NEW ADAPTIVE ITERATIVE IMAGE RESTORATION ALGORITHM

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ABSTRACT

It has been shown in literature that adaptive regularized image restoration is superior to non-adaptive one. However, the adaptivity introduced in most proposed iterative algorithms is based only on the application of space-variant smoothing operator. It is found that these adaptive algorithms suffer from insufficient smoothing of the flat image regions. In this paper, an adaptive iterative image restoration algorithm, which applies both techniques of space-variant smoothing and space-variant restoration, is proposed to overcome the stated problem. It is shown by experiments that the restored images obtained by the proposed algorithm are better in terms of both numerical measurement and visual quality.

1. INTRODUCTION

In many practical situations, image formation process can be adequately formulated by the following linear model [1].

$$y = Dx + n \tag{1}$$

where x is the original object, y is the observed image, and n is the additive noise due to the imaging system. When the above model is described in discrete form, (1) is a matrix-vector equation, where x, y and n are respectively vectors consisting of the lexicographically ordered elements of the object, image and noise. The matrix D represents the degradation associated with the image formation process, which is referred to as point spread matrix [1].

With this model, the purpose of image restoration is to obtain an image that resembles the original object x as closely as possible [1]. It is well-known that image restoration is an ill-posed inverse problem [2]. That is, existence, uniqueness and stability of the solution based on direct inversion is not guaranteed.

The ill-posedness of the restoration problem can be compensated by applying the regularization theory [2]. Most regularization approaches assume that the original image is reasonably smooth and utilize this information to obtain a solution which is a compromise between fidelity and smoothness of the image. However, typical linear space-invariant (LSI) regularization techniques introduce other undesirable artifacts, such as ringing in the vicinity of sharp intensity transitions. It has been recently shown that these artifacts can be suppressed by introducing adaptivity in the regularized restoration methods [3].

Most proposed adaptive regularized restoration algorithms [3-7] are based on the utilization of a space-variant smoothing operator. The amount of smoothing, which is determined by a scalar smoothing parameter, is made space-variant according to the local spatial activity of the image. It was found that these adaptive algorithms can reduce the undesirable artifacts to a great extent. Moreover, the restored image obtained with adaptive algorithms is visually better than those with non-adaptive algorithms [3-5]. However, there exists a problem in these adaptive restoration algorithms, namely, inadequate smoothing of the flat image regions. In this paper, we propose an adaptive image restoration algorithm that can overcome the problem stated above. In Section 2.1, the reason for why the problem aroused in the algorithms using solely smoothing operator is discussed. The derivation of the proposed algorithm, which is based on the set theoretic approach to the restoration problem, is given in Section 2.2. The main differences between the proposed algorithm and the adaptive algorithms formulated in weighted Hilbert space [3,6] are described in Section 2.3. Detailed experimental results reporting the performance of the proposed algorithm are given in Section 3. Finally, our conclusion is given in Section 4.

2. AN ADAPTIVE ITERATIVE IMAGE RESTORATION ALGORITHM

2.1. Problem Description

Common blur sources, such as defocusing, motion

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and atmospheric turbulence, perform like a low-pass operator. That is, most blur operators have relatively larger effect in the image regions with sharp intensity transitions. In more gradual image regions, the effect of blur operators is little. For a very flat image region, blur operators have almost no effect on it. Hence, image restoration can generally be viewed as a process that attempts to reconstruct the high-frequency components of the image.

Owing to the presence of noise, the restoration on a very flat region based on the regularization using smoothing operator is actually a process that amplifies the noise and then suppresses it again. However, a smoothing operator cannot suppress the noise magnification error completely. Therefore, restoration of a very flat image region is not only unnecessary but also produces errors in this sense.

Adaptive restoration methods applying only space-variant smoothing operator are inadequate to obtain a good restored image. The noise magnification error in the flat regions of the restored image still gives biase to the solution. Applying a smoothing operator in regularized restoration is just one of the methods to suppress the noise magnification. Another way to suppress the noise magnification is by reducing the amount of restoration. The amount of restoration in the flat image regions should be small so that the noise will not be magnified to have dominant effect. In regions with sharp intensity transitions, the amount of restoration should be large so that sharp edges and lines can be reconstructed. Based on these arguments, it is our opinion that the amount of restoration should be space-variant in order to obtain a closer approximation to the original image.

2.2. Formulation of the algorithm

The derivation of our algorithm is based on the set theoretic approach to the image restoration problem [4, 5]. In the first place, according to the local spatial activity of the image, it is partitioned into L segments. The restoration problem can then be considered as restorations of these L segments. For each segment $i(i = 1, 2, \cdots, L)$, the restoration problem is formulated as the selection of a vector x in the intersection of the two ellipsoids defined by

$$\|I_i(Dx - y)\|^2 \leq {\varepsilon_i}^2 = f_i\varepsilon^2(\frac{N_i}{N^2}) \qquad (2)$$

$$\|I_iCx\|^2 \leq E_i = e_iE^2(\frac{N_i}{N^2}) \qquad (3)$$

where N^2 is total number of image pixels, N_i is the number of pixels in segment i and I_i is a diagonal matrix with N_i elements equal to one and the rest equal to zero. The position of ones in matrix I_i correspond to the location of pixels belonging to segment i. Note that

$$I_1 + I_2 + \cdots + I_L = I(identity\ matrix) \qquad (4)$$

In (2) and (3), E and ε are two values satisfying the following:

$$\|Dx - y\|^2 \leq \varepsilon^2 \qquad (5)$$

$$\|Cx\|^2 \leq E^2 \qquad (6)$$

and f_i is the value of noise visibility function associated with segment i [5]. f_i for a smooth segment is larger than that for a segment with sharp edges, because noise is more visible in flat regions than in the vicinity of edges [5]. The matrix C in (3) and (6) is generally a high-pass filter imposing smoothness constraint on the solution. Since C is a high-pass operator, the value of e_i for the segment with low spatial activity is smaller than that for the segment with high spatial activity. It is practically difficult to obtain e_i as the original image is unavailable. A simple method for estimating its value is presented in Section 3.

It is reasonable to take the center of any bounding ellipsoid containing the intersection of the two ellipsoids defined by (2) and (3) as the best estimate of x [8]. The center of one of such bounding ellipsoids is given by

$$[\frac{D^TI_iD}{{\varepsilon_i}^2} + \frac{C^TI_iC}{{E_i}^2}]x = \frac{D^TI_i}{{\varepsilon_i}^2}y \qquad (7)$$

or

$$[D^Te_iI_iD + (\frac{\varepsilon^2}{E^2})C^Tf_iI_iC]x = D^Te_iI_iy \qquad (8)$$

There are totally L different equations of the form of (8) and x must satisfy all these equations. Owing to equation (4) and that I_i is a diagonal matrix with elements equal to one or zero, we can combine the L different equations into

$$\sum_{i=1}^{L}[D^Te_iI_iD + (\frac{\varepsilon^2}{E^2})C^Tf_iI_iC]x = \sum_{i=1}^{L}D^Te_iI_iy \qquad (9)$$

After some manipulation of (9), the solution is given by

$$x = [D^TRD + \alpha C^TSC]^{-1}D^TRy \qquad (10)$$

where $\alpha = \frac{\varepsilon^2}{E^2}$, $R = \sum_{i=1}^{L} e_iI_i$ and $S = \sum_{i=1}^{L} f_iI_i$.

An efficient method of obtaining x in (10) is to use steepest descent algorithm to approximate x iteratively

[3-5], which results in the following iterative restoration algorithm.

$$x_o = \beta D^T y$$

$$x_{k+1} = x_k + \beta D^T R(y - Dx_k) - \alpha\beta C^T S C x_k \quad (11)$$

where β is the contraction or relaxation parameter for the iteration. The iteration in (11) converges if β satisfies the following condition [4-5].

$$0 < \beta < \frac{2}{\lambda_{max}} \quad (12)$$

where λ_{max} is the largest eigenvalue of the matrix $(D^T RD + \alpha C^T SC)$.

2.3. Discussion

In formulation (11), the function of $\beta D^T R(y - Dx_k)$ is to reconstruct the high-frequency components of the image iteratively. The term $(-\alpha\beta C^T S C x_k)$ is used to suppress the noise magnification error at each iteration. Matrix R is applied to control the amount of restoration at each pixel and matrix S weights the amount of smoothing for each pixel. Therefore, the proposed adaptive algorithm applies both techniques of space-variant restoration and space-variant smoothing.

The proposed algorithm involves two diagonal matrices R and S. In the formulation of regularized image restoration in a weighted Hilbert space [3, 6], there are also two diagonal weighting matrices. In the context of weighted Hilbert space, diagonal matrix S weights the relative extent of regularization at every pixel in the restored image. Diagonal matrix R represents the weights in the importance of every pixel to be restored in the observed image [7,9]. R is typically chosen to be the identity as all pixels should be weighted equally [7]. However, in the formulation of the proposed algorithm, matrix R is related to the spatial activity of the image.

Although the derivation of the proposed algorithm is different from that based on the concept of weighted Hilbert space, the resulting iterative restoration algorithms of these two approaches are very similar. Both algorithms embed adaptivity in the diagonal matrices R and S. However, in the context of weighted Hilbert space, R and S are matrices containing weight coefficients with values between 0 to 1. In the proposed algorithm, elements in R and S can be any appropriate values. Therefore, the adaptive algorithms based on the weighted Hilbert space approach can be considered as a special case of the proposed algorithm.

3. EXPERIMENTAL RESULTS

Experiments have been carried out to test the performance of the proposed algorithm. The performance is evaluated by measuring the SNR improvement defined as

$$SNR\ Improvement = 10\log_{10}\frac{\|x-y\|^2}{\|x-x_k\|^2} \quad (13)$$

where x_k is the estimate of x after k iterations using (11).

The image "lenna" of size 256×256, shown in Figure 1(a), was artificially blurred with two blur operators, namely, defocusing blur with COC(circle of confusion) equal to 5 pixels and horizontal motion blur over 9 pixels, separately. White Gaussian noise was then added to these two blur images with BSNR (Blurred Signal-to-Noise Ratio) equal to 20, 30 and 40 dB, where BSNR is defined as

$$BSNR = 10\log_{10}\left(\frac{blurred\ image\ variance}{noise\ variance}\right) \quad (14)$$

The proposed algorithm was then performed to restore these degraded images and SNR improvements were measured after restorations. The results were compared to the restoration results of adaptive regularized restoration based solely on space-variant smoothing operators [5]. The matrices R and S in the proposed algorithm are both related to the spatial activity of the original image. However, since the original image is unavailable, matrices R and S are computed from an estimate of the spatial activity of the image. In our experiments, a restored image is first obtained by a non-adaptive restoration algorithm. Matrices R and S are then computed based on that image. In particular, the weight matrix S was obtained by computing the noise visibility function of the image, which is identical to that described in [3-5]. The weight matrix R was obtained by the following steps

1). Computing $\hat{e}(i,j)$ which is defined as

$$\hat{e}(i,j) = \frac{\|I_{ij}C\hat{x}\|^2}{\|C\hat{x}\|^2}N^2 \quad (15)$$

where $\hat{x}$ is the estimate of x and I_{ij} is a diagonal matrix with only one non-zero value equal to 1 at the ij-th diagonal position.

2). Obtaining the local mean of $\hat{e}(i,j)$, $m_e(i,j)$, which is given by

$$m_e(i,j) = \frac{1}{(2P+1)(2Q+1)}\sum_{k=i-P}^{i+P}\sum_{l=j-Q}^{j+Q}\hat{e}(k,l) \quad (16)$$

3). Computing the weights in diagonal matrix R, R_{ij}, given by

$$R_{ij} = \frac{m_e(i,j)}{1+m_e(i,j)} \quad (17)$$

Table 1: SNR Improvement for the Image Degraded by the Defocusing Blur, and with White Gaussian Noise Added of Various BSNRs.

BSNR /dB	Adaptive algorithm proposed in [5]	The proposed algorithm
20	2.6 dB	3.0 dB
30	4.1 dB	5.0 dB
40	6.1 dB	7.4 dB

Table 2: SNR Improvement for the Image Blurred by the Horizontal Motion, and with White Gaussian Noise Added of Various BSNRs.

BSNR /dB	Adaptive algorithm proposed in [5]	The proposed algorithm
20	4.1 dB	5.2 dB
30	6.3 dB	7.7 dB
40	7.8 dB	8.8 dB

In all experiments, the constraint operator C was set to be discrete Laplacian and β is equal to 1. P and Q are equal to 1 in computing weights for matrices R and S. The iterations for restoration were terminated when the changes in restoration result were smaller than a threshold value.

The experimental results obtained are reported in Table 1 and Table 2. It is shown in terms of SNR improvement that the proposed algorithm produces a better restored image compared to those produced by the adaptive algorithm using solely space-variant smoothing operator. By visual inspection, it is also shown that the restored images obtained by the proposed algorithm is smoother in flat regions, while it still preserves the sharp edges of the restored images. Figure 1 shows the experimental results for the horizontal motion blur with 20dB BSNR case. Figure 1(a) and 1(b) shows respectively the original image and the degraded image. Figure 1(c) shows the restored image obtained by the adaptive restoration algorithm proposed in [5] and, finally, Figure 1(d) shows the restored image obtained by the proposed algorithm.

4. CONCLUSIONS

In this paper, an adaptive iterative image restoration algorithm is proposed. The formulation of the algorithm is based on the set theoretic approach to the image restoration problem. Unlike other proposed adaptive regularized restoration algorithms in the literature [3-6], the proposed algorithm adopts both techniques of space-variant smoothing and space-variant restoration in performing adaptivity of the restoration algorithm. It is found that the proposed algorithm can obtain a restored image with better quality: sharp edges and lines are reconstructed while flat image regions remain smooth. The main differences between the proposed algorithm and the adaptive algorithm formulated in weighted Hilbert space are discussed. Detailed experimental results reporting the performance of the proposed algorithm are given. It is shown in the experiments that the restored images obtained by the proposed algorithm are better in terms of both numerical measurement and visual quality, as compared to those obtained by adaptive regularized restoration algorithms proposed in the literature.

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(a)

(b)

(c)

(d)

Figure 1

(a) The original image; (b) motion-blurred image with 20 dB BSNR; (c) restored image obtained by the adaptive restoration algorithm proposed in [5]; (d) restored image obtained by the proposed algorithm.