2009 3rd International Conference on Power Electronics Systems and Applications

Power Factor and K-factor in the Analysis of DC-DC Converters

B.P. DIVAKAR[1] K.W.E. CHENG[1*] Zhanghai SHI[1] K.F. KWOK[1]

EE Department, The Hong Kong Polytechnic University Hung Hom, Kowloon
*Email: eeecheng@inet.polyu.edu.hk

Abstract— In this work, a new performance parameter, namely power factor, introduced in [10] is used in the analysis of buck dc-dc converter operating in DCM. The power factor concept is now applied in the analysis of dc circuits. The derivation of the parameter and its application as one of the performance parameters in the analysis of the buck dc-dc converter operating in DCM and boost converter operating in discontinuous mode (DCM) has been carried out considering all the parasitic elements of the circuit. All the equations are normalized and expressed in terms of power factor so that a user can easily study the influence of the circuit elements on the performance of the converter. This will aid the designer to plot design curves for any loading conditions and choose the factor "K", which is used to calculate the inductance required to operate the converter in a desired mode.

Keywords-Power factor, K-factor and dc-dc converter

I. INTRODUCTION

The lack of normalized design expressions and analysis without parasitic in literatures make it impossible to compare different topologies for a given application. As the design expressions of each of the topologies are derived for a particular specifications it is not possible to compare two topologies with different specifications. The reason for this is that the design expressions of each circuit are not normalized with respect to any performance parameter that can be used for comparing different circuits. If the performance parameters of a circuit are expressed or normalized with respect to a design parameter different designs can be compared without any difficulty. The design parameters, in terms of which the performance parameters are expressed, should be so chosen that it permits the selection of circuit elements for any specifications. The advantage of normalizing the design expressions with the performance parameters is that it permits scaling to suit different operating conditions such as output power, input voltage and output voltage. The variation of the energy stored in the reactive components of a dc-dc converter was considered and defined as maximum variation of energy storage factor (MSEF) [1]. Although this factor is useful for comparing the relative sizes of reactive components in different designs, its application in the design of converters is limited. In order to compare different designs, the present paper proposes two factors namely, power factor and K-factor as goodness factors in the analysis and design of dc circuits.

A. Active and non-active component in ac circuits; Ia(t) and Iq(t) [2].

If V(t) and I(t) are the instantaneous input voltage and current, then $I_a(t)$ and $I_q(t)$ are the instantaneous active and non-active components. The active component is the one responsible for the transfer of active power and therefore it is defined as the component of input current having the same form as the input voltage and in phase with the voltage. On the other hand, the non-active component is obtained by subtracting the active component from the instantaneous component and hence it is a measure of the useless component present in the circuit. The following equations are used to calculate the two components.

$$I_a(t) = \frac{\frac{1}{T}\int_0^T V(t) * I(t)dt}{\frac{1}{T}\int_0^T V^2(t)dt} V(t) \tag{1}$$

$$I_q(t) = I(t) - I_a(t) \tag{2}$$

The concept of active component and the non-active component of the input current has given rise to a simplified expression for the active and non-active power as follows:

$$\text{Active power} = P = V_{rms} * I_a \tag{3}$$

$$\text{Non active power} = Q = V_{rms} * I_q \tag{4}$$

$$\text{Apparent power} = S = \sqrt{P^2 + Q^2} \tag{5}$$

$$\text{Power factor} = \text{pf} = \cos\phi = \frac{P}{S}. \tag{6}$$

B. Power factor in DC circuits.[10]

It is interesting to note that power factor which is one of the performance parameters in ac circuits is not defined for a dc circuit. So, the purpose of this section is to extend the concept of power factor and to understand the meaning of power factor in a dc circuit. The introduction of the concept of active and non-active components coupled with the fact that power factor is no longer referred to as the cosine of the angle between the current and the input voltage has made it is possible to interpret the meaning of power factor in a dc circuit. The main aim of this paper is to explore the meaning of power factor of a dc circuit and to study the application of this factor as one of the performance parameters in the study of a dc circuit. In the following section, a general expression for the power factor of a dc circuit is obtained.

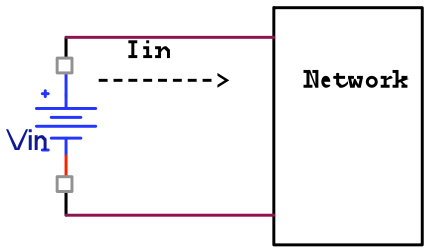


Fig. 1: DC circuit

Consider a dc circuit as shown in which the input current may be continuous or discontinuous. The instantaneous active current $I_a(t)$ and the non-active current Iq(t) of the input current can be obtained from (7) and (8) respectively by applying (1) and (2) as:

$$I_a(t) = \frac{\frac{1}{T}\int_0^T V_{dc} * I_{in}(t)dt}{\frac{1}{T}\int_0^T V^2{}_{dc}dt} V_{dc} = \frac{1}{T}\int_0^T I_{in}(t)dt = I_{inavg} \tag{7}$$

This implies that;

Ia = active component (rms) = Average input current (I_{inavg})

Iq = non-active component (rms)

$$= \sqrt{I^2_{inrms} - I^2_a} = \sqrt{I^2_{inrms} - I^2_{inavg}} \tag{8}$$

P = Avg Input power = V_{in} * I_a (9)

S=V_{in} * I_{inrms} (10)

Power factor = P/S = I_a/I_{inrms} = I_{inavg} / I_{inrms} (11)

The ratio of the average to rms value of the input current in a dc circuit can therefore be interpreted as power factor in dc circuits. As a result, power factor in a dc circuit is a measure of the deviation of the input current from the ideal dc level. This deviation is also a measure of the distortion of the input current and the degree to which the deviation is present depends on the topology of a circuit. Any change in the circuit topology is reflected onto the power factor. The paper attempts to incorporate the power factor in the design equations and use it as one of the performance parameters while making qualitative analyses of circuits.

The main aim of the topic of this paper is to explore how to incorporate this parameter into the expressions of the basic dc-dc converter and then to qualitatively judge the usefulness of this exercise in order to justify the introduction of the new parameter.

To start with an expression for the power factor for a buck converter operating in the discontinuous conduction mode is derived followed by the derivation of the normalized expression of the conversion ratio in terms of power factor by including all the parasitics of the circuit as well as taking into account of the contribution of conduction and switching losses in the circuit. Paper [3] has carried out the analysis of buck and boost dc-dc converters operating both in continuous and discontinuous mode. However, the paper has not included the capacitive turn-on loss which cannot be neglected at high input voltage. A detailed analysis of the PWM converter has been done in [4]&[5] only for the operation in continuous conduction mode. [6]-[8] either approximate or carry out analysis with all the parasitic elements under full load conditions. The selection of the inductor depends on the ripple current specification and hence on the mode of operation. The change in duty cycle, input voltage and loading can force the converter to operate in DCM. In order to ensure that the converter does not change its mode of operation for a wide change in load and line changes, papers [9] have used the factor K, which is equal to $2Lf_S/R$. For example, if the converter is expected to work only in CCM, then the condition K_{min} > (1-D_{min}) should be satisfied. The concept of power factor in dc circuit was first introduced in [10] where the results for buck converter operating in CCM were presented. In this paper, the analysis presented in [10] will be extended to buck converter operating in DCM.

In the analysis, the reverse recovery loss and magnetic losses are not considered. The voltage drop across the diode is considered to be a constant and the equivalent circuit of the diode is considered as a voltage source in series with the diode's on resistance.

The present paper is organized as follows. Power factor and other expressions are derived for the buck converter with parasitics and operating in continuous conduction mode in II, power factor for the boost converter in CCM is presented in III followed by concluding remarks.

II. Buck Converter in Discontinuous Conduction Mode

The classical buck converter with all the parasitics shown in fig. 2 where, the diode forward voltage drop is shown by a series dc source.

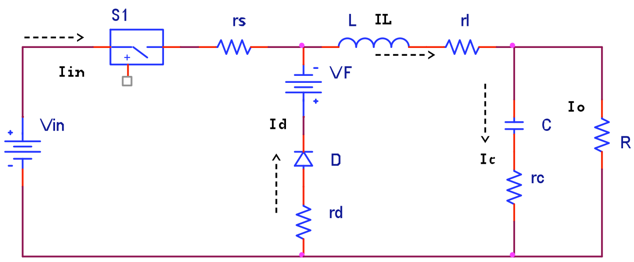
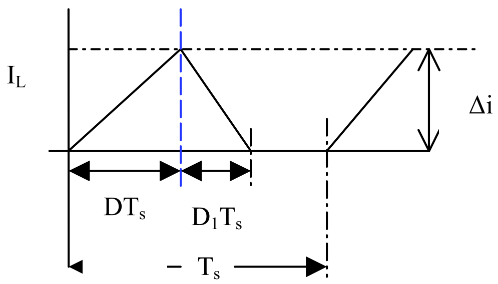


Fig. 2: A buck converter with its parasitic elements

The steady state expressions under discontinuous mode are derived in this section to derive the conversion ratio in terms of power factor of the circuit. It should be noted that the valid DCM points are those that satisfy the condition K < (1-D).



F ent waveform in D

Tl alys erter in DCM r the
fo ıg e

$$\Delta\imath = \frac{\;D_1}{K} \tag{12}$$

$$D_1 = \frac{-D+}{} \tag{13}$$

$$Iin_{avg} = \frac{\Delta i * D}{2} = \frac{I_{0}}{\;}K \tag{14}$$

$$Iin_{rms} = \Delta i\sqrt{\frac{D}{3}} = \frac{I_o D D_1}{K * pf} \tag{15}$$

$$I_{Lrms} = \frac{2I_o D_1}{K}\sqrt{\frac{D+D_1}{3}} = Iin_{rms}\frac{\sqrt{D_1}}{\sqrt{D+D_1}} = 2I_o\frac{\sqrt{D_1}}{\sqrt{3K}} \tag{16}$$

Power factor (pf) for the buck converter in DCM is given by (17):

$$pf = \frac{Iin_{avg}}{Iin_{rms}} = \frac{\sqrt{3D}}{2} \quad (17)$$

where f_S is the operating switching frequency and R is the resistance of the load.

The above general expression relates the power factor of buck converter with the circuit parameters as well as the duty ratio.

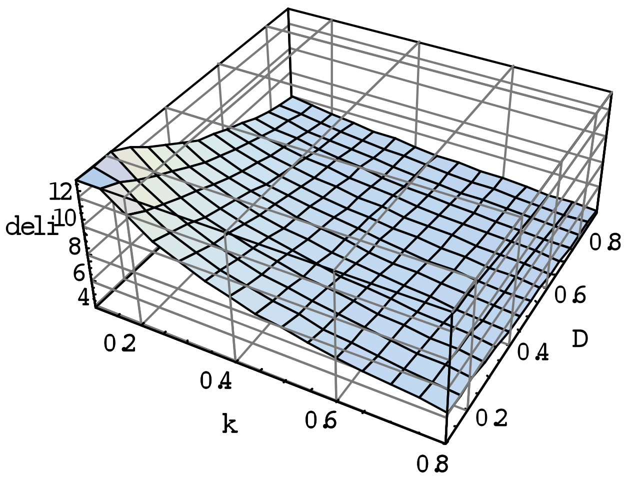


Fig. 4: Variation of inductor ripple current in DCM

Fig. 4 shows the plot of inductor current ripple against the variation of K and D. As expected, for low values of K, the inductor current ripple is larger compared to that for larger values of K. A plot of pf against the duty ratio and K gives an idea as to the influence of K on the power factor as in fig. 5.

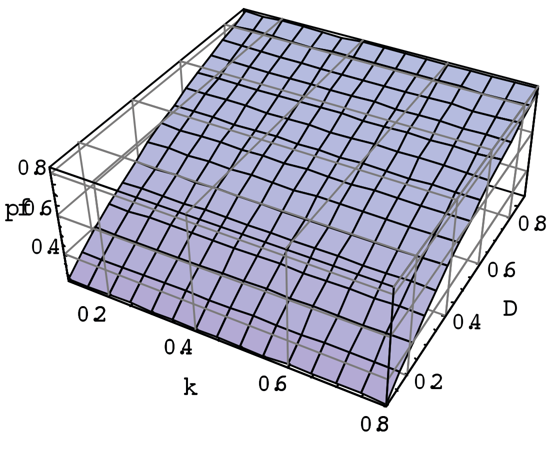


Fig. 5: Power factor versus K and D.

A. Derivation of conversion ratioin DCM

Conversion ratio for the buck converter shown in fig.2 can be derived by considering the power balance equation (18):

$V_{in}I_{in} = V_oI_o + \text{Losses}$ (18)

Loss in the switch is given in (19):

Losses = Inductor resistance loss + Switch conduction + switching losses + Loss in the diode + Loss in the capacitor

$$I_{inrms}^2 r_{ds} + \frac{5}{24}V_{in}\Delta i(t_f) f_S + \frac{CV_{in}^2 f_S}{2} = \left(\frac{I_o DD_1}{Kpf}\right)^2 r_{ds} + \frac{5}{24}\frac{2I_o D_1}{K}(t_f + t_r) f_S + \frac{CV_{in}^2 f_S}{2} \quad (19)$$

Conduction loss in the diode:

$$V_F I_{Davg} + I_{Drms}^2 r_F = \frac{V_F D_1^2 I_o}{K} + \left(\frac{2I_o D_1}{K}\sqrt{\frac{D_1}{3}}\right)^2 r_F \quad (20)$$

Loss in the capacitor is:

$$I_{crms}^2 r_c = I_o^2\left(\frac{4D_1}{3K} - I_o\right) r_c \quad (21)$$

Loss in the inductor:

$$I_{lrms}^2 r_L = \left(2I_o\sqrt{\frac{D_1}{3K}}\right)^2 r_L \quad (22)$$

By substituting the above equations in (18) and simplifying the expression for the conversion ratio is obtained as in (23):

$$\frac{V_o}{V_{in}} = m = \frac{-b + \sqrt{b^2 - 4ap}}{2a} \quad (23)$$

where

$$b = -\left(\frac{D}{D + D_1} - \frac{5D_1(t_f + t_r) f_s}{24K}\right) \quad (24)$$

$$a = 1 + \left(\frac{DD_1}{K * pf}\right)^2 \frac{r_{ds}}{R} + \frac{D_1^2}{K}\frac{V_F}{V_o} + \frac{4D_1^3}{3K^2}\frac{r_F}{R} + \left(\frac{4D_1}{3K} - \frac{V_o}{R}\right)\frac{r_c}{R} + \frac{4D_1}{3K}\frac{r_l}{R} \quad (25)$$

$$p = \frac{CRf_S}{2} \quad (26)$$

The expression for the voltage conversion ratio is arranged in such a way that the individual parasitic elements are normalized with respect to the output load. As a result the expression gives an insight into the contribution of every parasitic element of the circuit to the overall performance of the converter thereby aiding the designer to optimize the design. One such application where this information can become a tool for optimization is in the design of soft switching converters, which have higher conduction losses than the traditional hard switched converters. The designer can easily make a comparative study to justify the energy saved on account of soft switching at the cost of higher conduction loss, and also one can make a study to compare the zero voltage switching (which eliminates the capacitive turn-on loss) and the zero current switching technique for a specific application.

An example of the specifications of the buck converter operating in discontinuous mode considered in paper [3] is used to plot the following graphs. The specifications of the converter are V_i=28 V, V_o=10V, R = 3Ω, f_S = 50 kHz, r_{ds} = 55 mΩ, $t_f = t_r$ = 80ns, r_F = 20 mΩ, V_F = 0.5V, r_L = 50 mΩ, r_C = 50 mΩ and L = 10 uH. The valid points on the plot are the ones that satisfy the constraint ***K < (1-D).***

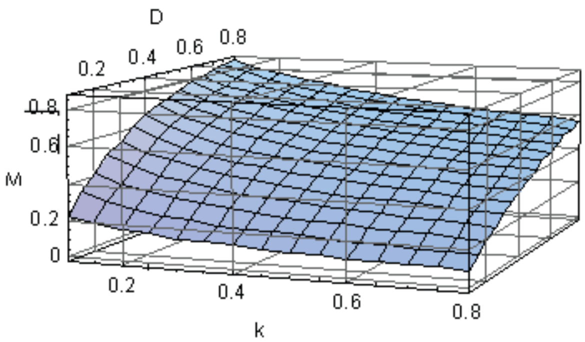


Fig. 6: Conversion ratio against K and D in DCM

The conversion ratio under DCM is a nonlinear function of D unlike in CCM where it exhibits linear relation with the duty cycle.

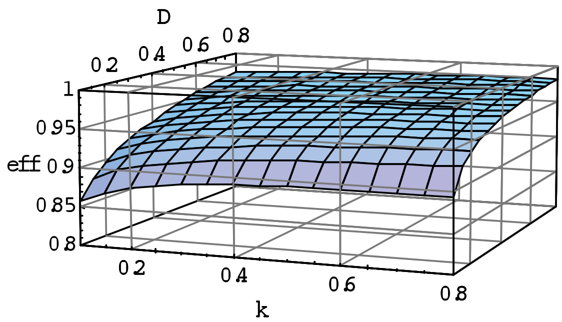


Fig. 7: Efficiency against K and D in DCM

The variation of efficiency with K can be seen in fig. 7. The improvement of the efficiency is expected at higher values of K as there is a reduction in the current ripple.

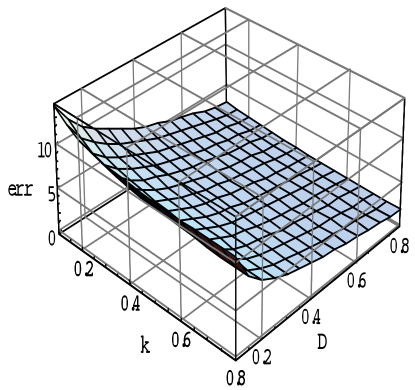


Fig. 8: %error between the ideal and actual conversion ratio in DCM

The effective use of the accurate expressions are depicted in fig. 8 where the %error between the ideal expression and the accurate expressions are plotted with respect to K and D. The waveform clearly confirms that error cannot be neglected for low values of duty cycle.

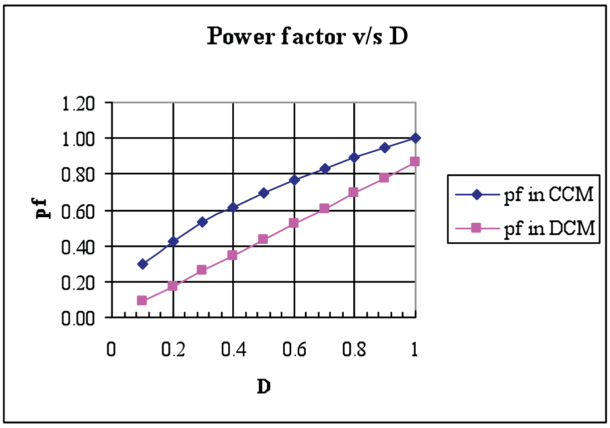


Fig. 9: Comparison of pf in CCM and DCM

Fig. 9 shows the plots of power factor versus duty ratio D for CCM and DCM operation. The plots clearly show that the buck converter operating in DCM has lower pf compared to the converter operating in CCM. This is due to the lower value of K in buck converter operating in DCM, which results in lower inductance value and hence higher ripple current. The higher ripple content increases the rms value of the current leading to lower power factor.

III. Boost Converter in Continuous Conduction Mode

A. *Power factor for boost converter*

Similar methodology applied in the case of the buck converter can be applied for the boost converter too and an expression for the power factor for the boost converter in CCM can be derived as follows:

$$I_{Lrms} = \sqrt{\frac{\Delta i^2}{12} + I_{inavg}^2} \quad (27)$$

$$I_{inavg} = \frac{I_0}{1-D} \quad (28)$$

$$pf = \frac{1}{\sqrt{1 + \frac{D^2(1-D)^4}{3K^2}}} \quad (29)$$

The power factor given in (29) can be plotted against K and D as in fig. 10.

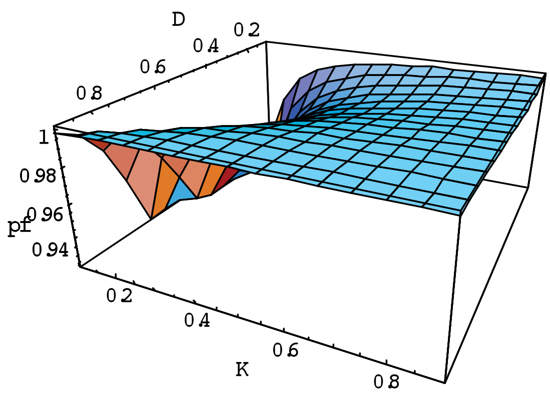


Fig. 10: Variation of pf with D and K in boost converter in CCM

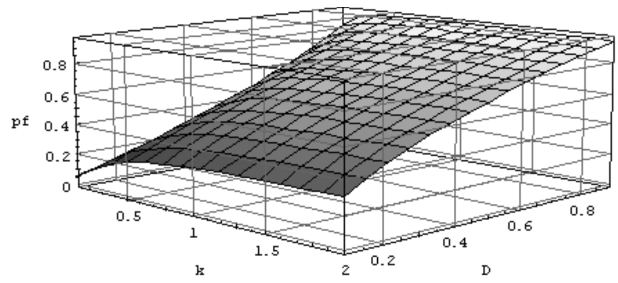


Fig. 11: Variation of pf with D and K in buck converter in CCM

Fig. 10 shows the variation of pf with K and D in the boost converter in CCM while fig 11 shows the corresponding waveform with the buck converter in CCM. It can be seen that the boost converter in general has higher pf due to the continuous input current on account of the input inductor.

IV. CONCLUDING REMARKS

In this work a new performance parameter namely power factor introduced in [10] has been applied to buck converter in DCM and boost converter operating in CCM. The use of these parameters as goodness factors in the steady state analysis of buck and boost converters is discussed. All the expressions are expressed in terms of power factor and the K-factor enabling the designer to evaluate different design options before optimizing the design. The derived expressions include the contribution from circuit's parasitic thereby assisting the comparative analysis of different converters. As the power factor in a dc circuit indicates the deviation of the average input current from its ideal dc value, the power factor can be used as an indicator of the distortion of the input current. It is envisaged that the new performance parameter can be applied to compare the performance of a new topology with that of the classical converters. Further research is process to apply this concept for comparing various soft-switched converters.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of the Research Committee of The Hong Kong Polytechnic university for this project (Project No: 1-BB86)

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