Texture Discrimination by Local Morphological Multifractal Signatures

Yong Xia, Rongchun Zhao, Yanning Zhang
School of Computer, Northwestern Polytechnical University
yxia@ieee.org

(David) Dagan Feng
Dept. of Electronic and Information Engineering, Hong Kong Polytechnic University; School of Information Technologies, University of Sydney
feng@it.usyd.edu.au

Jian Sun
School of Automatic Control, Northwestern Polytechnical University
kevnsun@hotmail.com

Abstract

Both the fractal dimension (FD) and the multifractal dimensions (MFD) have been widely used to describe natural textures in image processing community. However, due to the essential difference between the fractal reality of digital images and the mathematical fractal model, most FD/MFD estimation algorithms intrinsically produce less accurate results. In this paper, the idea of fractal signature is adopted and extended to the morphological multifractal estimation. As a result, a novel texture descriptor, namely the local morphological multifractal signatures (LMMS), is proposed to characterize the local scaling property of textured images. The LMMS depict the behavior of the morphological MFD over a wide range of spatial scales. The proposed LMMS feature, together with the fractal signature and the morphological MFD, has been applied to the discrimination of Brodatz textures. The comparison results demonstrate that our LMMS feature can differentiate natural textures more effectively.

1. Introduction

Natural textures can be discriminated by a great variety of texture descriptors, which may be categorized, roughly, as statistical, frequency domain based, model-based, and structural [1]. Among them, the descriptors based on the fractal model have drawn much attention from researchers in recent years.

The study of fractal geometry can be traced back to the provocative work of Mandelbrot [2], [3], where the description of complex and erratic shapes was introduced in terms of self-similarity. Since Pentland [4] presented evidence that most natural surfaces are spatially isotropic fractals and their intensity images are also fractals, fractal analysis has been successfully applied to many fields of digital image processing.

For texture discrimination, the fractal dimension has been widely used [5]-[8]. This choice is mainly motivated by the observation that the fractal dimension is relatively insensitive to scaling transform [4], and shows a strong correlation with human judgment of surface roughness. Theoretically, the fractal dimension should be a powerful tool to differentiate various textures. However, in practical cases its performance in texture discrimination is miles away from the optimum. The reason may lie in the fact that, because of the limited bit depth and spatial resolution used in digitalizing an image, most digital images are merely semi-fractals and have anisotropic and inhomogeneous scaling properties [9]. The huge gap between the fractal reality of digital images and the mathematical fractal model raises many difficulties for fractal dimension based texture discriminations. An important problem is that the power law, which should be followed by the scales and the corresponding measurements, is only approximately satisfied over a very limited range of scales [8]. Such inhomogeneous scaling property may cause an inaccurate estimation of the fractal dimension. This explains why the estimated fractal dimensions of textures with obvious visual differences may remain quite identical [10]. To improve the discrimination ability, many researches extended various fractal dimensions to the multifractal dimensions [9], [11], [12], which are known as a continuous spectrum of exponents. Nevertheless, the straightforward way to avoid inaccurate estimation is to substitute the fractal signature [13] for the fractal dimension. In Serafim's fractal signature estimation approach, the local fractal dimension at each scale is calculated by using only

1-4244-0549-1/06/$20.00 ©2006 IEEE.

three successive scales, which are obviously apt to satisfy the power law.

This work is inspired by the idea of the morphological multifractal estimation and the fractal signature. According to our previous work [12], the local morphological multifractal exponents (LMME) have a better ability to differentiate natural textures than popular box-counting based multifractal dimensions. In this paper, we generalize the LMME to a new texture descriptor, called the local morphological multifractal signatures, which is defined as the variation of the LMME over spatial scales. In our approach, we omit the step of linear fit and directly calculate the LMME of each scale by using the measurements of two successive scales. We present the results of our LMMS feature when used to discriminate various Brodatz textures [14]. We also compare them with the results obtained by using the fractal signature [13] and the LMME feature [12].

2. Local Morphological Multifractal Signatures

Fractal theory offers the potential of unifying and simplifying various two dimensional (2D) texture descriptions, as well as the possibility of interpreting them in terms of the three dimensional (3D) structure of an image. As a 3D extension of the approach proposed by Mandelbrot to calculate the length of British coastline [2], the morphological multifractal estimation algorithm measures the image surface by dilating it with a group of structure elements. Morphological dilation can overlook all irregularities of the surface, whose scale is less than that of the structure element, and thus obtain the observation of the image surface at a given scale.

An image is a 2D array of pixels defined on a $M \times N$ rectangular lattice $S = \{(i, j) : 1 \le i \le M, 1 \le j \le N\}$, which is indexed by the coordinate (i, j). An image can also be viewed as a 3D surface X and denoted by a set of triplets $\{(i, j, f(i, j))\}$, where $(i, j) \in S$ represents the 2D position and the third coordinate is the gray level of the corresponding pixel. At scale ε, the structure element Y_ε is given as a set of triplets $\{(i_{\varepsilon k}, j_{\varepsilon k}, z_{\varepsilon k}); k = 1,2,\cdots,P_\varepsilon\}$, where P_ε is the number of the elements in Y_ε. Dilating the image surface X with Y_ε, the resulted surface is denoted by $\{(i, j, f_\varepsilon(i, j))\}$, where

$$f_\varepsilon(i, j) = \max_{k=1,2,\cdots,P_\varepsilon} \{f(i + i_{\varepsilon k}, j + j_{\varepsilon k}) + z_{\varepsilon k}\}. \quad (1)$$

Within an estimation window of size $W \times W$, the q th-order local measurement of the surface patch is defined as [12]

$$I_{q,\varepsilon} = \alpha \sum_{i,j}^{W} \mu_\varepsilon(i, j)^q, \quad (2)$$

where

$$\mu_\varepsilon(i, j) = \frac{|f_\varepsilon(i, j) - f(i, j)|}{\sum_{i,j}^{W} |f_\varepsilon(i, j) - f(i, j)|} \quad (3)$$

is a natural measurement, and the coefficient

$$\alpha = \frac{\sum_{i,j}^{W} |f_\varepsilon(i, j) - f(i, j)|}{\varepsilon} \quad (4)$$

is added to ensure that the morphological fractal dimension presented by Samarabandu [6] can be acquired by setting $q = 1$.

For ideal multifractals, the measurement $I_{q,\varepsilon}$ and the scale ε must satisfy the following power law

$$I_{q,\varepsilon} \sim \varepsilon^{-D(q)}, \quad -\infty < q < \infty. \quad (5)$$

Consequently, the plot $\ln(I_{q,\varepsilon})$ versus $\ln(1/\varepsilon)$ over all scales will demonstrate a line. Calculating the gradient of the line, we will get the q th-order component of the LMME [12]. However, since a digital image is only a semi-fractal, practically most plots show approximate linearity over a very limited scale range only. Moreover, the linear scale range is decided by the essence of the texture and can hardly be predicted beforehand. When estimating the fitted line, if the scale range used surpasses the linear scale range of that texture, the fitting error will be relatively large. As a result, the estimated fractal feature may not be able to discriminate different texture patterns accurately.

To avoid such inaccurate estimation, we abandon the idea of estimating a dimension over all scales, and instead use the multifractal signature to characterize the property of a texture. In a given order q, the multifractal signature is defined as the plot connecting each two successive points of the measurements at all scales. For the sake of computation, the q th-order multifractal signature can be numerically represented by the gradients of all lines that span the corresponding plot, shown as follows

$$S_q = \{L_{q,\varepsilon} : \varepsilon = 1,2,\cdots,\infty\}, \quad (6)$$

where the gradient

$$L_{q,\varepsilon} = \frac{\ln(I(q,\varepsilon+1)) - \ln(I(q,\varepsilon))}{\ln(1/(\varepsilon+1)) - \ln(1/\varepsilon)} \quad (7)$$

can be viewed as the LMME component in the order q and at the local scale ε. The novel texture descriptor, called the local morphological multifractal signatures

(LMMS), is defined as a family of the multifractal signatures $\{S_q : -\infty < q < \infty\}$.

3. Experimental Results

In this section, the proposed texture descriptor LMMS has been applied to the discrimination of natural textures. For each pixel of the image under consideration, a window of size 11×11 is centered on it. Within the window, the LMMS feature

$$x_s = \{S_{-2}, S_{-1}, S_2\} \quad (8)$$

has estimated over the scale range of 1 to 5 by using the method described in the previous section. To demonstrate its improved performance, the LMMS feature has been compared with the fractal signature [13] and the LMME feature [12]. In our comparative experiments, the fractal signature describes the variation of the blanket-based fractal dimension over the scale range 1 to 5 and the LMME feature consists of three components of the LMME spectrum $\{L_{-2}, L_{-1}, L_1\}$, which are estimated over the scale range 2 to 6.

In the first experiment, three fractal features are compared in terms of cluster separability. For a given pair of textures, the feature space separability is measured by the Fisher criterion [15], which is a ratio of the between-class separability and the within-class variation. Large Fisher criterion values demonstrate better separability of two clusters. To make a comprehensive comparison, 60 homogeneous textures selected from the Brodatz album [14] are used in this experiment. Table Ⅰ gives the average Fisher criterion value (AFCV) of each feature over ${}_{60}C_2 = 1770$ texture pairs. It is clear that the AFCV of the LMMS feature is much higher that those of other two features. That means the LMMS feature has the best ability to discriminate those textures.

The second experiment compares those three features by applying them to an MRF-based texture segmentation algorithm [16]. To make a quantitative comparison, all test images used in this experiment are four-class texture mosaics generated by using twelve Brodatz textures, which are listed in Table Ⅱ. Three test cases, together with their corresponding segmentations, are shown in Fig. 1. The border of each

TABLE Ⅰ
AVERAGE FISHER CRITERION VALUES OF THREE FEATURES

FEATURE	FRACTAL SIGNATURE	LMME	LMMS
AFCV	0.6833	0.6783	0.8778

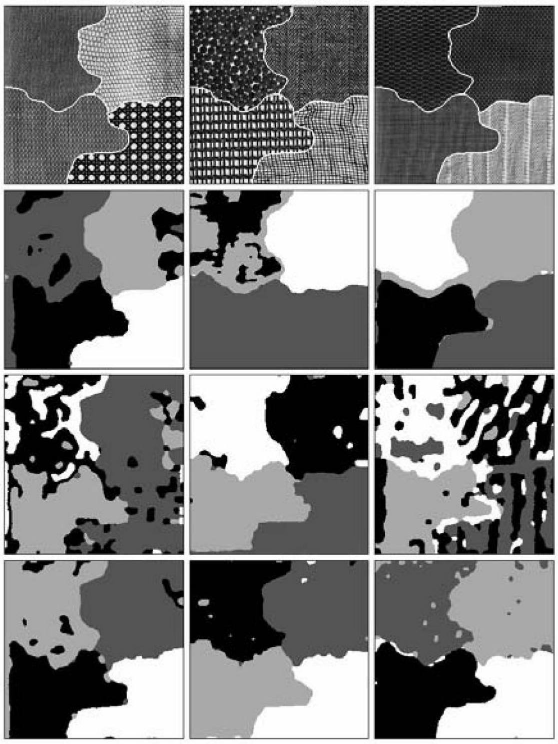

Fig. 1. Three test cases of mosaics of four textures (MⅣ1 – MⅣ3) and their segmentation by applying (the 2nd row) the fractal signature, (the 3rd row) the LMME feature, and (bottom row) the proposed LMMS feature.

TABLE Ⅱ
12 NATURAL TEXTURES FROM BRODATZ ALBUM

INDEX	DESCRIPTION	INDEX	DESCRIPTION
D3	reptile skin	D53	oriental straw cloth
D6	woven aluminum wire	D66	plastic pellets
D17	herringbone weave	D77	cotton canvas
D20	French cancas	D85	straw matting
D21	French canvas	D102	cane
D34	netting	D104	loose burlap

TABLE Ⅲ
SEGMENTATION ERRORS OF TEST CASE MⅣ1 – MⅣ3

FEATURE	FRACTAL SIGNATURE	LMME	LMMS
MⅣ1	13.49 %	41.20 %	6.71 %
MⅣ2	34.68 %	5.89 %	3.00 %
MⅣ3	8.09 %	37.51 %	5.04 %

TABLE Ⅳ
AVERAGE SEGMENTATION ERRORS OF 495 TEST CASES

FEATURE	FRACTAL SIGNATURE	LMME	LMMS
Mean Error	19.99 %	31.98 %	7.80 %

texture region is drawn with white color in the original images and all segmentation results are shown by using arbitrarily selected gray level to highlight different regions. The percentage of mis-segmented pixels is counted as the segmentation error and used to evaluate the obtained results. The segmentation errors of the cases shown in Fig 1 are given in Table III. And the average segmentation error of each feature over all ${}_{12}C_4 = 495$ cases is presented in Table IV. Apparently, the proposed LMMS feature achieves, on a whole, the most accurate segmentations. This is completely in accordance to the results reported in the previous experiment.

4. Conclusion

In this paper, we have described a texture descriptor called LMMS that can be considered as a generalization of the LMME. Three components of the LMMS have been estimated over five scales and applied to the discrimination of Brodatz texture. The performance has been compared with that of the fractal signature and the LMME feature. The experimental results demonstrate that the novel feature can provide an accurate and robust differentiation of various natural textures.

5. Acknowledgment

This research is partially supported by the NSFC under Grant No. 60141002, the Aeronautical Science Foundation of China under Grant No. 02I53073, the HK-RGC grant, and the ARC grant.

6. References

[1] T.R. Reed, and J.M. Hans du Buf, "A review of recent texture segmentation and feature extraction techniques", *CVGIP: Image Understanding*, vol. 57, no. 3, May, pp. 359-372, 1993.

[2] B.B. Mandelbrot, "How long is the coast of Britain? Statistical self-similarity and fractal dimension," *Science*, 156, pp. 636-638, 1967.

[3] B.B. Mandelbrot, The Fractal Geometry of Nature. New York: W. H. Freeman, 1983.

[4] A.P. Pentland, "Fractal Based Description of Natural Scenes," *IEEE Trans. Pattern. Anal. Machine Intell.*, vol. PAMI-6, no. 6, pp. 661-674, Jun. 1984.

[5] S. Peleg, J. Naor, R. Hartley, and D. Avnir, "Multiple resolution texture analysis and classification," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-6, no. 4, pp. 518-523, Jun. 1984.

[6] J. Samarabandu, R. Acharya, E. Hausmann, and K. Allen, "Analysis of Bone X-Rays Using Morphological fractals," *IEEE Trans. Medical Imaging*, vol. 12, no. 3, pp. 466-470, Sep. 1993.

[7] N. Sarkar and B.B. Chaudhuri, "An efficient differential box-counting approach to compute fractal dimension of image," *IEEE Trans Syst. Man Cybernet.*, vol. SMC-24, no. 1, pp. 115-120, Jan. 1994.

[8] X.C. Jin, S.H. Ong, and Jayasooriah, "A practical method for estimation fractal dimension," *Pattern Recogn. Lett.*, vol.16, no. 5, pp. 457-464, May, 1995.

[9] B.B. Chaudhuri and N. Sarkar, "Texture segmentation using fractal dimension," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-17, no. 1, pp. 72-77, Jan. 1995.

[10] R. Voss, Random fractals: characterization and measurement. In Scaling Phenomena in Disordered Systems. Plenum, New York, 1986.

[11] G. Du and T.S. Yeo, "A Novel Multifractal Estimation Method and Its Application to Remote Image Segmentation," *IEEE Trans. Geosci. Remote Sensing*, vol. 40 no. 4, pp. 980-982, Apr. 2002.

[12] Y. Xia, D. Feng, and R.C. Zhao, "Morphology-Based Multifractal Estimation for Texture Segmentation," *IEEE Trans. on Image Processing*, vol. 15, no. 3, pp. 614-623, Mar. 2006.

[13] A.F.L. Serafim, "Fractal Signatures for complex natural textures recognition," In Proc. the 24th Annual Conference of the IEEE Industrial Electronics Society, pp. 1252-1257, 1998.
[14] P. Brodatz, Texture: A Photographic Album for Artists and Designers. New York: Dover, 1966.

[15] R.O. Duda, P.E. Hart, and D.G. Stork, Pattern Classification (2nd Edit), John Wiley, New York, 2001.

[16] H. Deng, and D.A. Clausi, "Unsupervised segmentation of synthetic aperture radar sea ice imagery using a novel Markov random field model," *IEEE Trans. Geosci. Remote Sensing*, vol. GRS-43, no. 3, pp. 528-538, Mar. 2005.