# Design and Analysis of Switched-Capacitor-Based Step-Up Resonant Converters

K. K. Law, K. W. E. Cheng, and Y. P. Benny Yeung

*Abstract*—A switched-capacitor-based step-up resonant converter is proposed. The voltage conversion of the converters is in step-up mode. By adding a different number of switched-capacitor cells, different output voltage conversion ratios can be obtained. The voltage conversion ratio from 2 to any whole number can therefore be generated by these switching-capacitor techniques. A resonant tank is used to assist in zero-current switching hence the current spike, which usually exists for classical switched-capacitor can be eliminated. Both high-frequency operations and high efficiency are possible. Generalized analysis and design method of the converters are also presented. Experimental results verified the theoretical analysis.

*Index Terms*—Capacitor switching, charge transfer, resonant converter, switched capacitor.

## I. INTRODUCTION

**I** N RECENT YEARS, switched-mode power supplies (SMPS) become very popular for power conversions and conditionings. Some soft-switching resonant converters can operate in very high frequency so that the power density is high [1]. Energy storage of most SMPS circuits is based on large inductors or transformers. The weight and the size of the circuits are usually dominated by these magnetic components.

For DC–DC power conversion, a kind of switched-mode converters was proposed [2]–[4]. Voltage control investigation can be found in [5], [6]. These kinds of converters have no magnetic components. They use capacitors for storing energy so that the size of the converter is small. Also, it can be fabricated in integrated circuit chips. However, high current spikes are usually occurred in all devices in these circuits for charging or discharging the switching-capacitors. As a result, these kinds of converters are usually used on low power conditions.

In this paper, a family of novel resonant converters is presented. These converters have both the advantages of traditional SMPSs and switched-capacitor converters. The circuits consist of two switches, some diodes, and a number of switching-capacitor cells. Energy is stored by the switching-capacitors. All switching devices inside circuit are operated under zero-current switching condition by the resonance of the switching-capacitors and a very small resonant inductor. The circuit is also similar to the classical resonant circuit [7] where the resonant cur-

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 $V_{s} \xrightarrow{+} Q_{2}$ 

Fig. 1. Double-mode switched-capacitor resonant converter.



Fig. 2. Triple-mode switched-capacitor resonant converter.

rent/voltage is used as the power transfer. However, in the proposed circuit, the resonant current is used as assisting zero-current switching and is a quasiresonant manner [8]. These circuits are in step-up voltage mode. By adding numbers of proposed switching-capacitor cells, different step-up voltage conversion ratio can be obtained without adding any other active switches. Current spike problem of the conventional switched-capacitor converter is improved. Figs. 1–3 show the double, triple, and *n*-mode step-up voltage conversions ratio, respectively. After detailed perusal and circuits comparison, the general switchingcapacitor cell for step-up the voltage conversion ratio was found (Fig. 4).

# II. PRINCIPLES OF STEP-UP SWITCHED-CAPACITOR RESONANT CONVERTERS

The triple-mode switched-capacitor resonant circuit is like a two double-mode switched-capacitor resonant converters combined together. Similar to double-mode circuit, when  $Q_2$ is turned on while  $Q_1$  is turned off,  $C_{1a}$  and  $L_r$  are connected in series resonance through  $D_{1a}$ .  $C_{1a}$  is charged from source  $V_S$ . Both  $C_{2a}$  and  $C_{2b}$  are very large capacitors for keeping the voltage to be constant.  $C_{2a}$  is like another voltage source with its voltage equals to  $2V_S$ .  $C_{2a}$  discharges to  $C_{1b}$  through  $D_{1b}$  while  $C_{1b}$  and  $L_r$  are connected in series resonance. When  $Q_1$  is turned on while  $Q_2$  is turned off,  $V_S$  and  $C_{1a}$  are connected in series resonance with  $L_r$  and their polarities are in same direction and add up. Since  $C_{1a}$  has a dc component

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Fig. 3. *n*-mode of switched-capacitor resonant converter.



Fig. 4. Proposed switching-capacitor cell.

equal to  $V_S$ . They charge  $C_{2a}$  so that the voltage across  $C_{2a}$  equals to  $2V_S$ .  $C_{1b}$  and  $L_r$  are connected in series resonance as well with  $V_S$  in the same direction of polarity.  $C_{1b}$  has a dc component equal to  $2V_S$ .  $V_S$  and  $C_{1b}$  release energy to the load together.  $C_{1a}$  is not necessary equal to  $C_{1b}$ , but is usually made them equal in order to ensure zero-current switching can be controlled easily. The voltage conversion ratio of the converter is 3. Actually,  $C_{2a}$  can be considered to be another output with voltage conversion ratio equal to 2 so that this circuit can be a multiple output circuit.

## **III. MATHEMATICAL ANALYSIS**

The computer simulation waveforms of the triple-mode circuit using the parameters of  $(V_s = 40 \text{ V}, C_{1a} \text{ and } C_{1b} = 0.22 \,\mu\text{F}, C_{2a} \text{ and } C_{2b} = 0.22 \,\mu\text{F}, L_r = 1 \,\mu\text{H}$ , output power = 100 W) are shown in Fig. 5(a) and (b). For analyzing the circuit, it would be divided into four states in each switching period. Fig. 6(a)–(d) shows the equivalent circuits of the triple mode switched-capacitor resonant coverter for the four states.

## A. State I [ $t_0$ to $t_1$ ]

 $Q_2$  is turned on and  $Q_1$  is turned off in this state.  $C_{1a}$  and  $L_r$  resonate together with  $V_S$  while  $C_{1b}$  and  $L_r$  resonate together with  $C_{2a}$ . They all start resonating at  $t_0$  from the current equal to zero in sinusoidal manner. Since the current increases gradually at  $t_0, Q_2$  is turned on under zero-current switching condition. They stop resonating when the current reaches zero again at  $t_1$  by the reverse biased  $D_{1a}$  and  $D_{1b}$ . Let  $L_r = L$  and  $C_{1a} = C_{1b} = C$ . Assume that  $C_{2a}$  and  $C_{2b}$  are large enough to keep the voltage to be constant, and the circuit is lossless, the equations of this state can be derived by classical circuit equation

$$v_{C1a} = V_S - \frac{3}{4} I_o T_S \omega_0 Z_0 \cos \omega_0 (t - t_0)$$
(1)

$$i_{Lr} = \frac{3}{4} I_o T_S \omega_0 \sin \omega_0 (t - t_0)$$
 (2)



Fig. 5. Simulation waveforms of triple-mode switched-capacitor resonant converter.  $V_{\rm ds}$  and  $V_{\rm g}$  stand for the drain–source and gain voltages of the Mosfet.  $V_{\rm ct}$  is the interim voltage developed on the capacitor  $C_{2b}$ .

$$v_{C1b} = 2V_S - \frac{3I_o T_S \omega_0 Z_0}{4} \cos \omega_0 (t - t_0)$$
(3)

944

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Fig. 6. Equivalent circuits of the triple-mode switched-capacitor resonant converter. (a) State I. (b) State II. (c) State III. (d) State IV.

where

$$\omega_0 = \frac{1}{\sqrt{2LC}} \tag{4}$$

$$Z_0 = \sqrt{\frac{L}{2C}}.$$
(5)

## B. State II $[t_1 \text{ to } t_2]$

Both  $Q_2$  and  $Q_1$  are still turned off in this state. The resonance stops at  $t_1$ . The inductor current is equal to zero. The voltages of  $C_{1a}, C_{1b}$ , and  $C_{2a}$  are unchanged.  $C_{2b}$  discharges to the load. At  $t_2, Q_1$  is turned on and  $Q_2$  is turned off. Hence,  $Q_2$  is turned off under zero-current condition. The equations of this state are

$$v_{C1a} = V_S + I_o T_S \omega_0 Z_0 \tag{6}$$

$$v_{C1b} = 2V_S + \frac{I_o I_S \omega_0 Z_0}{2} \tag{7}$$

$$i_{Lr} = 0.$$

# C. State III $[t_2 \text{ to } t_3]$

 $Q_1$  is turned on and  $Q_2$  is turned off in this state.  $L_r$  and  $C_{1a}$  resonate in series together and connected with source  $V_s$  to  $C_{2a}$  while  $L_r$  resonates with  $C_{1b}$  and connected together to  $C_{2b}$  and the load. Similar to State I, the resonant currents are in sinusoidal manner. They increase from zero at  $t_2$  gradually so that zero-current turn-on of  $Q_1$  is achieved. The current reaches zero at  $t_3$ . The resonance stops by the reverse biased diodes  $D_{2b}$  and  $D_{2a}$ . The equations of this state are

$$w_{C1a} = V_S + \frac{3}{4} I_o T_S \omega_0 Z_0 \cos \omega_0 (t - t_2)$$
(9)

$$i_{Lr} = -\frac{3}{4} I_o T_S \omega_0 \sin \omega_0 (t - t_2)$$
(10)

$$v_{C1b} = 2V_S + \frac{3I_o T_S \omega_0 Z_0}{4} \cos \omega_0 (t - t_2).$$
(11)

# D. State IV $[t_3 \text{ to } t_4]$

In this state, both  $Q_1$  and  $Q_2$  are still turned off. Similar to State III, the resonance stops at  $t_3$ . The instantaneous inductor current is equal to zero. The voltage of  $C_{1a}$  and  $C_{1b}$  is unchanged.  $C_{2b}$  discharge to the load again. At  $t_4, Q_2$  is turned on and  $Q_1$  is turned off. Hence,  $Q_1$  is turned off in zero-current condition. The equations of this state are

$$v_{C1a} = V_S - I_o T_S \omega_0 Z_0 \tag{12}$$

$$v_{C1b} = 2V_S - \frac{I_o I_S \omega_0 Z_0}{2}$$
(13)

$$i_{Lr} = 0. (14)$$

The voltage conversion ratio has been derived by using balancing of the input and output energies. Also with the condition of continuity of the inductor current and capacitor voltages, the conversion ratio has been proved to be 3 and the coefficients of [(1)-(3), (6)-(7), (9)-(10), (12)-(13)] has been derived as shown above.

## IV. GENERALIZED EQUATIONS OF CONVERTERS

The voltage across  $C_{1a}, C_{1b}, C_{1c}, \ldots$  are denoted by  $V_{Ci}$ where  $i = a, b, c, \ldots$  Also define  $\varphi(i) = 1, 2, 3, 4 \ldots$  for  $i = a, b, c, d, \ldots$  respectively. All the resonant capacitor currents also pass through  $L_r$ . Hence, for an *n*-mode, there are  $n - 1C_1$  capacitors. Their general equations of an *n*-mode converter can be described as follows.

*A. State*  $I[t_0-t_1]$ 

$$v_{C_i} = \varphi(i) V_s - \frac{n\pi I_0 T_S Z_o}{2T_0} \cos \omega_0 (t - t_0)$$
(15)

$$i_L = \frac{n\pi I_0 T_S}{2T_0} \sin \omega_0 (t - t_0)$$
(16)

where

$$\omega_0 = \sqrt{\frac{1}{L_r n C_1}} = \frac{2\pi}{T_o} \tag{17}$$

$$Z_0 = \sqrt{\frac{L_r}{nC_1}}.$$
(18)

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(8)



Fig. 7. Measured waveforms of triple-mode switched-capacitor resonant converter (time base:  $5 \ \mu$ s/div). (a) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $I_L$ , 5 A/div. (b) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Middle trace:  $V_{ds}$  of  $Q_1$ , 40 V/div, Lower trace:  $Q_1$  current, 5 A/div. (c) Upper trace: Gate signal of  $Q_2$ , 20 V/div, Middle trace:  $V_{ds}$  of  $Q_2$ , 40 V/div, Lower trace:  $Q_2$  current, 5 A/div. (d) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div. (e) Upper trace: Gate signal of  $Q_2$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div. (f) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div. (f) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div. (f) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div. (f) Upper trace: Gate signal of  $Q_1$ , 20 V/div, Lower trace:  $V_{C1a}$ , 40 V/div.

B. In State II  $[t_1-t_2]$ 

$$v_{C_i} = \varphi(i)V_s + \frac{n\pi I_0 T_S Z_0}{2T_0}$$
 (19)

$$i_L = 0. (20)$$

C. In State III  $[t_2-t_3]$ 

$$v_C = \varphi(i)V_s + \frac{n\pi I_0 T_S Z_o}{2T_0} \cos\omega_o (t - t_2) \tag{21}$$

$$i_L = -\frac{n\pi I_0 T_S}{2T_0} \sin \omega_o (t - t_2).$$
 (22)

D. In State IV  $[t_3-t_4]$ 

$$v_C = \varphi(i)V_s - \frac{n\pi I_0 T_S Z_o}{2T_0} \tag{23}$$

$$i_L = 0. (24)$$

It could also be noted that the voltage rating of the transistors  $Q_1$  and  $Q_2$  are equal to  $V_s$ . The voltage ratings of all the diodes are also approximately equal to  $V_s$ .

## V. DESIGN

The design of a triple-mode converter is simple and can be shown in the following steps.

1) Define the specification

 $V_s = 40 \text{ V}, P_o = 100 \text{ W}, \text{ switching frequency} = 215 \text{ kHz}.$ 

2) The period of resonant frequency is chosen to be less than the period of switching frequency so that a zero-current switching can be naturally achieved. Usually

$$T_o = 0.9T_s.$$

This gives the switching frequency  $f_o = 240$  kHz.

3) The ripple voltage on the resonant capacitor  $C_{1a}$  and  $C_{1b}$  must be small compared to their dc level. The peak-topeak ripple voltage is derived from (1), (3), (9), or (11))

$$\Delta v_{C1} = \frac{3}{2} I_o T_S \omega_0 Z_0.$$

A maximum voltage ripple of one-third of the dc component is allowed on the resonant capacitor. Hence, it gives  $Z_o = 1.51 \ \Omega$ .

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Fig. 8. Measured efficiency and output power of the triple-mode switched-capacitor resonant converter.

4) The resonant components can therefore be calculated by angular frequency and impedance from (4)–(5)

$$C_1 = \frac{1}{2Z_o\omega_0} = 0.22 \ \mu\text{F}$$
$$L_r = \frac{Z_o}{\omega_0} = 1 \ \mu\text{H}.$$

5)  $C_2(C_{2a} \text{ and } C_{2b})$  is a larger capacitance and is to maintain constant voltage for each stage. Its value can be estimated by the basic capacitor ripple voltage calculation

$$\Delta V_{C2} \cdot \frac{I_0}{C_2} \left\{ \frac{\cos \theta}{f_s} \cdot \frac{1}{2f_0} \left( 1 \cdot 2\frac{\theta}{\pi} \right) \right\}$$

where  $\theta = \sin^{-1}(f_s)/(\pi f_o)$  and  $I_o = (P_o)/(V_o)$ .

For 0.025-V ripple voltage, this gives  $C_2 = 100 \ \mu\text{F}$ . This is to guarantee very good dc voltage on  $C_2$  for the experiment. In fact, 10  $\mu\text{F}$  is good enough for practical circuit.

6) The transistor rating voltage is at least equal to  $V_s$ . The diode rating voltage is also equal to  $V_s$  but with a tolerance of ripple voltage on  $C_1$ . Therefore, they are chosen a rating of at least 100 V.

## VI. EXPERIMENTAL RESULTS

The circuit of step-up triple-mode converter shown in Fig. 2 has been tested in laboratory. The specification and component values of the prototype is as follows:

Input voltage	40V		
Output voltage	120V		
Output power	20W - 100W		
Switching frequency	215kHz		
$Q_1$ and $Q_2$	IRF530		
$D_1$ and $D_2$	MBR10100		
C <sub>1a</sub> and C <sub>1b</sub>	0.22µF		
C <sub>2a</sub> and C <sub>2b</sub>	100µF		
L <sub>r</sub>	1µH		

TABLE I Measured Efficiency Input and Output Value of Triple-Mode Switched-Capacitor Resonant Converter

Input		Output			Efficiency	
Volt(V)	I(A)	P(W)	Volt(V)	I(A)	P(W)	%
40	0.35	14	120.9	0.1	11.6	82.9
40	0.65	26	118.4	0.2	23.9	91.9
40	0.95	38	116.5	0.3	34.7	91.3
40	1.24	49.6	115.8	0.4	46.8	94.4
40	1.55	62	113.8	0.5	56.9	91.8
40	1.84	73.6	112.6	0.6	67.1	91.2
40	2.15	86	111.8	0.7	78.5	91.3
40	2.44	97.6	110.1	0.8	87.8	90
40	2.78	111.2	109.3	0.9	98.8	88.8
40	2.88	115.2	108.5	0.95	103.3	89.7
40	3	120	107.4	1	106.3	88.6

The measured waveforms at 100 W are shown in Fig. 7. The waveforms are very clean with no serious parasitic oscillation. From both Fig. 7(a) and (b), it is seen that both  $Q_1$  and  $Q_2$  are zero-current switching during turning on and off in a sinusoidal manner. The waveforms agree with the simulation as shown in Fig. 5 and the theoretical characteristics as shown in Sections III and IV. From the graph of characteristic of efficiency shown in Fig. 8 and Table I, it can be seen that the efficiency of the converter is around 90% at 100-W power output. It should be noted that a closed-loop voltage control is not necessary to add to the proposed converters. The voltage conversion ratio is fixed according to the topologies and varied slightly with the load as shown in Table I. The concept of these topologies is based on complete resonance energy transfer among the switching capacitors and hence the voltage conversion ratio is about fixed. They are working under a different concept as compared to the classical switched-capacitor converters.

Fig. 7(d) and (e) shows the voltage of  $C_{1a}$  and  $C_{1b}$ . It can be seen that there is a dc component of  $V_s$  and  $2V_s$  on the volt-

ages of  $C_{1a}$  and  $C_{1b}$  respectively. The resonant component is small as expected. The amplitude of the resonant voltage and current have been examined and confirmed that they agree with the (1)–(11) derived, with a small deviation because of the energy loss in the circuit.

## VII. CONCLUSION

This paper has introduced a family of step-up-mode resonant switched-capacitor converter including a 100-W output power triple-mode step-up experimental results. Mathematical modeling, generalized equation, computer simulation, and experiments have been presented. There is only a very small inductor providing resonance in each circuit. No large inductor is needed for energy storage. All switching devices in these circuits are under zero-current switching condition. Both switching loss and EMI have been reduced. High efficiency can then be obtained. From the experiment, it is shown that the efficiency of the converter can be around 90%. Also, current spike problem does not exist in all these circuits.

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