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Integral Sliding Mode Control with Integral Switching Gain for Magnetic Levitation Apparatus

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Abstract – This paper presents an integral sliding mode control method with integral switching gain for the position trajectory of magnetic levitation apparatus. The magnetic levitation system is open loop instable, and it is a nonlinear dynamic system in nature. Many kinds controllers are proposed to face this challenge. Those controllers shall be robust enough to overcome the variations of system parameters and external disturbances, this is especially important in engineering applications. In this paper, a magnetic levitation system is firstly modeled. After that, an integral sliding mode controller with integral switching gain is proposed, the integral sliding mode control can guarantee the robustness to variations of system parameters and external disturbances, the integral switching gain can reduce the chattering obviously. Both the simulation results and the experimental results confirm the effectiveness of the controller.

Keywords – Integral sliding mode control, integral switching gain, magnetic levitation apparatus, real time implementation

I. INTRODUCTION

Magnetic levitation apparatus are contactless systems, they can eliminate the mechanical components such as gears, guide, ball bearings, also avoid generating mechanical dusts and pollution. Those virtues make them have vast untapped potential for industrial applications, for instance, magnetic levitation technologies have been world-wild investigated for advanced ground transportation systems [1]-[2]. The magnetic levitation systems are inherent nonlinear and are unstable in open loop. Those characteristics impede them for engineering applications. Hence, robust controllers shall be proposed to counteract the disadvantages, and the system performances won't be deteriorated by the variations of the system parameters and external disturbances.

Study on feedback linearization control of the magnetic levitation system has been conducted by many researches [3]-[4]. The findings in [3] revealed that the feedback linearization method were superior to the classical PID control in a large air gap of the magnetic levitation system; however, the disturbance of the system was not taken into account. While the disturbance was considered in [4], the experimental results indicated that there were static errors due to the magnetic levitation mass perturbation. In [5], an adaptive nonlinear control composed of feedback linearization method was introduced, and the experimental results reflected its robustness to system parameters uncertainties. However, adaptive control is computational intensive and time consuming.

The conventional Sliding Mode Control (SMC) is robust to variations of system parameters and external disturbances only after the occurrence of sliding mode. Integral Sliding Mode Control (ISMC) can eliminate the reaching phase by enforcing sliding mode throughout the entire system response [6].

Due to the above merits, ISMC is one of the effective candidates for the uncertain systems [7]-[8]. It is suitable to adopt this method in the magnetic levitation systems. On the other hand, the switching gain is used to hamper the effects of perturbations and uncertainties. However, the high gain will probably produce chattering, which undermines the system performance. Hence, the integral switching gain is used to avoid the gain becoming high.

In this paper, an ISMC with integral switching gain is designed for the magnetic levitation system. First, the magnetic levitation system is modeled. Second, the ISMC with switching gain is proposed for this system. Then, the results of ISMC are simulated to verify the availability of the controller. The last area of concern is to validate ISMC by real time implementation, and the experimental results matched the simulation ones very well.

The organization of this paper is as follows. The modeling of the magnetic levitation system was discussed in Section II. In Section III, the ISMC with integral switching gain was proposed and analyzed for the magnetic levitation system. In Section IV, the results of ISMC were simulated through Matlab and Simulink. The real time implementations of the magnetic system were carried out to verify the proposed algorithms in Section V. Finally, Concluding remarks were given in Section VI.

II. MODEL OF THE MAGNETIC LEVITATION SYSTEM

Figure 1 shows the experimental setup of the magnetic levitation system. The experimental setup is only an example for the magnetic levitation system, and the control method discussed later can be applied to any other typical magnetic levitation system. The magnetic levitation system is manufactured by Googol Technology Limited. The magnetic levitation system consists of an electromagnet, a steel ball, a light source, a position sensor, a data acquisition AD/DA board, a control computer and a drive circuit. The steel ball can be suspended at the desired set point by the electromagnetic force, which can be adjusted by the input current. The feedback apparatus includes LED light source and optoelectronic sensor. There is a slot in the light receiver panel to detect the light intensity, and it can be generated to relate voltage signals with the range from -10V to 0V. The output photo-voltage signals from optoelectronic sensor are transferred to controller through circuit's signal process and AD board data collection. By analyzing those data, the controller regulates the input current to implement on the electromagnet to match the levitation requirements. The above is the basic operational principle of this magnetic levitation system, and the system model is considered as follows.

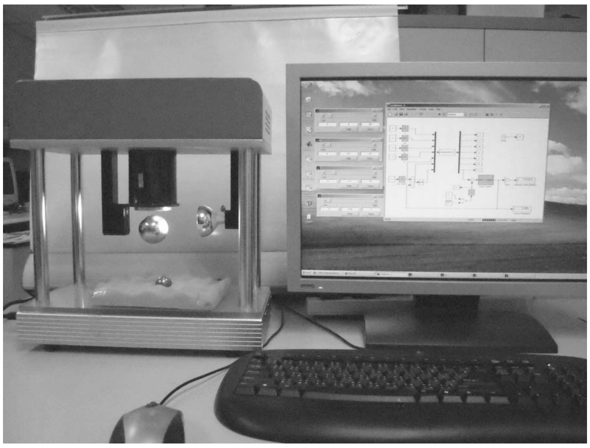

Fig. 1. Experimental setup of the magnetic levitation system

2.1 Mathematic equations of model

The model of the magnetic levitation system can be divided to three parts: mechanical kinetics model, electromagnetic force model and electrical model.

2.1.1 Mechanical kinetics model

The differential equation can be written as:

$$m\frac{d^2z(t)}{dt^2} = F(i,z) + mg \tag{1}$$

Where m is the quality of the steel ball, z is the air gap distance between the electromagnet and steel ball, F is the electromagnetic force, i is the current, g is the gravitational acceleration.

2.1.2 Electromagnetic force model

The electromagnetic force can be written as:

$$F(i,z) = -\frac{\mu_0 N^2 A}{4}\times\left(\frac{i}{z}\right)^2 = k\left(\frac{i}{z}\right)^2 \tag{2}$$

Where μ_0 denotes the permeability of air, $\mu_0 = 4\pi\times10^{-7}$ H/m, N denotes the number of coil turns, i denotes the current through the electromagnet, A denotes the air gap cross section crossed by the magnetic flux.

The equation (2) shows the nonlinearity of the magnetic levitation system since the electromagnetic force is the nonlinear function of the air gap distance and current.

2.1.3 Electrical model

The differential equation can be represented as follows:

$$u(t) = R_c i(t) + \frac{d\lambda(i,z)}{dt} \tag{3}$$

Where u denotes the terminal voltage on electromagnet, Rc denotes the resistance of coil, $\lambda = L(z)\cdot i(t)$ denotes the flux linkage.

In this paper, only the mechanical kinetics model and electromagnetic force model are considered in the magnetic levitation system. Although the proposed model is slightly different from the reality for overlooking the effect of inductance, the proposed model is simplified and is easy to design controller. Since the model is inherent nonlinear, it needs to be linearized to carry out the linear control theory.

2.2 Linearized model of the magnetic levitation system

The equation (2) can be rewritten by Taylor series expansion at the nominal operating point when the error between variation and nominal point is very small:

$$F(i,z) = F(i_0,z_0) + K_i(i-i_0) + K_z(z-z_0) \tag{4}$$

$$K_i = \frac{\partial F(i,z)}{\partial i}\Big|i = i_0, z = z_0$$

$$K_z = \frac{\partial F(i,z)}{\partial z}\Big|i = i_0, z = z_0$$

Where z_0 is the desired air gap distance, i_0 is the corresponding current with stability of the magnetic levitation system.

When the magnetic levitation system is stable, the electromagnetic force equals to gravity of the steel ball:

$$F(i_0,z_0) + mg = 0 \tag{5}$$

Then, the following equation can be conducted:

$$\frac{d^2z}{dt^2} = -\frac{2g}{i_0}i + \frac{2g}{z_0}z \tag{6}$$

2.3 Model of the controlled system

In this controlled system, the terminal voltage on electromagnet is defined as the input variation of system, the photo-voltage from the position sensor is defined as the output variation of system.

The current passes through the power amplifier to produce the input voltage. The power amplifier is approximate a proportional amplifier since it almost dose not delay in this circuit. Hence, the mathematic function of input voltage and current can be written as:

$$u_{in} = K_{in}i \tag{7}$$

Where K_{in} is a constant value decided by the circuit.

The output voltage of position sensor can be experimentally measured at different air gap distance. The mathematic function of output voltage and air gap distance can be least squares fitted as:

$$u_{out} = K_{out}(z + z_{out}) \tag{8}$$

Where K_{out} and z_{out} are constant values determined by the least squares fit.

Combination of the above equations offers:

$$\frac{d^2u_{out}}{d^2t} + au_{out} + l = bu_{in} \tag{9}$$

$$a = -2g / z_0$$

$$b = -\frac{2gK_{out}}{i_0K_{in}}$$

$$l = \frac{2gK_{out}z_{out}}{z_0}$$

The controlled system is a second-order time-invariant system from the above equation, and it is an unstable system in open loop. Therefore designing effective controller is necessary for this system.

The specifications and system parameters of the proposed magnetic levitation system are listed in Table I.

TABLE I Specifications and system parameters of system

Mass of the steel ball	**0.11kg**
Diameter of electromagnet	0.055m
Number of coil turns	2450
Reference air gap distance at steady state	0.03m
Reference current at steady state	1.1509A
Cross section of the air gap	7.3322e-004m^2
Gravitational acceleration	9.81m/s^2
K_{in}	5.893
K_{out}	-448
a	-654
b	1296
l	3603.54

III. THE INTEGRAL SLIDING MODE CONTROLLER

One robust controller referred as ISMC is proposed for this magnetic levitation system. The state space equations and the ISMC method are depicted as follows.

3.1 State space equations

Fig. 2 shows the control structure of the magnetic levitation system, r is the reference input, y is the output, e is the error between r and y, and $e=r-y$, u is the control law, f is the outside disturbance.

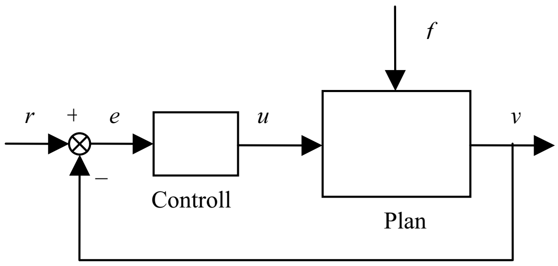

Fig. 2. Control structure of the magnetic levitation system

The differential equation of the system shown in figure 4 can be conducted from (9):

$$\frac{d^2}{dt^2}y(t) + ay(t) + l = b[u(t) + f(t)] \tag{10}$$

The system state-space equations are as follows by adopting error e as the state variation, suppose $x_1=e$, $x_2=de$, then:

$$\dot{X}(t) = AX(t) + B(u(t) + F(t)) \tag{11}$$

Where

$$X = [x_1, x_2]^T,\ A = \begin{bmatrix} 0, & 1 \\ -a, & 0 \end{bmatrix},\ B = [0, -b]^T,$$

$$F(t) = -\frac{1}{b}\left(\frac{d^2}{dt^2}r(t) + ar(t) + l - bf(t)\right),$$

suppose $|F(t)| < \varepsilon$

3.2 Integral Sliding mode control

An ISMC is adopted for the magnetic levitation system. Firstly, the sliding surface is chosen. Secondly, a sliding control law is designed to force the system state trajectories toward the sliding surface and stay on it in small vicinity bound.

3.2.1 Sliding surface

In a second-order time-invariant system, the sliding surface is a switching line in the phase space. This switching line can be represented by:

$$S = B^T X - \int_0^t \left(B^T AX + B^T Bu_0\right)dt \tag{12}$$

3.2.2 Sliding control law

Designing the sliding control law as follows:

$$u = u_0 + u_1 \tag{13}$$

Where u_0 is predetermined such that system $\dot{X} = AX + Bu_0$ follows a given trajectory with desired accuracy; and u_0 can be designed as linear static feedback control:

$$u_0 = -P^T X \tag{14}$$

In which gain vector P can be decided by pole placement method. Using *Ackermann's formula*, the control gain P can be determined explicitly by assuming that the desired eigenvalues for system $\dot{X} = AX + Bu_0$ are λ_1 and λ_2 :

$$P^T = [0,1][B, AB](A - \lambda_1 I)(A - \lambda_2 I) \tag{15}$$

In equation (13), u_1 can be designed as:

$$u_1 = -\eta|\rho| sat(S) \tag{16}$$

Where η is constant parameter, $\eta>0$, ρ is the integral switching gain, $\eta|\rho| > \varepsilon$, as well:

$$\rho = \int_0^t (\zeta\rho + S)dt \tag{17}$$

ζ is constant parameter, $\zeta<0$. When the sliding mode is occurred, S tends to zero, then, the integral of S also tends to zero, which cause the switching gain tend zero so as to eliminate chattering. When it is reaching phase, it can be seen from (17) that the mathematical signs of $\zeta\rho$ and ρ are always contrary. This can limit the switching gain in a low range.

In equation (16), *sat(S)* is saturation function:

$$sat(S) = \begin{cases} 1, & S > \Delta \\ S/\Delta, & |S| \le \Delta \\ -1, & S < -\Delta \end{cases} \tag{18}$$

Where Δ denotes a small vicinity of the origin to define the boundary layer, $0<\Delta<1$.

The saturation function can turn the discontinuous control to continuous control. That is, the system state trajectories are bounded in a small vicinity of the switching line S=0, in place of the exactly ideal mode. Since the switching action is replaced by a continuous approximation, the chattering problem can be undermined.

3.2.3 Stability analysis

Employing the positive definite Lyapunov function candidate:

$$V = \frac{1}{2} S^2 \tag{19}$$

Since the system behaviour is not determined within the small vicinity, the system will be stable if the phase trajectory can converge into the small vicinity. For this purpose, the saturation function can be replaced by sign function, therefore:

$$\begin{aligned} \dot{V} &= S\dot{S} = B^T B\left(-\eta\left|\rho\right|\left|S\right| + FS\right) \\ &< B^T B\left(-\eta\left|\rho\right|\left|S\right| + \varepsilon S\right) < 0 \end{aligned} \tag{20}$$

That verifies the system is stable.

IV. Simulation Results

The proposed system is simulated by using Matlab Simulink. The fixed sample time is T=0.003s. A constant value disturbance is adopted in the simulation, and the disturbance will not be triggered until the simulation time comes to 3 seconds. Fig.3 depicts the simulation results of voltage response curve on condition that r=-5V, λ_1=-10, λ_2=-20, η=0.0002, ζ =-10, Δ=0.01, f=1. Fig.4 shows the simulation results of control law with the same parameters of fig.3.

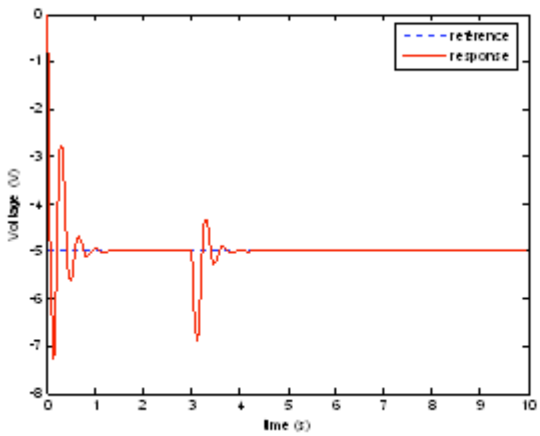

Fig. 3 Voltage response curve

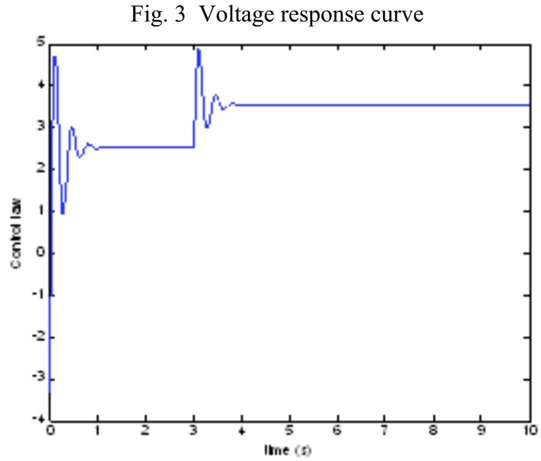

Fig. 4 Control law

As are shown in fig.3, after the interim period, at about 1 second, the response voltage matches the reference voltage. When time runs to 3 seconds, the response voltage is deviated from the reference voltage because of the external disturbance, after about one second self adjustment, the response voltage can trace the reference voltage very well again. The same results can be obtained from fig.4, which is corresponding to fig.3. These results are identical with the control algorithms of the above section. It can be concluded that the proposed ISMC is valid.

V. Experimental Results

Experiments are also carried out to verify the control algorithms. The control algorithms are developed under the Matlab/Simulink environment. The real time implementation of the magnetic levitation system is executed with the Real Time Workshop (RTW) of Matlab. One data capture and processing card, plugged into a PCI bus of the host AMD Athlon XP 1223MHz Computer, is a 16 digital I/O channels and 12 bit A/D converter card. The fixed sample time is T=0.003s.

After the power of the magnetic levitation system is switched on and the control software Matlab/Simulink is running, the controlled steel ball is put into the electromagnetic field. When the steel ball is levitated stably, another steel ball of 0.05kg, used as the disturbance, is added to the electromagnetic field in around 17 seconds.

Fig.5 represents experimental results of voltage response curve on condition that u_{in}=-5V, λ_1=-10, λ_2=-20, η=0.0002, ζ =-2, Δ=0.01. The control law in the same condition is shown in fig.6.

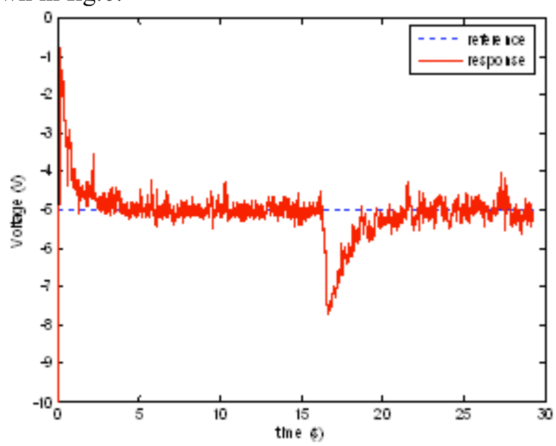

Fig. 5 Voltage response curve

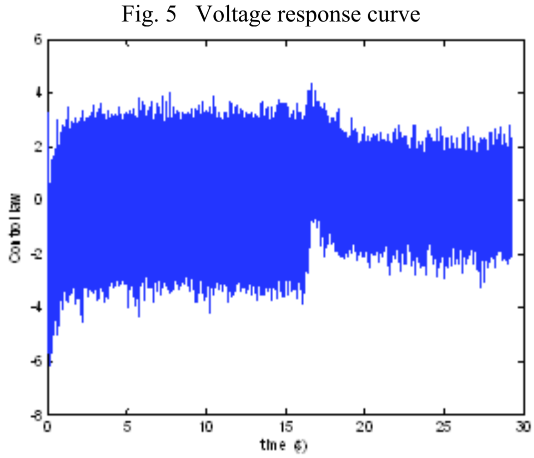

Fig. 6 Control law

As are shown in fig.5, through the interim period, the response curve tracks the reference trajectory at 4 seconds or so, the system reaches a steady state. When the disturbance steel ball is added to the system, the response voltage mismatches the reference voltage, but the response voltage tracks the reference voltage within several seconds. Fig.6 shows the same trends, and it also shows the control law switches fast. One possible reason is that the LED light is sensitive to the environment, this uncertainty will result in the continuous change of output voltage, and the magnetic levitation system is a high nonlinear system, the control law is correlative to the output voltage. Another possible reason is from the neglect of electrical model, which will affect the accurate control.

The experimental results show that the magnetic levitation system can be stable regardless of the external disturbance, which validate the effectiveness of the proposed ISMC.

VI. Conclusions

In this paper, the model the magnetic levitation apparatus is built, and an ISMC with integral switching gain is proposed for this system to eliminate the effects of external disturbance. The system characteristics and stability of the controller are analyzed. Both the simulation results and experimental results agree with the control algorithms and prove the feasibility of the proposed controller.

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References

[1] B. V. Jayawant, *Electromagnetic Levitation and suspension techniques*, Edward Arnold (Publishers) Ltd, 41 Bedford Square, London WC1B 3DQ, 1981.

[2] H.-W. Lee, K.-C. Kim, and J. Lee, "Review of Maglev Train Technologies," IEEE Trans. on Magnetics, vol.42, no.7, pp.1917-1925, July 2006.

[3] A. El Hajjaji, and M. Ouladsine, "Modeling and nonlinear control of magnetic levitation systems," IEEE Trans. on industrial electronics, vol.48, no.4, pp.831-837, August 2001.

[4] S. J. Joo, and J. H. Seo, "Design and analysis of the nonlinear feedback linearizing control for an electromagnetic suspension system," IEEE Trans. on control systems technology, vol.5, no.1, pp.135-144, January 1997.

[5] Z.-J. Yang, and i M. Tateish, "Adaptive robust nonlinear control of a magnetic levitation systems," Automatica, vol.37, pp.1125-1131, 2001.

[6] Vadim Utikin, Jurgen Guldner, and JingXin Shi, *Slidng mode control in electromechanical systems*, Taylor & Francis Ltd, 11 New Fetter Lane, London EC4P 4EE, 1999.

[7] A. Poznyak, L. Fridman, and F. J. Bejarano, "Mini-max integral sliding mode control for multimodel linear uncertain system," IEEE Trans. on automatic control, vol.49, pp.97-102, January 2004.

[8] Q. Zong, Z. S. Zhao, L. Q. Dou, and L. K. Sun, "Integral sliding mode control for a class of nonlinear mismatched uncertain systems" Proc. of the 2nd international symposium on systems and control in aerospace and astronautics, pp.1-4, Dec. 10-12, 2008.

Biographies

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