Experimental Confirmation of Frequency Correlation for Bifurcation in Current-Mode Controlled Buck-Boost Converters

Ka Wai Eric Cheng, Mingjian Liu, and Yiu Lun Ho

***Abstract*—Frequency scaling in pulse-width modulated buck-boost power converters can be used to predict the first bifurcation point at various frequency operations. This letter first derives novel frequency correlation for the power converter. This is then examined by experimental results at high-frequency operation.**

***Index Terms*—Chaos bifurcation, converters, current control, DC-DC power conversion.**

I. INTRODUCTION

BASIC topologies of the pulse-width modulated dc/dc converters including buck, boost and buck-boost are commonly used for power application. They also have been observed to have a phenomenon in bifurcation and chaos [1], [2]. Their analysis is based on a time-step simulation to obtain the condition of the bifurcation. Their subharmonics during the bifurcation can also be observed by the time-domain waveforms and the i-v trajectory. Stroboscopic map is also a common method to understand the transition from Period-1 to Period-2, and so on. Experimental verifications are usually presented in kHz range with careful handling of the parasitic effect of the leakage inductance, parasitic capacitance, and conductive resistance.

As the frequency is changed, the circuit parameters for the occurrence of the bifurcation cannot be predicted easily without going through the simulation again. Is it possible to obtain a frequency correlation for the dc/dc converter? This letter presents the analysis for the buck-boost converter under the peak-current mode control.

II. DERIVATION OF FREQUENCY CORRELATION

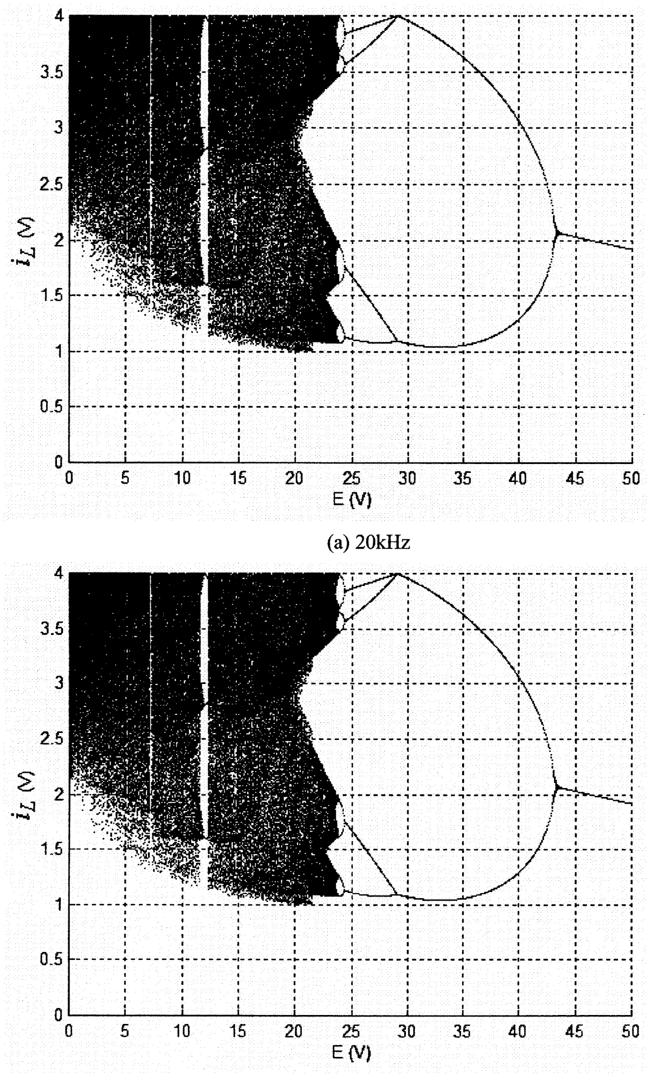
The turn-on time for the transistor t_{ONn} is defined by the time for i_L reaching the reference current $\mathrm{I_{ref}}$: At the $\mathrm{n^{th}}$ time step

$$t_{\mathrm{ON}_n} = \frac{L}{E}(I_{\mathrm{ref}} - i_{L_n}) \tag{1}$$

where L is the inductance and E is the input voltage. When the circuit enters into the first bifurcation, $t_{ON_n} > \mathrm{T}$ where T is the switching period, the time-step equations for the inductor current i_L and capacitor voltage v_C can be expressed as

$$i_{L_{n+1}} = i_{L_n} + \frac{E}{L}T \tag{2}$$

$$v_{C_{n+1}} = v_{C_n} \cdot e^{-(T/(\mathrm{RC}))} \tag{3}$$



(a) 20kHz

(b) 50kHz

Fig. 1. Bifurcation diagram of the buck-boost converter.

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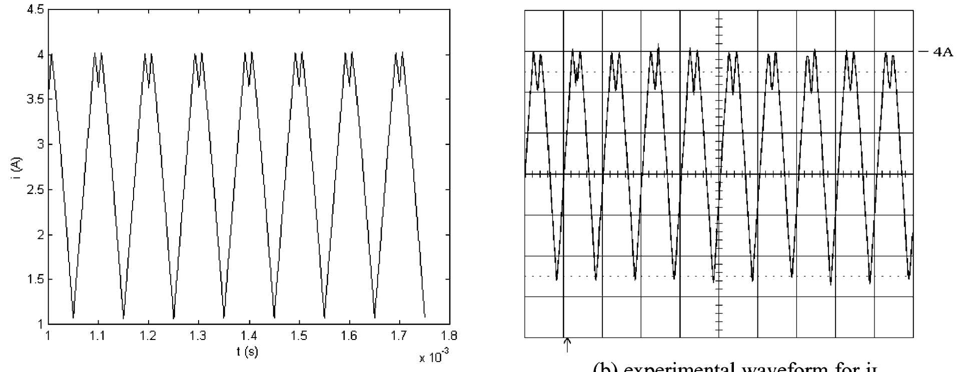
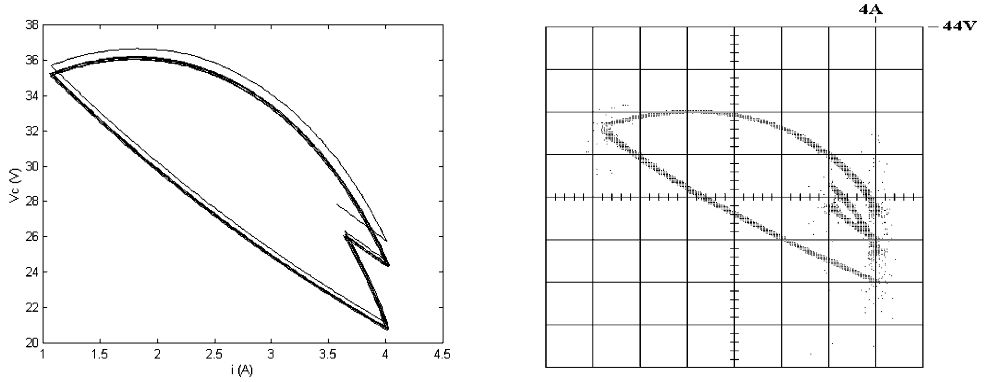


(a) simulated waveform for i_L

(b) experimental waveform for i_L (0.5A/div, 100 µs/div)



(c) simulated phase portrait

(d) experimental phase portrait (Vert: 4V/div; Hor: 0.5A/div)

Fig. 2. Period-2 waveforms and the phase trajectory of the buck-boost converter with $\mathrm{E} = 35\ \mathrm{V}$ operated at 20 kHz.

where R is the load. When Period-1 operation, $t_{\mathrm{ON}_n} < \mathrm{T}$, and the iterative equations for i_L and v_C at n^{th} time step can be expressed as [3]

$$\begin{aligned} i_{L_{n+1}} =& e^{\alpha(T-t_{\mathrm{ON}_n})}[c_1\cos\beta(T-t_{\mathrm{ON}_n}) \\ &+ c_2\sin\beta(T-t_{\mathrm{ON}_n})] \quad (4) \\ v_{C_{n+1}} =& -Le^{\alpha(T-t_{\mathrm{ON}_n})}[(c_1\alpha+c_2\beta)\cos\beta(T-t_{\mathrm{ON}_n}) \\ &+ (c_2\alpha-c_1\beta)\sin\beta(T-t_{\mathrm{ON}_n})] \quad (5) \end{aligned}$$

where

$$\alpha = -\frac{1}{2\mathrm{RC}},\ \beta = \frac{1}{2\mathrm{RC}}\sqrt{\frac{4\mathrm{R}^2\mathrm{C}}{L}-1}$$
$$c_1 = I_{ref},\ c_2 = -\frac{1}{\beta}\left(\frac{v_{C_n}\cdot e^{-(t_{\mathrm{ON}_n})/(\mathrm{RC})}}{L}+I_{ref}\alpha\right).$$

Observed from the independent variables, (4) and (5) can be rewritten as

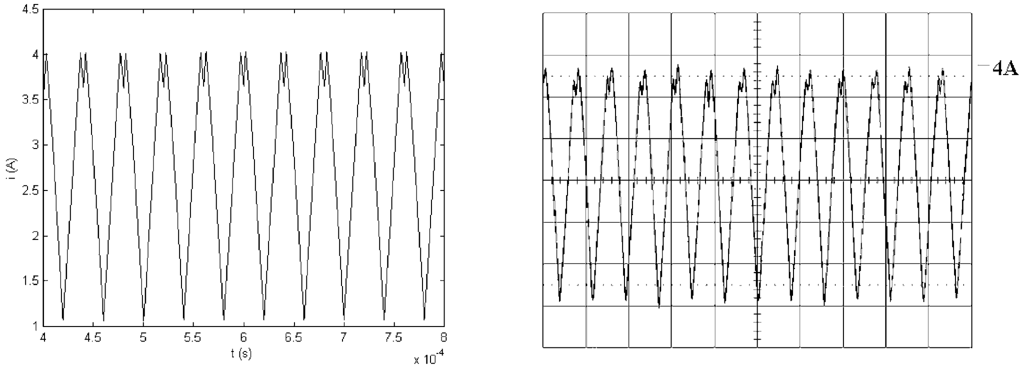
$$\begin{aligned} i_{L_{n+1}} &= f_1(i_{L_n}, v_{C_n}, \mathrm{R}, \mathrm{I}_{ref}, E, Z, Z_C) \quad (6) \\ v_{C_{n+1}} &= f_2(i_{L_n}, v_{C_n}, \mathrm{R}, \mathrm{I}_{ref}, E, Z, Z_C) \quad (7) \end{aligned}$$

where $Z = \sqrt{L/C}$ and $Z_C = T/2\pi C$

Equations (2), (3), (6), and (7) all share the same characteristics of frequency scaling. Therefore if T/C and T/L are fixed, the condition for the bifurcation is unchanged. As the frequency of operation is varied, the impedance of the passive components is kept fixed and the equations will also be unchanged. This confirms theoretically the frequency scaling of the bifurcation characteristics. Note that this condition is not unique for all the electronic circuits because the passive component may not be arranged to exhibit the same iterative equation for all the transitions of the circuit operation. It is interesting to discover this phenomenon in the power converter. Fig. 1(a) shows the bifurcation diagram of the converter at $\mathrm{f} = 20$ kHz. The circuit parameters are fixed at the following values. i.e., $\mathrm{I}_{ref} = 4$ A, $\mathrm{L} = 0.5$ mH, $\mathrm{R} = 20\ \Omega$, $\mathrm{C} = 4\ \mu$F, and $\mathrm{T} = 50\ \mu$s. The bifurcation diagram is recalculated at 50 kHz. All the parameters are unchanged except that the passive component is changed to $\mathrm{L} = 0.2$ mH and $\mathrm{C} = 1.6\ \mu$F, i.e., their impedances are unchanged. Exactly the same bifurcation as shown in Fig. 1(b) is obtained.

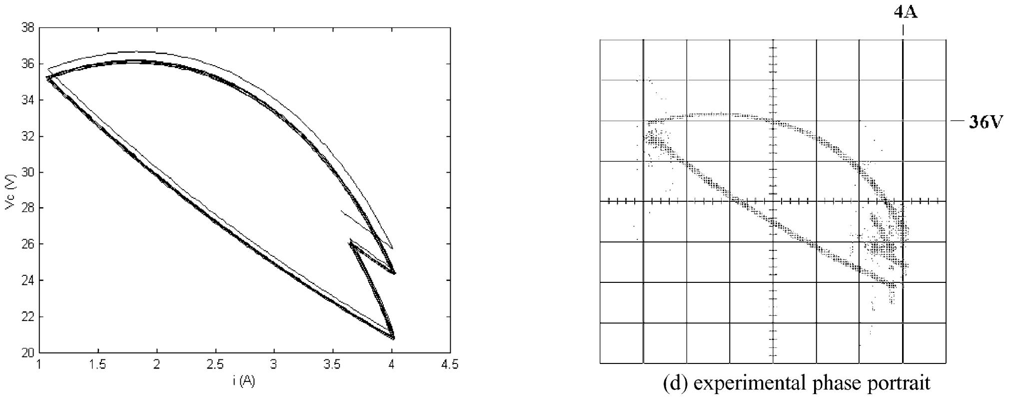
III. EXPERIMENTAL VERIFICATION

A prototype buck-boost converter has been built in order to verify the above theory. The circuit parameters are the same as the simulation. Because the test is examined at high frequency, the PCB is designed with care with low parasitic impedance. The transistor is selected to be low for on-state resistance and the diode is the Schottky type to reduce the on-state loss. The inductor is wound with multistranded wire with low dc resistance. Figs. 2 and 3 show the Period-2 operation at switching frequencies of 20 kHz and 50 kHz respectively. It can be confirmed that good agreement has been obtained between 20 kHz and 50 kHz simulations and experimental results. The Period-2 operations are also the same for 20 kHz and 50 kHz. These verify the frequency correlation of the bifurcation for the power converter.



(a) simulated waveform for i L

(b) experimental waveform for iL (0.5A/div, 50μs/div)



(c) simulated phase portrait

(d) experimental phase portrait (Vert: 4V/div; Hor: 0.5A/div)

Fig. 3. Period-2 waveforms and the phase trajectory of the buck-boost converter with $\mathrm{E} = 35\ \mathrm{V}$ operated at 50 kHz.

IV. CONCLUSION

This letter derives the basic time-step iterative equation for the buck-boost converter. The equation can be used to simulate the bifurcation phenomenon of the buck-boost converter. Experimental results at 20 kHz and 50 kHz have been used to examine the frequency scaling for the passive components variation by keeping the impedance fixed. The analysis developed can be used to predict the occurrence of the first flip bifurcation at another frequency for the basic power converter when one bifurcation point at a certain frequency is given. The letter describes the equations and the experimental results for the buck-boost converter. In fact the frequency scaling can also be applied to other power converters such as buck and boost.

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