  
18 slice reconstructed using SDW with  $\alpha=2.0$  looks the most smoothed (Fig. 3(b)). Though  
19 the speckles were effectively suppressed, the tissues boundaries were also blurred. In  
20 comparison, the slice reconstructed using ASDW well presented both enhanced tissue  
21 boundaries and suppressed speckles.

22 Fig. 4 illustrates quantitative comparisons of the voxel intensities along column  
23 124 on the three slices. In Fig. 4(a), reconstructions were performed using ASDW as well

1 as SDW with  $\alpha=0.1$  to preserve the sharp edges. For regions without sharp edges as  
2 indicated by the circle, ASDW performed better in speckle suppression. In Fig. 4(b),  
3 ASDW worked as well as SDW with  $\alpha=2.0$  in speckle reduction, as indicated by the  
4 circle. Moreover, ASDW was more capable of enhancing boundaries. It was  
5 demonstrated that ASDW was good at both edge preservation and speckle suppression.

6 For results of the averaged local contrast measure, averaged local contrast of the  
7 volume reconstructed using SDW with  $\alpha=0.1$  was 0.1947, that using SDW with  $\alpha=2.0$   
8 0.1734, and that using ASDW 0.1899. This implied that ASDW produced more  
9 homogeneous regions than SDW with  $\alpha=0.1$ , and preserved more significant edges than  
10 SDW with  $\alpha=2.0$ . It was also demonstrated that ASDW offered a good trade-off between  
11 edge preservation and speckle suppression.

12

#### 13 **4. Discussion and conclusion**

14 In this paper, we introduced an adaptive SDW interpolation method for the  
15 volume reconstruction in freehand 3D US. The proposed method could adaptively adjust  
16 the parameter  $\alpha$  in SDW, which controls the weight distribution, according to the ratio of  
17 local variance and mean in the spherical neighbourhood of each voxel. For the voxels  
18 with higher contrast in their neighbourhoods, the parameter  $\alpha$  was assigned smaller  
19 values to enhance the edges. For those voxels with lower contrast in their neighbourhoods,  
20 the parameter  $\alpha$  was assigned relatively larger values to reduce speckles in homogeneous  
21 regions. An exponential function was used to automatically determine the parameter  $\alpha$   
22 according to the statistics of a voxel's neighbourhood. The simulation results successfully  
23 demonstrated the performance of the new method. In comparison with SDW method, this

1 new adaptive technique could offer a good trade-off between the edge preservation and  
2 speckle suppression. This feature is important, especially for segmentation of tissues in  
3 3D US data, as the enhanced edges and suppressed speckles and other artefacts can lead  
4 to an accurate trace of tissue boundaries, which are essential to volume estimation [4] and  
5 multi-volume combination [12].

6 In ASDW method, there were two positive parameters ( $a$  and  $b$ ) that needed to be  
7 set by the operator. According to Eq. (4), the parameter  $a$  determines the interpolation  
8 effect on a completely homogeneous region. If a voxel is located in such a region, the  
9 ratio of local variance and mean ( $\sigma^2/\mu$ ) in its neighbourhood is 0, and ASDW becomes a  
10 SDW with  $\alpha=a$ . The parameter  $b$  determines the effect of  $\sigma^2/\mu$ . A larger parameter  $b$   
11 results in a faster change of the exponential component ( $ae^{-b\sigma^2/\mu}$ ) in ASDW. In this study,  
12 the two parameters were empirically determined. Nevertheless, the properties of the two  
13 parameters are important for operators, and they will be systemically studied in our future  
14 study.

15 In conclusion, we have introduced an adaptive interpolation method based on  
16 SDW in this paper. According to the simulation results, this new method demonstrated a  
17 good performance in both edge preservation and speckle suppression. Future study on  
18 this new method will be conducted to further improve the quality of volume  
19 reconstruction in freehand 3D US systems.

20

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1 **Figure captions**

2 Fig. 1. A 2D representation of the volume reconstruction using SDW interpolation. For  
3 each voxel in the volume, a sphere region centred about it is defined. The squared  
4 inverse distance of each pixel falling into the region to the centre of the voxel is  
5 used to determine the weighted contribution of the pixel to the voxel.

6 Fig. 2. Two slices of 3D reconstruction using SDW with different values of  $\alpha$ : (a)  $\alpha=0.1$ ,  
7 and (b)  $\alpha=10.0$ .

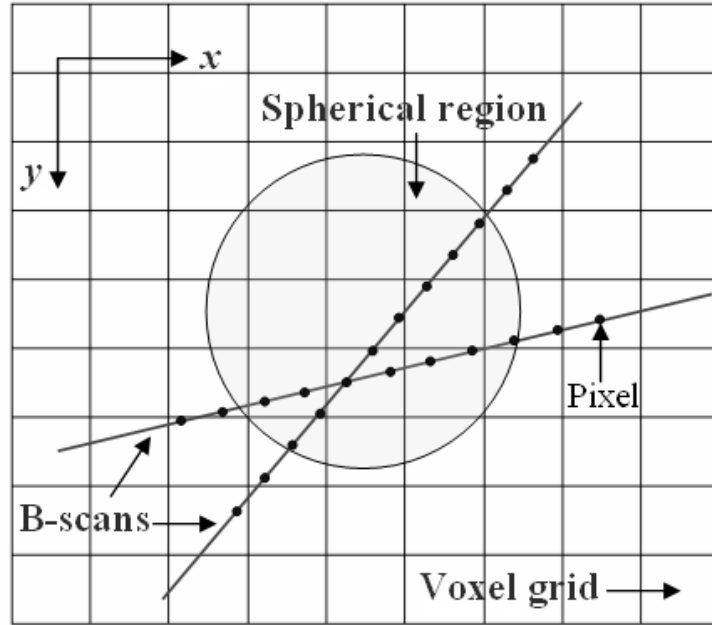
8 Fig. 3. The reconstructed slices using SDW and ASDW. (a) SDW ( $\alpha=0.1$ ), (b) SDW  
9 ( $\alpha=2.0$ ), and (c) ASDW ( $a = 1000.0, b=2.0$ ).

10 Fig. 4. The comparison of intensities along column 124 as indicated by the lines in Fig. 3.  
11 (a) The comparison between SDW ( $\alpha=0.1$ ) and ASDW ( $a = 1000.0, b=2.0$ ), and  
12 (b) the comparison between SDW ( $\alpha=2.0$ ) and ASDW ( $a = 1000.0, b=2.0$ ).

13

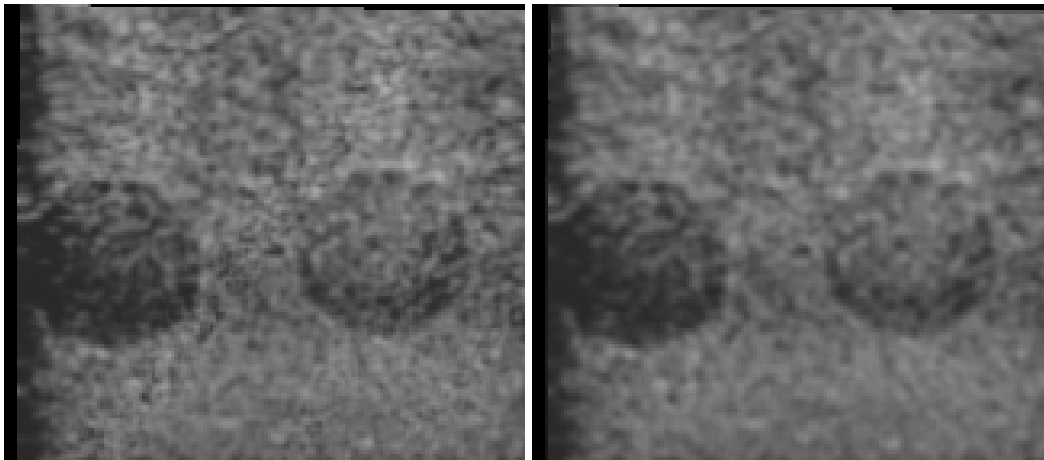
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Fig. 1

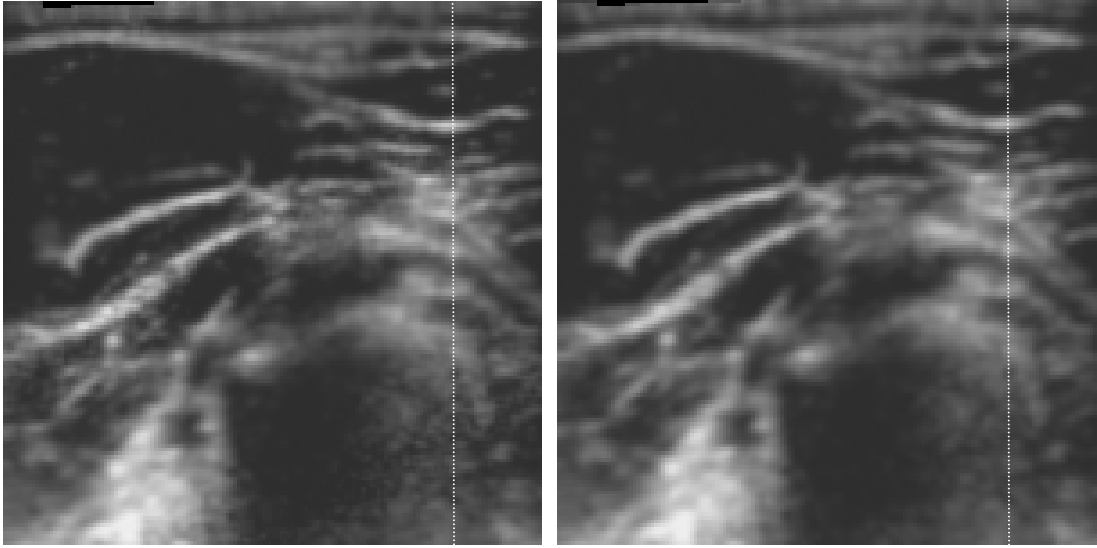


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(a)

(b)

Fig. 2

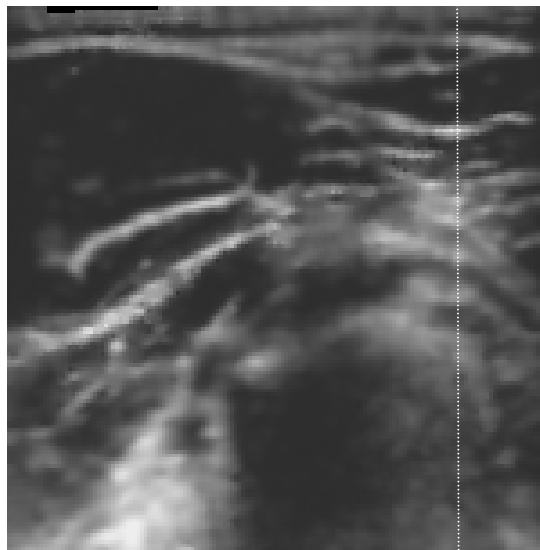


1

2

(a)

(b)



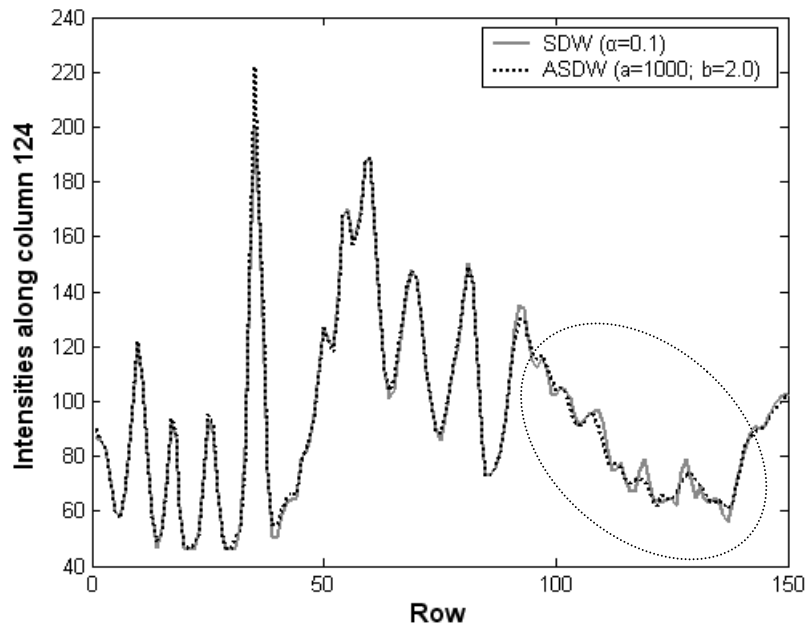
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(c)

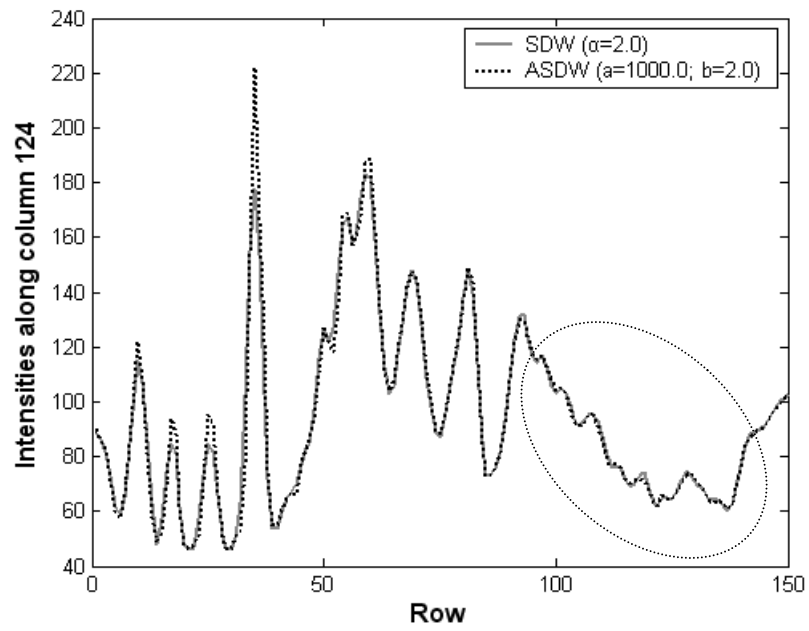
Fig. 3



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(a)



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(b)

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Fig. 4