

# Reduction of Block-Transform Image Coding Artifacts by Using Local Statistics of Transform Coefficients

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**Abstract**—This letter presents a new approach to reduce coding artifacts in transform image coding. We approach the problem in an estimation of each transform coefficient from its quantized version with its local mean and variance. The proposed method can reduce much coding artifacts of low bit-rate coded images, and at the same time guarantee that the resulting images satisfies the quantization error constraint.

## I. INTRODUCTION

IN low bit-rate block transform-based image coding, noise caused by the coarse quantization of transform coefficients is noticeable in the form of visible block boundaries. In order to remove these blocking artifacts while maintaining compatibility with current industrial standards, various post-processing techniques such as lowpass filtering (LPF) [1], [2], projections onto convex sets (POCS) [3], [4], and maximum *a posteriori* (MAP) methods [5], [6] have been proposed. However, some of them are basically image enhancement schemes [1], [2], and some of them are intrinsically iterative, which makes them impossible for real-time application [3]–[6]. Moreover, these conventional approaches generally devote their efforts to spatial domain processing of encoded images based on some *a priori* knowledge such as the intensity smoothness of images.

Without taking the channel error into account, the quantization of transform coefficients is the sole error source in transform based coding scheme. In view of this, tackling the coding artifacts problem in the transform domain would be much more efficient than in the spatial domain. In this letter, we formulate a weighted-least-squares estimation of the transform coefficients from their quantized versions. The computation of the estimate involves only local statistics of the quantized coefficients and the proposed algorithm is non-iterative, which allows real-time applications. The proposed method simultaneously reduces the coding artifacts and confines the reconstructed transform coefficients to their original quantization intervals by means of weighting values that are devised on the local statistics of the transform coefficients and the *a priori* quantizer information.

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## II. FORMULATION

Throughout this letter, a digital  $N \times N$  image, as well as its corresponding transform, is treated as an  $N^2 \times 1$  vector in the space  $R^{N^2}$  by lexicographic ordering. Block transform is then an  $N^2 \times N^2$  matrix, which carries out linear transformation from  $R^{N^2}$  to  $R^{N^2}$ . The lossy effects of the block transform-based compression can be modeled by  $Y = Q[Tx]$ , where  $x$  is the original image,  $T$  is the transform matrix,  $Y$  is the quantized coefficients, and  $Q[\bullet]$  represents the quantization process. The encoded image with blocking artifacts is given by  $y = T^{-1}Q[Tx]$ , where  $T^{-1}$  is the inverse transform. If uniform scalar quantizers are used,  $Y$  can be decomposed as

$$Y = X + n \quad (1)$$

where  $X$  is the transform of  $x$  and  $n$  is the additive zero-mean noise introduced by the quantizers.

The problem here is to estimate  $X$  from  $Y$  and the available information about the quantizers. Suppose the *a priori* mean of  $X$  is  $\bar{X}$ . One very reasonable estimation of  $X$ , by taking  $Y$  into account, is the weighted least-squares estimation [7], which is formulated as finding the  $X$  that minimizes the functional

$$J = (X - \bar{X})^t M (X - \bar{X}) + (Y - X)^t R (Y - X) \quad (2)$$

where  $M$  and  $R$  are weighting matrices. We denote this estimate as  $\hat{X}$  and it is given explicitly as

$$\hat{X} = \bar{X} + W(Y - \bar{X}) \quad (3)$$

with  $W = (M + R)^{-1}R$ . Here,  $(M + R)$  is assumed invertible.

To ensure that the method would be practical with light computational cost, it is desirable to choose both  $M$  and  $R$  to be diagonal. In this case, we have

$$\hat{X}_i = \bar{X}_i + w_i(Y_i - \bar{X}_i) \quad (4)$$

where  $w_i$  is the  $i$ th diagonal element of  $W$ , and  $[\cdot]_i$  represents the  $i$ th element of a vector. It can be derived that  $w_i$ 's that minimize the mean-square error of  $\hat{X}$  are given as

$$w_i = \frac{\sigma_{\hat{X}_i}^2}{\sigma_{\hat{X}_i}^2 + \sigma_{n_i}^2} \quad (5)$$

where  $\sigma_{\hat{X}_i}^2$  and  $\sigma_{n_i}^2$  are the variance of  $X_i$  and  $n_i$ , respectively.

The computation of  $\hat{X}$  requires the *a priori* mean and variance of  $X_i$ , as well as the quantization-error variance of  $X_i$ . By assuming that the quantization error  $n_i$  has a uniform

TABLE I  
PSNR IMPROVEMENTS OF THE IMAGES RECONSTRUCTED WITH VARIOUS ALGORITHMS

JPEG Encoded Image	bpp	PSNR (dB)	PSNR Improvement (dB)			
			LPF [1]	POCS [4]	WLS	WLS*
Baboon	0.449	21.244	-0.026	0.085	0.190	0.126
Cameraman	0.315	26.442	-0.288	0.119	0.405	0.251
Peppers	0.323	27.686	0.374	0.361	0.655	0.556
House	0.244	30.512	0.404	0.432	0.767	0.720
Lenna	0.318	27.879	0.447	0.389	0.734	0.597
Girl	0.231	30.502	0.633	0.486	0.822	0.747
Germany	0.238	29.287	0.487	0.357	0.645	0.618
Couple	0.233	30.597	0.433	0.479	0.649	0.521
Sailboat	0.374	25.693	0.222	0.287	0.491	0.399
Tiffany	0.232	29.328	0.477	0.398	0.715	0.674
Face	0.282	30.171	0.922	0.680	1.073	1.051
Hat	0.352	29.399	1.164	0.779	1.394	1.353

probability density function, the quantization-error variance is given as  $\sigma_{n_i}^2 = \frac{1}{12} q_i^2$ , where  $q_i$  is the known stepsize of the corresponding quantizer applied to  $X_i$ . Both the mean and variance of  $X_i$  can be estimated from  $Y_i$ . Since  $n_i$  is assumed zero mean and uncorrelated with  $X_i$ , from (1) it can be derived that

$$\bar{X}_i = \bar{Y}_i \quad (6)$$

and

$$\sigma_{X_i}^2 = \sigma_{Y_i}^2 - \sigma_{n_i}^2 \quad (7)$$

where  $\bar{Y}_i$  and  $\sigma_{Y_i}^2$  are the mean and variance of  $Y_i$ , respectively. (Note that in practice  $\sigma_{X_i}^2$  is determined as  $\max\{0, \sigma_{Y_i}^2 - \sigma_{n_i}^2\}$  to guarantee its positive nature.) As an approximation in practical realization, the mean and variance of  $Y_i$  are computed as the "local" mean and variance of  $Y_i$ . Our idea on local statistics of transform coefficients is illustrated in the following. Let  $y^{(m,n)}$  denote  $y$  that is shifted in the image domain by  $(m, n)$ . Its transform,  $Ty^{(m,n)}$ , is then denoted as  $Y^{(m,n)}$ . The local mean  $\bar{Y}_i$  and the local variance  $\sigma_{Y_i}^2$  are defined by

$$\bar{Y}_i = \frac{1}{(2L+1)^2} \sum_{m=-L}^L \sum_{n=-L}^L Y_i^{(m,n)} \quad (8)$$

$$\sigma_{Y_i}^2 = \frac{1}{(2L+1)^2} \sum_{m=-L}^L \sum_{n=-L}^L [Y_i^{(m,n)} - \bar{Y}_i]^2 \quad (9)$$

where  $(2L+1)^2$  is the extent of the analysis window.

However, in order to guarantee that  $\hat{X}_i$  satisfies the quantization error constraint, i.e.,  $|Y_i - \hat{X}_i| < q_i/2$ , there should be a bound for  $w_i$ . By substituting (4) and (6) into this constraint, we have

$$w_i > 1 - \frac{q_i}{2|Y_i - \bar{Y}_i|}. \quad (10)$$

Hence, in order to obtain a  $\hat{X}$  that satisfies the quantization error constraint,  $w_i$  is given by

$$w_i = \max \left\{ \frac{\sigma_{X_i}^2}{\sigma_{X_i}^2 + \sigma_{n_i}^2}, 1 - \frac{q_i}{2|Y_i - \bar{Y}_i|} \right\}. \quad (11)$$

Finally, we remark that both  $\bar{Y}_i$  and  $\sigma_{Y_i}^2$  can be approximated to speed up the whole process quite a bit after sacrificing a little bit of its restoration performance. In fact, it is clear that  $\bar{Y}_i = [T\bar{y}]_i$ , and we found empirically in our simulation that a close performance could still be provided by approximating  $\sigma_{Y_i}^2$  with  $|[T\sigma_y^2]_i|$ , where  $\bar{y}$  and  $\sigma_y^2$  denote the local mean and variance of image  $y$ , respectively.

### III. PERFORMANCE EVALUATION

Experiments were carried out to evaluate the performance of the proposed method. In order to show its robustness, a number of *de facto* standard 256 gray-level test images of size  $256 \times 256$  each were encoded with the block discrete-cosine-transform-based (DCT-based) Joint Photographers Expert Group (JPEG) compression algorithm, and then restored. All of them were encoded with the same quantization table that was used in [3] and [4]. The LPF algorithm [1], the POCS algorithm [4], and the proposed algorithm, denoted WLS, were used to reconstruct the encoded data. As we have remarked before,  $\sigma_{Y_i}^2$  can be approximated by  $|[T\sigma_y^2]_i|$ . This approximated version of WLS, denoted WLS\*, was also implemented for comparative studies. Images reconstructed with these algorithms were then compared with each other. Table I shows their peak signal-to-noise ratio (PSNR) improvement with respect to the JPEG-encoded image. The proposed approach is obviously superior. Our experimental results also shown that it can provide subjectively better reconstructed images.

### IV. CONCLUSIONS

In this letter, we proposed a new technique that was based on the weighted least-squares estimation of transform coefficients from their quantized versions and the available information about the quantizers used. The proposed method can reconstruct an objectively and subjectively better image, and at the same time assure that the reconstructed image satisfies the quantization error constraint. Finally, the proposed method is noniterative and, thus, allows real-time applications. Though

in this letter only a uniform quantizer is considered, it can be easily generalized to the case with a nonuniform quantizer.

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