Adaptive Image Noise Filtering Using Transform Domain Local Statistics

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Abstract

Image noise filtering has been widely perceived as an estimation problem in the spatial domain. In this paper, we deal with it as an estimation problem in an uncorrelated transform domain. This idea leads to a generalization of the adaptive LMMSE estimator for filtering noisy images. In our proposed method, the transform-domain local statistics obtained from the noisy image are exploited. Due to the fact that the transform-domain local statistics carry more information about the image than the spatial-domain local statistics do, improvement in noise filtering is gained overall and particularly significant in the vicinity of edges.

Subject terms: *image restoration; image noise smoothing; adaptive LMMSE estimation; local statistics; decorrelation.*

1 Introduction

Image noise filtering, as a fundamental task in image processing, has received significant attention in the image processing literature [1]. A variety of techniques has been proposed and developed over the last two decades to remove noise in digital images. Noise filtering techniques can be broadly classified into *estimation-based methods* and *heuristic methods*. In the first category, noise filtering is considered as an estimation of an ideal image from its distorted rendition, and stochastic formulations of both the problem and the solution are employed in the algorithmic development. The minimum mean-square-error (MMSE) is often applied as the optimality criterion in the estimation. If we impose a linear constraint on the estimator structure, then we have the well-known *linear minimum mean-square-error* (LMMSE) filter as the optimal method for solving the problem.

The LMMSE filter is optimal merely in a theoretical setting based on the knowledge about the statistical properties of an image up to second order. Early techniques for LMMSE filtering assume a wide-sense stationary image model and apply simple and spatially invariant image correlation function in realization [2]-[3]. However, the filters developed accordingly are spatially invariant and blur edges where stationarity is not justified.

[0]To appear in *Optical Engineering*, August 1998. This work was supported by the Hong Kong Polytechnic University Research Grant: Project account 340.813.A3.420

In order to overcome the drawback of blurring edges, various spatially adaptive techniques were proposed for improving the performance of the LMMSE filtering [4]-[7]. These techniques were developed from nonstationary image models and utilize local statistics of the image to make improvement on the filtering performance. A number of refined techniques had also been proposed, such as Refs. [8]-[12], which obtained an improved filtering result by refining the estimation of local statistics of the image.

As alternatives to estimation-based methods, a host of adaptive or nonlinear techniques derived from heuristic approaches have also been proposed in the literature, some of which are highlighted in Refs. [13]-[19]. In general, these techniques operate in the spatial domain and apply certain type of local operation to perform noise smoothing. They would not make use of any specific assumption about signal and noise models. Their common concerns are the suppression of noise corruption and the preservation of image details in the distorted observation.

Although of great variety in the existing image noise filtering techniques, nearly all of them are based on spatial-domain processing of the distorted image. They generally process image data in the spatial domain to diminish the noise while preserve important image details such as edges and lines. Image noise filtering has been widely perceived as the spatial-domain estimation and processing. In this paper, we present a new perspective on image noise filtering. We deal with it as an estimation problem in an uncorrelated transform domain. This idea leads to a generalization of the adaptive local LMMSE filter which includes the well-known *spatially adaptive LMMSE filter* [6],[10] as one of its examples.

In conventional local LMMSE filtering, image pixels are considered to be jointly independent. Instead of minimizing the overall MSE of the image, the MSE of each individual pixel is minimized independently with the use of the statistics about that pixel. The local LMMSE estimator can well approximate the optimal LMMSE estimator when the correlations among image pixels are low. However, it is generally recognized that image pixels are highly correlated. There is no fundamental reason to treat pixels as being jointly independent in devising a noise smoothing filter.

To comply with the fact that images are highly correlated random fields, and at the same time not to complicate the noise filtering process too much, we suggest first decorrelating the image pixels into less-correlated components by making use of the image transform theory [1] and then performing a local LMMSE estimation in the new uncorrelated domain. By doing so, two advantages can be gained. First, since the transform components are less correlated as compared to image pixels, performing local LMMSE estimation in the transform domain can better approximate the optimal LMMSE filter. Another advantage also comes from the decorrelation property of the image transform. The local LMMSE filter in practice necessitates the estimation of the statistical mean and variance of each component under processing. These statistics have to be estimated from the distorted image, and, therefore, there must be some estimation errors. Since in the transform domain the components are less correlated, the estimation of their statistics is less sensitive to noise as compared to that in the spatial-domain. This will lead to a filter that is less sensitive to the estimation error and, as a result, certain amount of improvement can be gained in noise filtering.

In this paper, inspired by the aforementioned idea, we provide a generalized version of the adaptive local LMMSE filter. We eventually come to a noise smoothing filter where filtering is adapted to the local characteristics about the transform components. This filter preserves the simplicity of point processors in the transform domain, and improves the noise filtering performance over the spatially adaptive filter in the neighborhood of edges. Improvement gained by the proposed filter is due to the exploitation of the local statistics in the uncorrelated transform domain rather than those in the spatial-domain.

2 Noise Filtering as an Estimation in Transform Domain

Throughout this paper, a digital image $f(m,n)$, where (m,n) are the spatial coordinations, is represented as a $N\times 1$ vector $\mathbf{f} = [f_1, f_2, \cdots, f_N]^t$ by lexicographical ordering. Matrix-vector notation will be used in our formulation for the sake of simplicity.

Consider the observation equation

$$\mathbf{g} = \mathbf{f} + \mathbf{n}, \tag{1}$$

where $\mathbf{g}$ is the degraded observation, $\mathbf{f}$ is the ideal image, and $\mathbf{n}$ is a zero-mean white noise with covariance matrix $\mathbf{C}_n = \sigma_n^2\mathbf{I}$. The LMMSE estimation of $\mathbf{f}$ is well-known to be [1]

$$\hat{\mathbf{f}} = E(\mathbf{f}) + \mathbf{C}_f(\mathbf{C}_f + \mathbf{C}_n)^{-1}\left[\mathbf{g} - E(\mathbf{g})\right], \tag{2}$$

where $E(\bullet)$ is the expectation operator and $\mathbf{C}_f$ is the covariance matrix of $\mathbf{f}$.

The image covariance matrix is usually over-simplified to be diagonal in order to reduce the complexity of the LMMSE estimator. This simplification is accompanied by the assumption that images are white (statistically uncorrelated) random fields [10]. Suppose $\sigma_{f_i}^2$ denotes the ensemble variance of $\mathbf{f}$ at spatial position $i = (m,n)$. The covariance matrix of $\mathbf{f}$ is then given by

$$\mathbf{C}_f = \mathrm{diag}\left(\sigma_{f_1}^2, \sigma_{f_2}^2, \cdots, \sigma_{f_N}^2\right). \tag{3}$$

This diagonal form of covariance matrix simplifies the LMMSE estimator from the matrix-vector processing to the following scalar processing

$$\hat{f}_i = E(f_i) + \frac{\sigma_{f_i}^2}{\sigma_{f_i}^2 + \sigma_n^2}\left[g_i - E(g_i)\right], \tag{4}$$

where f_i, g_i and $\hat{f}_i$ are respectively the i-th element of $\mathbf{f}$, $\mathbf{g}$ and $\hat{\mathbf{f}}$. Therefore, by imposing a white assumption on $\mathbf{f}$, the LMMSE estimation of $\mathbf{f}$ decomposes into independent LMMSE estimations of f_i's. This form of LMMSE estimation in essence minimizes the local MSE rather than the overall MSE. Because of its local nature in estimation, (4) is commonly referred to as *local* LMMSE filter [10]. To emphasize that this local LMMSE estimation is performed in the spatial domain, in this paper we would call it *spatial local* LMMSE filter.

Although the assumption that images are white random fields can simplify the LMMSE estimator to a very desirable structure, it goes against the common recognition that images are highly correlated random fields [11]. To remedy the situation, in the following part of this section, we will formulate image noise filtering as a LMMSE estimation in an uncorrelated transform domain instead of the highly-correlated spatial domain.

Let us define the residual of the ideal image as

$$\mathbf{f}'=\mathbf{f} - E(\mathbf{f}). \tag{5}$$

Suppose that matrix $\mathbf{T}$ represents a unitary transformation that can decorrelate $\mathbf{f}'$. Here, decorrelating $\mathbf{f}'$ means making elements of $\mathbf{Tf}'$ much less correlated than those of $\mathbf{f}'$. By applying $\mathbf{T}$ to $\mathbf{f}'$, we have

$$\mathbf{F}' = \mathbf{F} - E(\mathbf{F}), \tag{6}$$

where $\mathbf{F} = \mathbf{Tf}$ and $\mathbf{F}' = \mathbf{Tf}'$. Note that $\mathbf{F}'$ rather than $\mathbf{f}'$ is more appropriate to be modeled as a white process. Then, the covariance matrix of $\mathbf{F}$ can be well approximated by

$$\mathbf{C}_F = \mathrm{diag}\left(\sigma^2_{F_1}, \sigma^2_{F_2}, \cdots, \sigma^2_{F_N}\right), \tag{7}$$

where $\sigma^2_{F_i}$ represents the variance of the i-th element of $\mathbf{F}$.

Similarly, let us define the residual of the observed image as

$$\mathbf{g}'=\mathbf{g} - E(\mathbf{g}). \tag{8}$$

Applying $\mathbf{T}$ to $\mathbf{g}'$, together with the knowledge derived from (1) that $E(\mathbf{g}) = E(\mathbf{f})$, we have

$$\mathbf{G}' = \mathbf{G} - E(\mathbf{G}) = \mathbf{F}' + \mathbf{N}, \tag{9}$$

where $\mathbf{G} = \mathbf{Tg}$, $\mathbf{G}' = \mathbf{Tg}'$ and $\mathbf{N} = \mathbf{Tn}$.

The LMMSE estimate of $\mathbf{F}$ is given as

$$\hat{\mathbf{F}} = E(\mathbf{F}) + \mathbf{C}_F(\mathbf{C}_F + \mathbf{C}_N)^{-1}\left[\mathbf{G} - E(\mathbf{G})\right]. \tag{10}$$

Here, $\mathbf{C}_N = E\{(\mathbf{Tn})(\mathbf{Tn})^{\mathbf{t}}\} = \mathbf{T}\mathbf{C}_n\mathbf{T}^{\mathbf{t}} = \mathbf{T}\sigma_n^2\mathbf{I}\mathbf{T}^{\mathbf{t}} = \sigma_n^2\mathbf{I}$. Since both $\mathbf{C}_F$ and $\mathbf{C}_N$ are diagonal, the above matrix-vector expression can be simplified to the following scalar form

$$\hat{F}_i = E(F_i) + \frac{\sigma^2_{F_i}}{\sigma^2_{F_i} + \sigma^2_n}\left[G_i - E(G_i)\right], \tag{11}$$

where F_i, G_i and $\hat{F}_i$ denote respectively the i-th element of $\mathbf{F}$, $\mathbf{G}$ and $\hat{\mathbf{F}}$. Having the estimate $\hat{\mathbf{F}}$, the estimate of $\mathbf{f}$ is then obtained with $\hat{\mathbf{f}} = \mathbf{T}^{-1}\hat{\mathbf{F}}$. Different from the spatial local LMMSE estimator (4), the estimator (11) is "transform-domain local" in the sense that the estimation of $\mathbf{f}$ is achieved via the local LMMSE estimation of each transform component. In view of this, we term it as *transform-domain local LMMSE filter.*

The reason for decorrelating the image before performing the estimation is, as we have stated before, to make the elements to be estimated uncorrelated with each other so that the LMMSE estimation can be simplified to be a local LMMSE estimation. This simplification is based on the assumption that $\mathbf{F}'$ is a white random field. The validity of this assumption depends on the choice of transform $\mathbf{T}$. Theoretically, the optimal $\mathbf{T}$ is the Karhunen-Loeve transform (KLT) [1]. However, from a practical point of view, the KLT is not recommended as performing a KLT is computationally very expensive. Appropriate alternatives are those transforms which have fast realization algorithms and yet can decorrelate images to a large extent. For instance, the discrete cosine transform (DCT) is such an appropriate candidate [20]. Throughout our empirical justification and simulation studies, the decorrelation transform $\mathbf{T}$ would be approximated with a periodic 8×8 two-dimensional DCT kernel. Specifically, to decorrelate an image, we first partition it into a number of non-overlapped subimages of size 8×8 and then performed an 8×8 DCT to each of them. We exploit this block-based DCT as it can take advantage of the spatial local characteristics of an image and requires little computational effort. Hereafter, we use $\mathbf{T}_B$ to denote this block-based DCT.

3 Transform Domain Local Statistics

The implementation of the proposed transform-domain local LMMSE filter necessitates the estimation of the unknown ensemble statistics, namely, $E(F_i)$ and $\sigma^2_{F_i}$. Before presenting our approach to this problem, it behooves us to review the conventional solution for the spatial local LMMSE filter.

In implementing the spatial local LMMSE filter shown in (4), the required ensemble statistics are usually replaced with the statistics obtained by a spatial averaging over a uniform window. Specifically, the ensemble mean $E(f_i)$ is replaced with

$$\bar{f}_i = \bar{f}(m,n) = \frac{1}{(2L+1)^2} \sum_{p=-L}^{L} \sum_{q=-L}^{L} f(m+p, n+q) \tag{12}$$

and the ensemble variance $\sigma^2_{f_i}$ is replaced with

$$\nu_{f_i} = \nu_{f(m,n)} = \frac{1}{(2L+1)^2} \sum_{p=-L}^{L} \sum_{q=-L}^{L} [f(m+p, n+q) - \bar{f}(m,n)]^2. \tag{13}$$

In above, $(2L+1)^2$ is the extent of the analysis window. The above two statistics, $\bar{f}_i$ and ν_{f_i}, are widely used in image enhancement and restoration [1],[6],[8],[10],[12]. However, in the literature there is an inconsistency in terming them. They are termed as "sample", "local" or "local spatial" statistics. Throughout this paper, we would use the term "spatial-domain local statistics" to refer to them. Having the ensemble statistics replaced with the spatial-domain local statistics, the spatial local LMMSE filter becomes

$$\hat{f}_i = \bar{f}_i + \frac{\nu_{f_i}}{\nu_{f_i} + \sigma_n^2} \left[g_i - \bar{g}_i \right]. \tag{14}$$

This filter is effectively an adaptive filter where image noise filtering is adapted to the spatial-domain local statistics, and it is commonly referred to as *spatially adaptive* LMMSE filter.

As for the proposed transform-domain local LMMSE filter, the information required is the ensemble statistics (mean and variance) of the transform coefficient F_i. In the following we would define "transform-domain local statistics", which are computed from a single image. We then use them to replace the ensemble statistics of F_i. To distinguish the local statistics from the ensemble statistics, the local mean and variance are respectively denoted as $\bar{F}_i$ and Υ_{F_i}. Our idea on the local statistics of the transform coefficients is illustrated in the following.

Let $\mathbf{f}^{<m,n>}$ denote the shift version of $\mathbf{f}$ obtained by shifting all its elements m steps up and n steps right in the spatial domain. Its transform, $\mathbf{T}(\mathbf{f}^{<m,n>})$, is denoted as $\mathbf{F}^{<m,n>}$. The transform-domain local mean $\bar{F}_i$ and local variance Υ_{F_i} are defined as

$$\bar{F}_i = \frac{1}{(2L+1)^2} \sum_{m=-L}^{L} \sum_{n=-L}^{L} F_i^{<m,n>} \tag{15}$$

$$\Upsilon_{F_i} = \frac{1}{(2L+1)^2} \sum_{m=-L}^{L} \sum_{n=-L}^{L} \left[F_i^{<m,n>} - \bar{F}_i\right]^2 . \tag{16}$$

Here, $(2L+1)^2$ is the total number of shifted images used in obtaining the statistics.

It is worthwhile to note that, when $\mathbf{T}$ is particularly set to be the identity matrix, $\bar{F}_i$ and Υ_{F_i} are respectively equivalent to the spatial-domain local mean and variance. Hence, the spatial-domain local statistics are only special cases of the transform-domain local statistics we defined.

By replacing $E(F_i)$ and $\sigma^2_{F_i}$ in (11) with $\bar{F}_i$ and Υ_{F_i}, we have a filter which adapts filtering to the transform-domain local statistics. Explicitly, it is written as

$$\hat{F}_i = \bar{F}_i + \frac{\Upsilon_{F_i}}{\Upsilon_{F_i} + \sigma_n^2} \left(G_i - \bar{G}_i\right) . \tag{17}$$

Note that this form of adaptive filter includes the spatially adaptive LMMSE filter stated in (14) as a special case. Specifically, when the concerned transform is set to be the identity transform, (17) is just equivalent to (14). We finish up generalizing the spatially adaptive LMMSE filter to transform domain. For the sake of reference, we hereafter term (17) as *transform-domain adaptive* LMMSE filter.

4 Empirical Justification

Before discussing the matters of practical realization, we first use a few empirical studies to justify that the idea we have brought forward is useful for image noise filtering.

In the first place, let us use an example to illustrate that the transform-domain local statistics (TDLS) carry more information about an image as compared to the spatial-domain local statistics (SDLS). Figure 1(a) shows a testing image for our example. Figure 1(b) is its residual where the information carried by the SDLS were deducted from the testing image. Explicitly, its i-th pixel is given as $(f_i - \bar{f}_i)/\nu_{f_i}$ with L set to be 1 in computing $\bar{f}_i$ and ν_{f_i}. This deduction transforms some regions of the image into white noise. However, in edge regions, substantial visible correlations still exist. This indicates that using the SDLS to describe an image is inadequate in the vicinity of edges. In order to demonstrate that the TDLS can retain more information about the image, we first apply $\mathbf{T}_B$ to decorrelate the testing image and then deduct the information carried by the TDLS from the testing image. The i-th transform component of the residual is given as $(F_i - \bar{F}_i)/\Upsilon_{F_i}$ with L set to be 1 in computing $\bar{F}_i$ and Υ_{F_i}. Figure 1(c) shows the deduction result in the spatial domain. It is observed that edges retained in Figure 1(c) are less visible as compared with those in Figure 1(b). This implies that more structural information about the image is carried by the TDLS than by the SDLS.

Next, we use two noise filtering examples to demonstrate that the adaptive filter using TDLS can perform better than that using SDLS. A distorted image, shown in Figure 2(b), was produced by adding white noise of variance $\sigma_n^2 = 400.0$ to the testing image shown in Figure 2(a). Noise smoothing was then carried out by implementing (17) with L set to be 1. Note that we use the undistorted image to compute $\bar{F}_i$'s and Υ_{F_i}'s with (15) and (16). Figure 2(c) shows the filtered image obtained with $\mathbf{T} = \mathbf{T}_B$ while Figure 2(d) shows that with $\mathbf{T} = \mathbf{I}$. Note that the latter is actually the result provided by the spatially adaptive filter shown in (14). It was found that the adaptive filter using TDLS outperforms that using SDLS. Their performance difference is substantial especially in the vicinity of edges. Figure 3 shows the same set of filtering results of another testing image. These experimental results confirm that the TDLS carry more information about an image and are therefore more useful for effective image noise smoothing.

5 Practical Considerations

In practical cases, since the undistorted image is unavailable, the statistics $\bar{F}_i$ and Υ_{F_i} have to be estimated from the noisy data. In this section, we discuss how to approximate $\bar{F}_i$ and Υ_{F_i} solely based on the given distorted image $\mathbf{g}$.

From (1), we have

$$G_i^{<m,n>} = F_i^{<m,n>} + N_i^{<m,n>}. \tag{18}$$

Hence, by definition, the transform-domain local mean and variance of $\mathbf{g}$, denoted as $\bar{G}_i$ and Υ_{G_i} respectively, are given as

$$\bar{G}_i = \bar{F}_i + \bar{N}_i, \tag{19}$$

$$\Upsilon_{G_i} = \Upsilon_{F_i} + \Upsilon_{N_i} - 2\xi_i \tag{20}$$

where

$$\xi_i = \frac{1}{(2L+1)^2} \sum_{m=-L}^{L} \sum_{n=-L}^{L} \left\{ (F_i^{<m,n>} - \bar{F}_i)(N_i^{<m,n>} - \bar{N}_i) \right\}, \tag{21}$$

and $\bar{N}_i$ and Υ_{N_i} are respectively the transform-domain local mean and variance of $\mathbf{n}$. Since $\mathbf{N}$ is zero-mean white, $\bar{N}_i$ should be very close to zero and its value is insignificant as compared with that of $\bar{F}_i$. Hence, we can approximate $\bar{F}_i$ as

$$\bar{F}_i = \bar{G}_i. \tag{22}$$

Based on the assumption that $\mathbf{N}$ is independent of $\mathbf{F}$, it is expected that the value of ξ_i is also much smaller than those of Υ_{F_i} and Υ_{N_i}. Moreover, since $\mathbf{N}$ is stationary with variance σ_n^2, Υ_{N_i} can be well approximated by σ_n^2. Consequently, we can approximate Υ_{F_i} as

$$\Upsilon_{F_i} = \max\left\{\Upsilon_{G_i} - \sigma_n^2, 0\right\}. \tag{23}$$

Note that the maximum value is taken to guarantee the positive nature of the variance.

6 Simulation Results

Here, we present our experimental results where all local statistics required were estimated from the distorted image itself.

For the distorted image shown in Figure 2(b), the filtering result obtained with the *transform-domain adaptive filter* (TAF) is shown in Figure 4(a), while the result of the *spatially adaptive filter* (SAF) is shown in Figure 4(b). For the distorted image shown in Figure 3(b), the two filtered images obtained respectively with TAF and SAF are shown in Figures 5(a) and 5(b).

For objective comparison, we provide in Table 1 the signal-to-noise ratio improvement (SNRI) of the above filtered images. To offer a detailed quantitative comparison, we also segmented the image into edge and level regions and then computed the SNRIs in these two regions separately. These figures reveal that, although SAF has a better SNRI in the level regions, its performance in the edge regions is poor; whereas, for TAF, there is a large SNRI in the edge regions. Since the SNRI is usually criticized to be an inappropriate measure in evaluating the image restoration performance, we also used the performance measure proposed in [21] in our comparative study. This measure is termed *Restoration Score* (RS) and was found to be more appropriate than the SNRI as a performance index for image restoration. The measurement is based on the weighted sum of fidelity improvement of each image pixel, and the main properties of the human visual system are incorporated through the weighting factors. The RS's of the performed filtering experiments are reported in Table 1. It is also shown that TAF is superior to SAF.

By visual inspection, one can get an overall impression that Figures 4(a) and 5(a) are of visually better quality than Figures 4(b) and 5(b). When looking into detail, one can observe some annoying artifacts in the neighborhood of the edges in Figures 4(b) and 5(b). Whereas, in Figure 4(a) and 5(a), these artifacts are reduced and sharp edges are preserved. This subjective observation along with the above objective comparison justifies that TAF does a better job of image noise smoothing.

Finally, we remark that all the experimental results presented in this paper were obtained with $L = 1$. From our other noise filtering experiments which applied larger values of L, such as $L = 2$ and $L = 3$, we got the following observation. By increasing the value of L, although noise smoothing is improved to some extent, the performance on preservation of image details is not so good as that with smaller value of L. In view of this and the fact that increasing L

in the estimation will also increase the complexity of the filtering algorithm, it is in general not worthwhile to apply a larger value of L.

7 Conclusions

Image noise filtering has been widely perceived as an estimation problem in the spatial domain. However, the fact that image pixels are highly correlated makes filtering techniques based on spatial-domain estimation ineffective. In this paper, we tackled the noise filtering problem as an estimation in an uncorrelated transform domain. We then formulated a local LMMSE estimation of the uncorrelated image components, and ended up with a *transform-domain adaptive* LMMSE filter. The potential superiority of the proposed approach over the conventional spatial-domain approaches has been demonstrated through a number of experiments. The improvement gained by the proposed approach is due to the exploitation of the *transform-domain local statistics* rather than the *spatial-domain local statistics.*

The main idea we put forward in this paper is that image decorrelation can help to improve the performance of image noise filtering. The introduction of image decorrelation provides an additional means to incorporate useful *a priori* information about the solution into the filtering process, and therefore offers a valuable opportunity to improve the filtering performance. The proposed approach to noise filtering is derived from this idea. The noise filtering algorithm we devised, however, represents just a simple paradigm of the proposed approach, in which the computation of the transform-domain local statistics is a natural adaptation from the spatial-domain. As in spatially adaptive LMMSE filtering techniques, where filtering performance can be improved by refining the estimation of the spatial-domain local statistics, there is still room for advancement of the proposed transform-domain adaptive LMMSE filtering.

References

[1] A. K. Jain, *Fundamentals of Digital Image Processing,* Prentice-Hall, Englewood Cliffs, New Jersey, (1989).

[2] W. K. Pratt, 'Generalized Wiener filtering computation techniques,' *IEEE Trans. Comput.*, **C-21**, 636-641 (1972).

[3] W. K. Pratt, *Digital Image Processing*, Wiley, New York, (1978).

[4] B. R. Hunt and T. M. Cannon, 'Nonstationary assumptions for Gaussian models of images,' *IEEE Trans. Syst., Man, Cybern.*, **SMC-6**, 876-881, (1976).

[5] H. T. Trussell and B. R. Hunt, 'Sectioned methods for image restoration,' *IEEE Trans. Acoust., Speech, Signal Processing*, **ASSP-26**, 157-164, (1978).

[6] J. S. Lee, 'Digital image enhancement and noise filtering by use of local statistics,' *IEEE Trans. Pattern Anal. Mach. Intell.,* **PAMI-2**(2), 165-168 (1980).

[7] S. A. Rajala and R. J. P. de Figueiredo, 'Adaptive nonlinear image restoration by a modified Kalman filtering approach,' *IEEE Trans. Acoust., Speech, Signal Processing*, **ASSP-29**, 1033-1042, (1981).

[8] J. S. Lee, 'Refined filtering of images using local statistics,' *Computer Graphics and Image Processing,* **15**, 380-389 (1981).

[9] P. Chan and J. S. Lim, 'One-dimensional processing for adaptive image restoration,' *IEEE Trans. Acoust., Speech, Signal Processing,* **ASSP-33**(2), 117-125 (1985).

[10] D. T. Kuan, A. A. Sawchuk, T. C. Strand and P. Chavel, 'Adaptive noise smoothing filter for image with signal-dependent noise,' *IEEE Trans. Pattern Anal. Mach. Intell.,* **PAMI-7**(2), 165-177 (1985).

[11] F.-C. Jeng and J. W. Woods, 'Inhomogeneous Gaussian image models for estimation and restoration,' *IEEE Trans. Acoust., Speech, Signal Processing,* **ASSP-36**(8), 1305-1312 (1988).

[12] B. Mahesh, W. J. Song and W. A. Pearlman, 'Adaptive estimators for filtering noisy images,' *Opt. Eng.,* **29**(5), 488-494 (1990).

[13] L. S. Davis and A. Rosenfeld, 'Noise cleaning by iterated local averaging,' *IEEE Trans. Systems Man Cybernet.*, **SMC-8**, 705-710, (1978).

[14] D. C. C. Wang and A. H. Vagnucci and C. C. Li, 'Gradient inverse weighted smoothing scheme and the evaluation of its performance,' *Compt. Graphics Image Processing*, **15**, 167-181, (1981).

[15] A. C. Bovick and T. S. Huang and D. C. Munson, 'A generalization of median filtering using linear combinations of order statistics,' *IEEE Trans. Acoust., Speech, Signal Processing*, **ASSP-31**, 1342-1350, (1983).

[16] Y. H. Lee and A. S. Kassam, 'Generalized median filtering and related nonlinear filtering techniques,' *IEEE Trans. Acoust., Speech, Signal Processing*, **ASSP-33**, 672-683, (1985).

[17] J. S. Lee, 'Digital image smoothing and the sigma filter,' *Computer Vision Graphics Image Processing*, **24**, 255-269, (1983).

[18] R. T. Chin and C.-L. Yeh, 'Quantitative evaluation of some edge-preserving noise-smoothing techniques,' *Computer Vision Graphics Image Processing*, **23**, 67-91, (1983).

[19] G. A. Mastin, 'Adaptive filters for digital image noise smoothing: an evaluation,' *Comput. Vision Graphics Image Processing,* **31**, 103-121 (1985).

[20] K. R. Rao and P. Yip, *Discrete Cosine Transform: Algorithms, Advantages and Applications,* Academic Press, (1990).

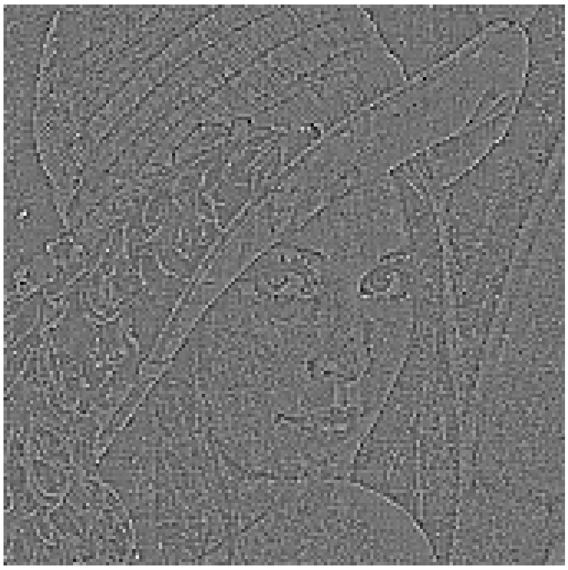
[21] S. S. O. Choy, Y.-H. Chan and W.-C. Siu, 'New image restoration performance measures with high precision,' *Opt. Eng.,* **36**(4), 1035-1043 (1997).

Image	SNR Improvement			Restoration Score [21]	
	Image Region	Filtering Using SDLS	Filtering Using TDLS	Filtering Using SDLS	Filtering Using TDLS
Figure 2(b)	overall	2.83 dB	3.71 dB	0.250	0.436
	edge	-0.16 dB	2.23 dB		
	level	7.33 dB	4.27 dB		
Figure 3(b)	overall	4.28 dB	4.25 dB	0.418	0.476
	edge	2.39 dB	3.15 dB		
	level	6.95 dB	5.48 dB		

Table 1: Performance Measurements of the Adaptive Filtering Using SDLS and TDLS.

(a) Original Image

(b) image where spatial-domain local statistics are deducted



(c) image where transform-domain local statistics are deducted

Figure 1:

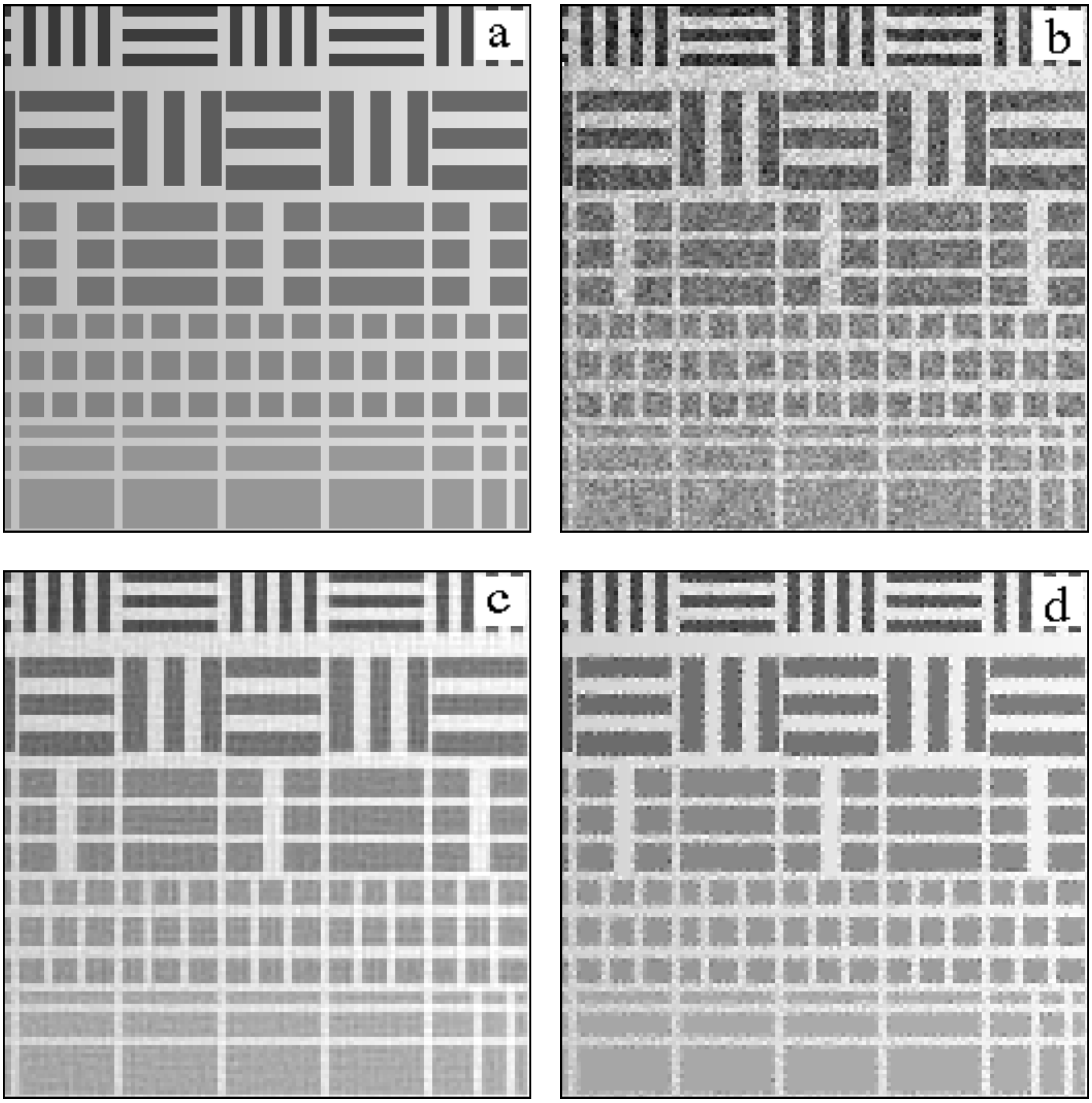


Figure 2: Adaptive noise filtering results for a testing chart, where local statistics were obtained from the undistorted image. **(a):** original image; **(b):** distorted image; **(c):** result of adaptive filtering using transform-domain local statistics; **(d):** result of adaptive filtering using spatial-domain local statistics.

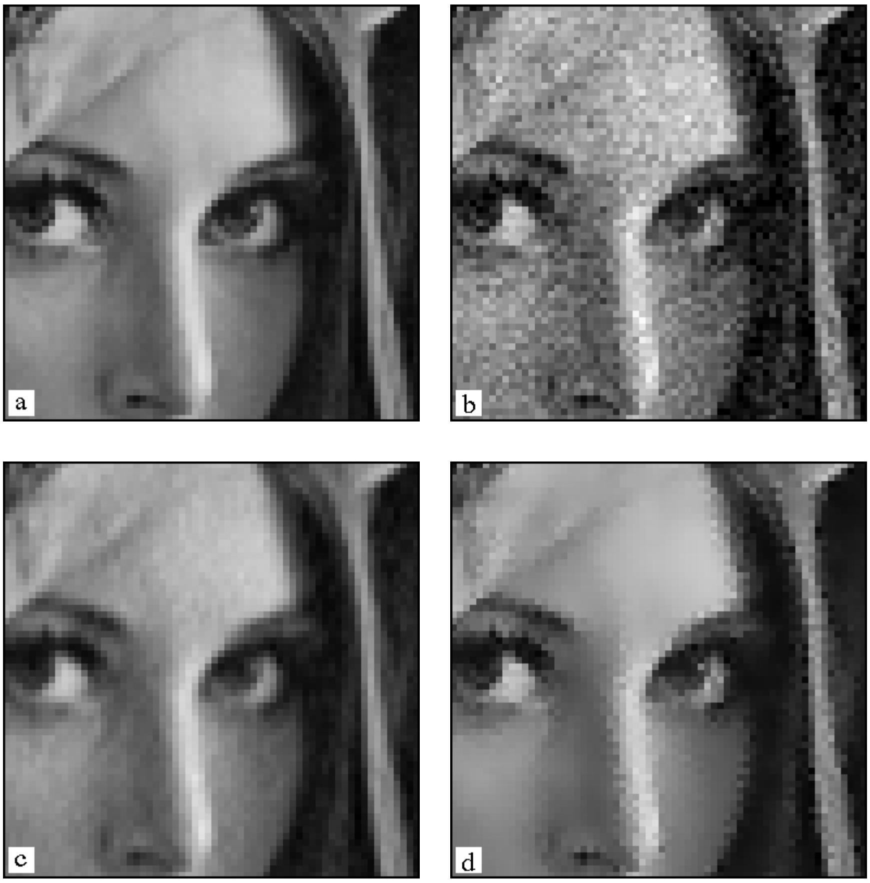


Figure 3: Adaptive noise filtering results for image ‘lenna’, where local statistics were obtained from the undistorted image. **(a):** original image; **(b):** distorted image; **(c):** result of adaptive filtering using transform-domain local statistics; **(d):** result of adaptive filtering using spatial-domain local statistics.

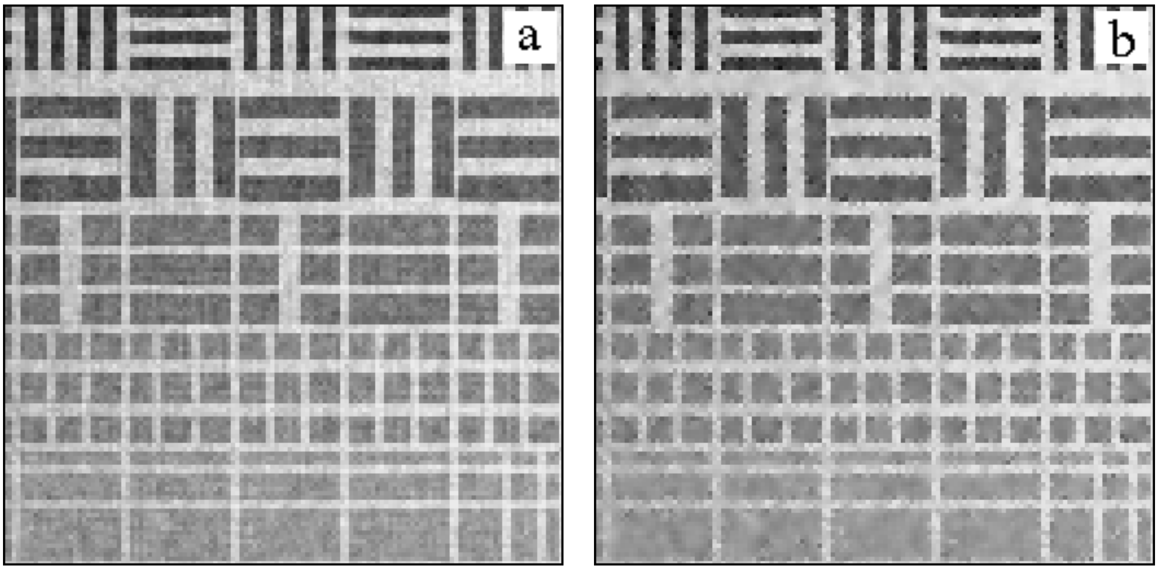


Figure 4: Practical results of adaptive noise filtering for a testing chart, where local statistics were estimated from the distorted image. **(a):** result of adaptive filtering using transform-domain local statistics; **(b):** result of adaptive filtering using spatial-domain local statistics.

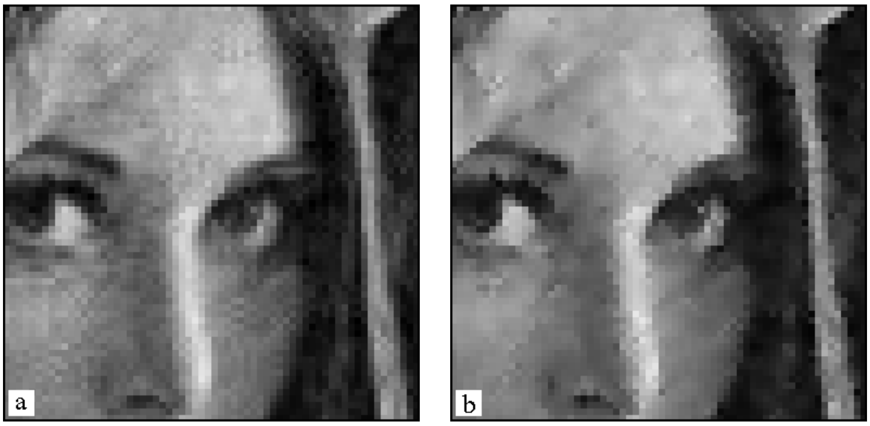


Figure 5: Practical results of adaptive noise filtering for image 'lenna', where local statistics were estimated from the distorted image. **(a):** result of adaptive filtering using transform-domain local statistics; **(b):** result of adaptive filtering using spatial-domain local statistics.