Restoring Halftoned Color-quantized Images with Simulated Annealing

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ABSTRACT

Restoration of color-quantized images is rarely addressed in the literature especially when the images are color-quantized with halftoning. Direct applications of existing restoration techniques are generally inadequate to deal with this problem. In this paper, a restoration algorithm based on simulated annealing is proposed to solve the problem. This algorithm makes a good use of the available color palette and the mechanism of a halftoning process to derive useful *a priori* information for restoration. It is shown to be able to provide a good restoration result.

1. INTRODUCTION

Color quantization is the process of reducing the number of colors in a digital image by replacing them with a representative color selected from a palette [1,2]. It is widely used nowadays as it lessens the burden of massive image data on storage and transmission bandwidth that are bottlenecks in many multimedia applications. When color quantization is performed, certain types of degradation are introduced due to the limited colors used to produce the output image. There are two most disturbing defects. One is the false contour which appears in smoothly changing regions and the other is the color shift in the color-quantized images. The smaller the color palette size, the more severe the defects are. Digital halftoning [3-6] would be helpful to eliminate these defects by making use of the fact that human eyes act as spatial low-pass filters. At the moment, the most popular halftoning method is error diffusion and several well-known error diffusion filters such as Floyd–Steinberg's filter [3] are generally used to achieve the goal.

Color quantization is a kind of degradation to the original full-color image. Image restoration is sometimes necessary for one to recover the original image from its color-quantized version. However, though there are a lot of reported works on the restoration of noisy and blurred color images [7]-[14], little effort has been seen in the literature for restoring halftoned color-quantized images. Obviously, the degradation models of the two cases are completely different and hence direct adoption of conventional restoration algorithms does not work effectively.

Simulated annealing [15] is a general adaptive heuristic and belongs to the class of non-deterministic algorithm. As compared with some other conventional methods, it accepts solution with deteriorated cost to a limited extent. This feature gives the heuristic the capability to escape from the local minimum. This paper is devoted to formulating the process of color quantization when error diffusion is involved and developing a simulated annealing restoration algorithm to restore corresponding degraded images.

2. IMAGE DEGRADATION IN COLOR QUANTIZATION WITH HALFTONING

A color image $\mathbf{X}$ generally consists of three color planes, say, $\mathbf{X_r}$, $\mathbf{X_g}$ and $\mathbf{X_b}$, which represents the red, the green and the blue color planes of the image respectively. Accordingly, the $(i,j)^{\text{th}}$ color pixel of a 24-bit full color image of size $N\times N$ consists of three color components. The intensity values of these three components form a 3D vector $\vec{\mathbf{X}}_{(i,j)}=(\mathbf{X}_{(i,j)r},\mathbf{X}_{(i,j)g},\mathbf{X}_{(i,j)b})$, where $\mathbf{X}_{(i,j)c}\in[0,1]$ is the intensity value of the c^{th} color component of the $(i,j)^{\text{th}}$ pixel. Here, we assume that the maximum and the minimum intensity values of a pixel are, respectively, 1 and 0.

Figure 1 shows the system which performs color quantization with error diffusion. The input image $\mathbf{X}$ is scanned in a row-by-row fashion from top to bottom and left to right. The relationship between the original image $\mathbf{X}$ and the encoded image $\mathbf{Y}$ is described by

$$\mathbf{U}_{(i,j)c}=\mathbf{X}_{(i,j)c}-\sum_{(k,l)\in S}\mathbf{H}_{(k,l)c}\mathbf{E}_{(i-k,j-l)c} \tag{1}$$

$$\vec{\mathbf{Y}}_{(i,j)}=Q_c[\vec{\mathbf{U}}_{(i,j)}] \tag{2}$$

and
$$\vec{\mathbf{E}}_{(i,j)}=\vec{\mathbf{Y}}_{(i,j)}-\vec{\mathbf{U}}_{(i,j)}=Q_c[\vec{\mathbf{U}}_{(i,j)}]-\vec{\mathbf{U}}_{(i,j)} \tag{3}$$

where $\vec{\mathbf{U}}_{(i,j)}$ is a state vector of the system, $\vec{\mathbf{E}}_{(i,j)}$ is the quantization error of the pixel at position (i,j) and $\mathbf{H}_{(k,l)c}$ is a coefficient of the error diffusion filter for the c^{th} color component. S is the corresponding casual supported region of $\mathbf{H}_{(k,l)c}$.

The operator $Q_c[\bullet]$ performs a 3D vector quantization. Specifically, the 3D vector $\vec{\mathbf{U}}_{(i,j)}$ is compared with a set of representative color vectors stored in a previously generated color palette $C=\{\hat{\mathbf{v}}_i: i=1,2,\ldots N_c\}$. The best-matched vector in the palette is selected based on the minimum Euclidean distance criterion. In other words, a state vector $\vec{\mathbf{U}}_{(i,j)}$ is

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represented by color $\hat{\mathbf{v}}_k$ if and only if $\|\vec{\mathbf{U}}_{(i,j)} - \hat{\mathbf{v}}_k\| \leq \|\vec{\mathbf{U}}_{(i,j)} - \hat{\mathbf{v}}_j\|$ for all j=1,2,... N_c. Once the best-matched vector is selected from the color palette, its index is recorded and the quantization error $\vec{\mathbf{E}}_{(i,j)} = \hat{\mathbf{v}}_k - \vec{\mathbf{U}}_{(i,j)}$ is diffused to pixel (i,j)'s neighborhood with eqn. (1). Note that, to handle the boundary pixels, $\vec{\mathbf{E}}_{(i,j)}$ is defined to be zero when (i,j) falls outside the image. The recorded indices will be used in the future to reconstruct the color-quantized image with the same color palette.

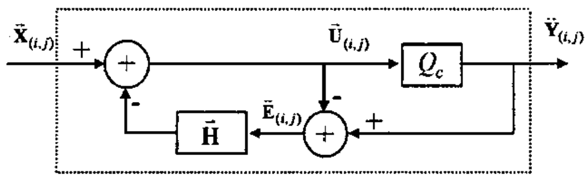

Figure 1. Color quantization with halftoning

3. FORMULATION OF THE PROPOSED ALGORITHM

Let $\mathbf{S}$ be the output of the restoration. Obviously, the restored image $\mathbf{S}$ must be equal to $\mathbf{Y}$ if it is color-quantized with error diffusion. In formulation, we have

$$\mathbf{Y} = Q_{ch}[\mathbf{S}] \tag{4}$$

where $Q_{ch}[\bullet]$ denotes the operator which performs color quantization with halftoning as shown in Figure 1. Accordingly, the cost function of a restored image can be defined as

$$E = \Sigma[\mathbf{Y} - Q_{ch}[\mathbf{S}]] \tag{5}$$

where $\Sigma[\mathbf{I}]$ denotes the total number of nonzero elements in image $\mathbf{I}$.

In our approach, $\mathbf{S}$ is searched with the simulated annealing algorithm to minimize the cost function E. The simulated annealing algorithm is a double-loop iterative algorithm that simulates the annealing process at a given temperature T. The core part of the algorithm is the Metropolis procedure. During the iteration of simulated annealing algorithm, the temperature T is reduced in a controlled manner as given by

$$T_{k+1} = \alpha T_k \tag{6}$$

where T_k is the temperature at outer iteration k and α is a constant used to achieve cooling. At a particular temperature T_k, the amount of time spent in annealing is gradually changed by

$$M_{k+1} = \beta M_k \tag{7}$$

where M_k is actually the number of inner iterations performed at temperature T_k and β is a constant used to do the adjustment. The algorithm is terminated at temperature T_m when $\sum_{k=0}^{m} M_k$ is larger than a predefined threshold $t_{\max}$. Here, we assume that the simulated annealing process starts at its initial stage k=0.

Let $\mathbf{S}_{cur}$ be the current estimate of the restored image at a particular inner iteration at temperature T_k and E_{cur} be its corresponding cost. The new estimate of the restored image is made with $\mathbf{S}_{cur}$ by

$$\mathbf{S}_{new} = \mathbf{S}_{cur} + \gamma(\mathbf{Y} - Q_{ch}[\mathbf{S}_{cur}]) \tag{8}$$

where γ is a controlling parameter used to control the amount of perturbation applied to the $\mathbf{S}_{cur}$.

The cost of $\mathbf{S}_{new}$, say, E_{new}, is then evaluated with eqn. (5). When $E_{new} < E_{cur}$ happens, $\mathbf{S}_{cur}$ is updated to be $\mathbf{S}_{new}$. Furthermore, if $E_{new} < E_{best}$ happens, where E_{best} is the cost of the best estimate so far ($\mathbf{S}_{best}$), then $\mathbf{S}_{best}$ will be replaced by $\mathbf{S}_{new}$. In formulation, we have

$$\mathbf{S}_{cur} = \begin{cases} \mathbf{S}_{new} & \text{if } E_{new} < E_{cur} \\ \mathbf{S}_{cur} & \text{otherwise} \end{cases} \tag{9}$$

and

$$\mathbf{S}_{best} = \begin{cases} \mathbf{S}_{new} & \text{if } E_{new} < E_{best} \\ \mathbf{S}_{best} & \text{otherwise} \end{cases} \tag{10}$$

When $E_{new} \geq E_{cur}$ happens, Metropolis will accept $\mathbf{S}_{new}$ by updating $\mathbf{S}_{cur}$ to be $\mathbf{S}_{new}$ if $r < e^{(E_{cur} - E_{new})/K_B T}$, where r is a randomly generated value which is uniformly distributed between 0 and 1, T denotes the current temperature and K_B is the Boltzmann constant. This criterion for accepting the new solution is known as the Metropolis criterion. At the beginning, temperature T is high. This permits many uphill moves and provides chances for the solution to leave a local minimum. As temperature T is reduced gradually, fewer and fewer uphill moves are permitted and only downhill moves are allowed eventually.

4. SIMULATIONS

Simulation has been carried out to evaluate the performance of the proposed algorithm on a set of color-quantized images. In our simulation, a number of *de facto* standard 24-bit full-color images of size 256×256 each were used. The images were color-quantized to produce $\mathbf{Y}$'s. The color palette used for quantization was of size 128 colors and was generated with the median cut algorithm [2]. In color quantization, halftoning was performed with error diffusion and the Floyd-Steinberg diffusion filter [3] was used. The proposed restoration algorithm was applied to restore the halftoned color-quantized images ($\mathbf{Y}$'s)

In the realization of the proposed algorithm, both $\mathbf{S}_0$, the initial estimate of $\mathbf{S}$, and $\mathbf{S}_{best}$ were initialized to be the filtered output of the observed image $\mathbf{Y}$. Specifically, a 3×3 Gaussian filter was used to generate $\mathbf{S}_0$ and $\mathbf{S}_{best}$. Parameter T_0 was selected to be $|E_{\mathbf{S}_0} - E_{\mathbf{S'}_0}|/(K_B \log(0.95))$, where $E_{\mathbf{S}_0}$ and $E_{\mathbf{S'}_0}$ were, respectively, the cost of $\mathbf{S}_0$ and $\mathbf{S'}_0$. Here, $\mathbf{S'}_0$ is the first estimate obtained with eqn. (8) based on $\mathbf{S}_0$. This allows reasonable amount of uphill move at the beginning. Parameter α was selected to be 0.9, the middle value of the selection range suggested in [15]. To simplify the algorithm, we selected β to be 1 such that we had $M_k = M$ for all $k \geq 0$. Different combinations of M and γ were evaluated to study their impact to the restoration performance. Figure 2 shows the simulation results of restoring the color-quantized "Lenna" with different combinations of M (=10, 50 and 100) and γ (=0.001, 0.002, 0.005, 0.015, 0.05, 0.2). Similar results were obtained with some other testing images. Here, SNRI is defined as

$$\mathrm{SNRI} = 10\log\frac{\sum_{(i,j)}\left\|\vec{\mathbf{X}}_{(i,j)} - \vec{\mathbf{Y}}_{(i,j)}\right\|^2}{\sum_{(i,j)}\left\|\vec{\mathbf{X}}_{(i,j)} - \vec{\mathbf{S}}_{best(i,j)}\right\|^2} \qquad (11)$$

where $\vec{\mathbf{X}}_{(i,j)}$, $\vec{\mathbf{Y}}_{(i,j)}$ and $\vec{\mathbf{S}}_{best(i,j)}$ are, respectively, the $(i,j)^{\text{th}}$ pixels of the original, the color-quantized and the restored images. In Figure 2, the dotted lines correspond to the cases of M=10 while the solid lines correspond to the cases of M=50 and 100. For a particular γ, the curve of M=50 overlaps with the curve of M=100. When $\gamma > 0.002$, using different values of M makes very little difference. Another observation we have is that, when $M \geq 50$, the smaller the value of γ, the more iterations are required for the estimate to converge but the better SNRI performance the converged output can be achieved.

Based on the evaluation results we have, it can be found that, for a selection of $t_{\max}$ =1000, the combination of M =50 and γ =0.002 is a reasonable choice for restoring color-quantized images. With this combination, the best SNRI performance can be achieved in around 1000 iterations. It is sufficient for the estimate of the iterative algorithm to converge. If complexity is a critical concern, one may select a combination of M =50 and γ =0.05, which can achieve a SNRI close to the one obtained with the previous selection in less than 100 iterations. In this paper, the presented results of the proposed algorithm were obtained with α =0.9, β =1, $T_0 = |E_{S_0} - E_{S'_0}|/(K_B \log(0.95))$, $t_{\max}$ =1000, M = 50 and γ =0.002.

For comparison, some other restoration algorithms that were originally proposed for restoring noisy and blurred color images were also been evaluated. They were simulated here for comparative study as few schemes had been proposed for restoring color-quantized images and they are typical examples of the type ([7]-[14]). In particular, Galatsanos's algorithm [9] is based on the constrained least square approach and Hunt's algorithm [10] is based on Wiener filtering. There were two algorithms presented in Altunbasak's work [11]. The first one makes use of the correlation among the color components of a pixel while the other one does not. They are, respectively, referred to as KL and IND in [11]. Both algorithms take the colorimetric aspects into account and try to minimize the error in CIELAB space [16].

In realizing Galatsanos's algorithm [9], the noise power of each channel was estimated with the original full-color image. In realizing Hunt's algorithm [10], three separate Wiener filters were used in three different channels and, during the design of the filters, the noise spectrum of each channel was estimated with the original full-color image. Similarly, the original full-color image was used to estimate the power spectra of different channels in realizing Altunbasak's algorithms [11]. In a practical situation, no original image is available and hence all must be estimated from the degraded image. In other words, in practice, the restoration results of [9], [10] and [11] may not as good as those presented in this paper. As it is not necessary to use the original full-color image to extract information for the proposed algorithm, additional credit should be added to the simulation results of the proposed algorithm indeed.

Table 1 shows the SNR improvement (SNRI) achieved by different algorithms. From Table 1, one can see that the performance of the proposed algorithm is better than that of the others. On average, with the proposed algorithm, a SNRI of 7.89 dB in image quality was achieved for images color-quantized with a palette of size 128.

Tables 2 and 3 show the performance of the evaluated algorithms in terms of the CIELAB color difference (ΔE) metric. A well accepted rule of thumb is that color error is visually detectable when $\Delta E > 3$ [11][17]. Table 2 shows the average of the ΔE values of all pixels in a restoration output and Table 3 shows the percentage of pixels whose color error is visually undetectable in a restoration output. Again, one can see that the proposed algorithm is superior to the others.

5. CONCLUSIONS

In this paper we have introduced a restoration algorithm for restoring halftoned color-quantized images. This algorithm makes a good use of the available color palette and the halftoning process to derive useful *a priori* information for restoration. Significant improvement in various aspects can be achieved as compared with other conventional algorithms.

6. ACKNOWLEDGEMENTS

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	SNR Improvement (dB)				
	Proposed	IND [11]	KL [11]	[9]	[10]
Lenna	8.51	4.94	4.54	2.44	3.83
Baboon	4.40	2.69	2.50	0.83	1.95
Boat	5.67	4.07	3.82	1.42	2.72
Peppers	10.21	6.24	6.12	1.79	4.59
Fruits	10.14	5.14	4.69	2.27	3.83
Couple	7.67	4.93	4.57	3.41	3.84
Girl	8.62	5.34	5.07	3.28	3.81
Average	7.89	4.76	4.47	2.21	3.51

Table 1. SNR Improvements of various algorithms

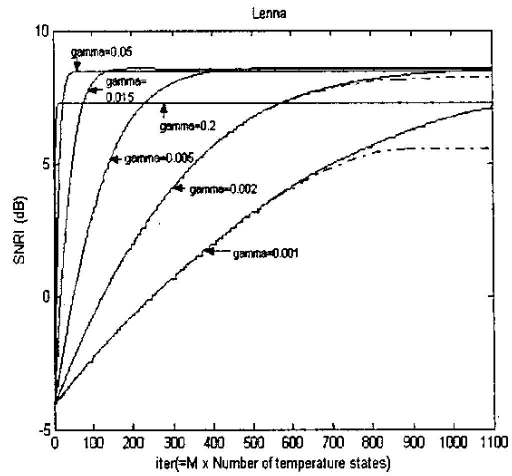

Figure 2. SNR Improvements achieved with various combinations of γ and M in restoring halftoned color-quantized "Lenna"

	Average of CIELAB difference ΔE					
	Observed **Y**	Proposed	IND [11]	KL [11]	[9]	[10]
Lenna	3.96	2.32	2.88	3.01	3.48	3.06
Baboon	5.73	3.94	4.60	4.65	5.49	4.79
Boat	5.15	3.36	3.97	4.03	4.89	4.37
Peppers	6.23	3.72	4.74	4.80	5.81	5.25
Fruits	4.41	2.53	3.29	3.39	4.01	3.57
Couple	10.01	5.07	7.31	7.56	7.81	7.74
Girl	8.46	4.56	6.05	6.20	7.55	6.83
Average	6.28	3.64	4.69	4.81	5.58	5.09

Table 2. CIELAB difference ΔE measurement of the outputs of various algorithms

	% of pixels whose CIELAB ΔE <3					
	Observed **Y**	Proposed	IND [11]	KL [11]	[9]	[10]
Lenna	41.76	76.32	62.18	59.46	48.48	58.32
Baboon	24.37	42.48	32.81	32.29	25.58	31.75
Boat	37.33	56.10	48.33	47.57	39.72	44.61
Peppers	26.59	61.00	43.92	43.11	30.13	35.93
Fruits	50.42	76.20	64.79	63.17	55.27	60.46
Couple	8.572	30.74	17.38	16.41	14.51	14.60
Girl	13.39	37.71	24.72	23.72	19.35	19.81
Average	28.92	54.36	42.02	40.82	33.29	37.93

Table 3. Percentage of pixels whose CIELAB difference ΔE is less than 3 after restoration