A Simple Adaptive Control Strategy for Regulated Switching DC-DC Converter Based on Grid-Point Concept[1]

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Abstract - **A simple, easy to implement adaptive control strategy for dc-dc converters based on a gird-point concept is presented in this paper. Multiple control laws are designed previously with respect to some defined operating points (grid-points). A parameter subspace associated with each grid-point can then be derived. The main idea is that the specified operation space is partitioned into small subspaces characterized by the grid-points, so that any disturbances can be localized to be small signals within a particular subspace. By switching the control law based on the associated grid-point best representing the actual operation, the operation range of the regulated converter is widened and the transient behavior is improved. Simulation results have shown its merit when large-signal disturbances are present in output load and input voltage.**

I. INTRODUCTION

Conventionally, the control law of dc-dc converter is designed based on a small-signal linearized model of the converter about a single operating point [1,2,3]. Unfortunately, dc-dc converters are highly non-linear by nature, and the parameters of the model (mainly, the input voltage and the output load resistance) are uncertain during operation. The regulation is often realized by a fixed-parameter PID controller of which the performance is good only when the disturbances in line voltage and output load are small. The regulation will become poor as the disturbances become large.

In this paper, a simple and easy to implement adaptive control strategy based on a grid-point modeling approach is proposed. Multiple operating points within the large operation space of the converter are defined. Each defined operating point (grid-point) spans an operation subspace, in accordance with a simple control law designed based on that grid-point. The grid points are chosen such that the union of their associated subspaces can completely cover the operation space of the switching converter. During operation, the control law is switched based on the associated grid-point best representing the actual operation.

Computer simulations have been performed on a regulated PWM boost converter. It is found that improvements can be achieved in terms of transient responses under large-signal disturbances, and the allowable range of operation.

II. SWITCHING DC-DC CONVERTER

Switching DC-DC Converter is the heart of Switch Mode Power Supplies (SMSP). Despite their advantages of high power density and efficiency, regulated switching converters are usually restricted to operate in a small range. It is because the regulation of output power against any disturbances to the system is realized by a simple compensator only. The design of such a compensator is usually based on a small-signal linearized model of the converter previously defined. Still, the parameters of the converter model are uncertain and subjected to inevitable and significant perturbations during operation. On applying averaging and small-signal linearization techniques about a single operating point, the problems of non-linearies and uncertainty of system parameters are neglected by assuming that the perturbations to the closed-loop system are very small. This conventional approach is usually found ineffective or inadequate in many applications when the range of operation and the signal of disturbances are large. Therefore, the operation range of the converters is usually small.

Under these conditions, the dc-dc converter is usually equipped with a sophisticated controller to tackle the large-signal disturbances, but the results are often unsatisfactory in terms of cost-effectiveness or complexity of the controller structure

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III. Adaptive Control Based on a Grid-Point Concept

In this paper, a Grid-Point Approach is adopted in order to cater for the control problem of regulated switching dc-dc converters systematically. By considering the small-signal linearization around a single operating point, an allowable parameter subspace (with respect to the input voltage, Vin and output load resistance, R) enclosing the operating point (grid-point) can be defined based on a designed simple control law. It is done in a way that within this parameter subspace, the disturbances causing parameter variations are small. In general, the parameter subspace is an n-dimensional hyper-space, where n is the number of varying parameters. Once a suitable parameter subspace is defined, another grid-point in the parameter space can be taken and another parameter subspace can be defined based on another simple designed control law. The procedure is repeated until the specified operation space of the regulated switching converter has been covered by the union of subspaces spanned by all grid-points defined. The idea is depicted in Figure 1 which shows that the specified operation range bounded by the dotted lines is completely covered by the union of subspaces spanned by grid-points G1 to G4.

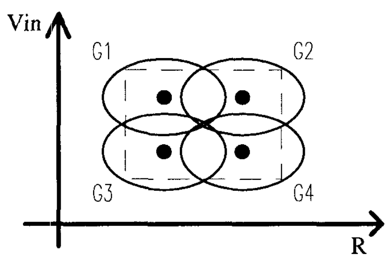

Figure 1. A diagram representing the grid-point modeling approach

In this way, the wide operation range of the regulated switching converter subject to large-signal disturbances can be localized to a narrow range subject to small-signal disturbances only. Adaptivity is achieved by switching the simple control law associated with the best grid-point representing the actual operating condition.

IV. Boundary of Parameter Space for a Grid-Point

The boundary of the parameter subspace for a given grid-point has to be determined so that a minimum number of grid-point can be defined to cover the whole operating range of the switching dc-dc converter. In this paper, the boundary of the parameter subspace is analyzed in z-domain.

A defined gird point can be generally characterized by the converter model parameter vector $\boldsymbol{a_o}$, of which the elements are the coefficients of a given linear discrete-time transfer function $G_o(z)$. $\boldsymbol{a_o}$, is a point defined on the parameter space. A controller of transfer function $H(z)$ can then be designed based on $G_o(z)$. The converter parameters are perturbed during operation. Let the perturbation vector be $\boldsymbol{q}$, the actual converter's model parameter vector $\boldsymbol{a}$ is then given by:

$$\boldsymbol{a} = \boldsymbol{a}_o + \boldsymbol{q} \qquad \text{-(1)}$$

The closed-loop system formed by the controller and the converter has a characteristic equation given by:

$$den[G_{cl}(z, \boldsymbol{a})] = 1 + G(z, \boldsymbol{a})H(z) = 0 \qquad \text{-(2)}$$

where, $den[G_{cl}(z, \boldsymbol{a})]$ is the denominator of the closed-loop transfer function $G_{cl}(z, \boldsymbol{a})$.

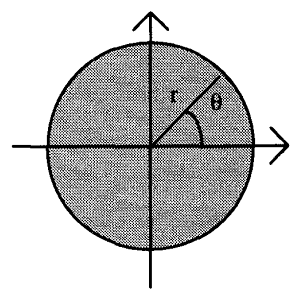

Figure 2. Circle of radius r on the z-plane

If all poles of $G_{cl}(z, \boldsymbol{a})$ (roots of $den[G_{cl}(z, \boldsymbol{a})] = 0$) are restricted inside a circle centred at the origin and of radius r ($r < 1$) on the z-plane, the stability of the system can be guaranteed. Based on this criterion of pole location on the z-plane, as shown in the Figure 2, the parameter subspace can be derived by considering the following three cases:

Case 1): $z = r$:

$$den[G_{cl}(r, \boldsymbol{a})] = 0$$

$$\Rightarrow \; c_1 + \boldsymbol{g}_1^{\mathrm{T}}\boldsymbol{a} = 0 \qquad \text{-(3)}$$

where c_1 is a scalar constant and $\boldsymbol{g}_1$ is a constant vector.

Case 2): $z = -r$:

$$den[G_{cl}(-r, \boldsymbol{a})] = 0$$

$$\Rightarrow \; c_2 + \boldsymbol{g}_2^{\mathrm{T}}\boldsymbol{a} = 0 \qquad \text{-(4)}$$

where c_2 is a scalar constant and $\boldsymbol{g}_2$ is a constant vector.

Case 3): $z = r(\cos\theta + j\sin\theta)$ where $\theta \in (-\pi,\pi)$ and $\theta \neq 0$

$$den[G_{cl}(r(\cos\theta + j\sin\theta), \boldsymbol{a})] = 0$$

$$\Rightarrow c_3(\theta) + g_3(\theta, \boldsymbol{a}) + j\,[c_4(\theta) + g_4(\theta, \boldsymbol{a})] = 0$$

$$\Rightarrow c_3(\theta) + g_3(\theta, \boldsymbol{a}) = 0 \quad \text{and } c_4(\theta) + g_4(\theta, \boldsymbol{a}) = 0 \qquad \text{-(5a, 5b)}$$

where c_3, c_4, g_3, g_4 are some scalar functions.

On eliminating θ, a linear or nonlinear function can be obtained which is given by:

$$g(\boldsymbol{a}) = 0 \qquad \text{-(6)}$$

From the above analysis, it can be seen that the boundary of the circle on the z-plane can be mapped to the boundary of a hyperspace of parameter $\boldsymbol{a}$. Such a hyperspace forms a basis for the parameter subspace spanned by the grid-point $\boldsymbol{a}_0$. Note that this hyperspace is a plane on a 2-dimensional parameter space.

IV. APPLICATION EXAMPLE

A PWM Type Boost Converter regulated by a digital P.I. controller is taken as an example. The circuit of the converter and a block diagram of the small-signal closed-loop system are shown in Figure 3 & Figure 4 respectively.

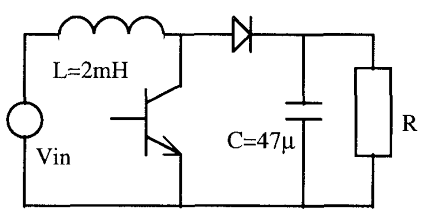

Figure 3. Circuit diagram of a boost converter

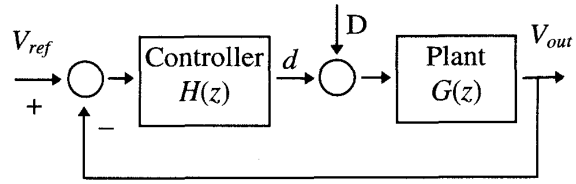

Figure 4. A block diagram representing the small-signal closed-loop system

The input voltage ***Vin*** and output load resistance ***R*** are the varying parameters constituting the operation space. The effects of other minor factors such as EMI and thermal reactions of components are neglected. In order to determine a suitable parameter subspace associated with a defined grid-point, the characteristic equation of the closed loop system in z-domain is investigated [4].

For a P.I. Controller of $H(z) = Kp + Ki \cdot \frac{T}{2} \cdot \frac{(z+1)}{(z-1)}$, the highest order of the characteristic equation is 3.

$$\mathrm{den}(G(z)) := A \cdot z^3 + B \cdot z^2 + C \cdot z + D \qquad \text{-(8)}$$

where A, B, C and D are functions of Vin and R

Based on the small-signal linearized model, if the closed-loop poles are restricted inside the circle of radius $\boldsymbol{r}$ (r < 1) in the z-plane [4] as shown in Figure 2, the parameter subspace spanned by the grid point can be found by following the steps described in section IV.:

$$den(Gc(z))\Big|_{\substack{z = r\cdot(\cos\theta + i\cdot\sin\theta) \\ z \neq r}} = 0 \qquad \text{-(9)}$$

$$\mathrm{den}(\mathrm{Gcl}(z)) := 1 + G(z)\cdot H(z)$$

$$:= 1 + \mathrm{Vin}\cdot(1-D)^2 \cdot \frac{\left[1 - \frac{2}{T}\cdot\frac{(z-1)}{(z+1)}\cdot\frac{L}{\left[R\cdot(1-D)^2\right]}\right]}{\left[\frac{4}{T^2}\cdot\frac{(z-1)^2}{(z+1)^2}\cdot L\cdot\frac{C}{(1-D)^2} + \frac{2}{T}\cdot\frac{(z-1)}{(z+1)}\cdot\frac{L}{\left[R\cdot(1-D)^2\right]} + 1\right]}\cdot\left[Kp + Ki\cdot\frac{T}{2}\cdot\frac{(z+1)}{(z-1)}\right]$$

$$:= 0 \qquad \text{-(7)}$$

Then a set of equations with *Vin* ad *R* as variables can be obtained:

$$\left.\begin{aligned} & r^6 \cdot A^2 + B \cdot r^2 \cdot D - D^2 - C \cdot r^4 \cdot A := 0 \\ & D = 0 \\ & A \cdot r^3 + B \cdot r^2 + C \cdot r + D := 0 \\ & -A \cdot r^3 + B \cdot r^2 - C \cdot r + D := 0 \end{aligned}\right\} \quad \text{-(10)}$$

Two sets of control law are defined and fine-tuned.

Control Law 1: Kp1=1.1901e-3, Ki1=4.030

Control Law 2: Kp2=1.2151e-3, Ki2=8.060

The switching frequency of the converter is 50kHz and the sampling frequency of controller is 2 kHz.

With reference to the values of converter parameters, A, B, C & D can be easily found as follows:

for control law 1,

$$\left.\begin{aligned} A &= 19.56 \cdot Vin \cdot R - 9.01e^2 \cdot Vin + 9.66e^5 \cdot R + 4.61e^6 \\ B &= 54.55 \cdot Vin \cdot R - 7.11e^2 \cdot Vin - 2.50e^6 \cdot R - 4.61e^6 \\ C &= 50.42 \cdot Vin \cdot R + 9.01e^2 \cdot Vin + 2.50e^6 \cdot R - 4.61e^6 \\ D &= 15.43 \cdot Vin \cdot R + 7.11e^2 \cdot Vin - 9.66e^5 \cdot R + 4.61e^6 \end{aligned}\right\} \quad \text{-(11)}$$

for control law 2,

$$\left.\begin{aligned} A &= 2.07 \cdot Vin \cdot R - 95.251 \cdot Vin + 9.66e^5 \cdot R + 4.61e^6 \\ B &= 2.07 \cdot Vin \cdot R + 95.15 \cdot Vin - 2.50e^6 \cdot R - 4.61e^6 \\ C &= -2.06 \cdot Vin \cdot R + 95.25 \cdot Vin + 2.50e^6 \cdot R - 4.61e^6 \\ D &= -2.07 \cdot Vin \cdot R - 95.15 \cdot Vin - 9.66e^5 \cdot R + 4.61e^5 \end{aligned}\right\} \quad \text{-(12)}$$

From the above coefficients for each control law and the boundary equations found before, a bounded area in the parameter space (a [Vin, R] -plane) can be found which is mapped by the region enclosed by the circle of radius r in the z-plane. Then other physical constraints and considerations of nonlinearites are added to refine the parameter subspace. In this example, the net result is the definition of two grid-points, each associated with a simple and fine-tuned P.I. controller to span a parameter subspace so that the specified operation range can be fully covered. The input voltage is ranged from 2.4V to 9.6V, and the output load resistance is ranged from 5 Ω to 50Ω.

The boost converter is started up at time t=0, and it is subjected to a Vin change from 5V to 3.6V at time t=0.03. The transient responses with fixed Control Law 1 and fixed Control Law 2 are shown in Figure 5 & 6 respectively. Although Control Law 1 can give a better transient response when the input voltage is 5V during start-up, the response is much poor as the converter is subjected to a change on Vin. On the other hand, Control Law 2 can speed up the rising time of the output voltage, but the transient response is poor when the input voltage is 5V.

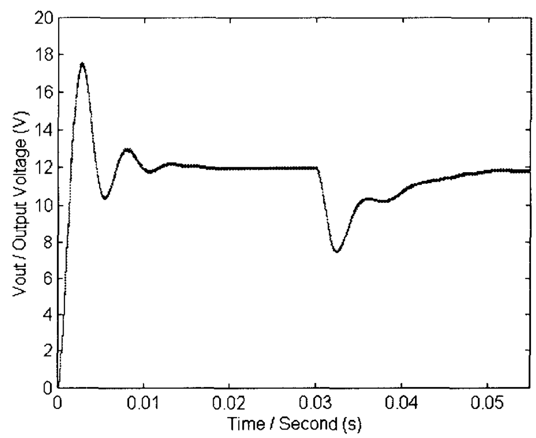

Figure 5. Vin changed with Control Law 1.

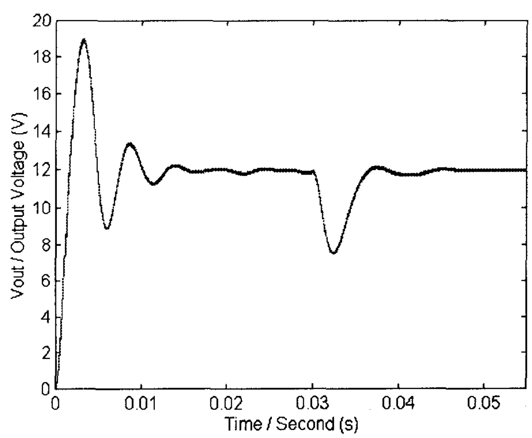

Figure 6. Vin changed with Control Law 2.

The advantages of both Control Law 1 and Control Law 2 can be combined by using the proposed grid-point concept. The result of the transient response of the converter is shown in Figure 7.

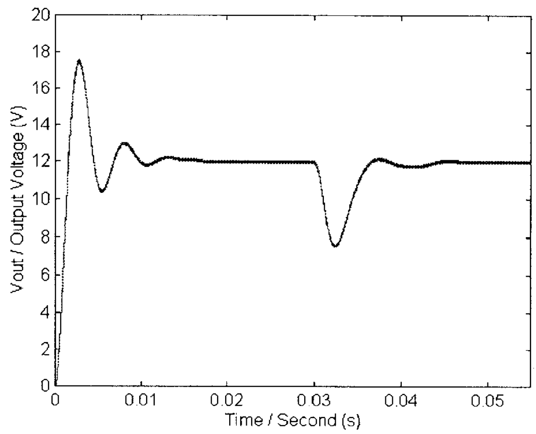

Figure 7. Vin changed with Control Law 1 & 2.

V. CONCLUSION

By using a grid-point concept, the large operation space of a dc-dc converter can be easily tackled by some very simple control laws. The controller of the dc-dc converter is designed according to the defined grid-points, and the small-signal linearized model of the converter. Simulation results have reported a great improvement in the transient response when large-signal disturbances are present in input voltage *Vin* or output load resistance *R*. A simple a P.I. controller is used as an example. When the increase of complexity is justified, a better improvement can readily be achieved by using PID controllers associated with more grid-points with the same adaptive control strategy. It is the systematic design methodology which is to present. The actual performance of the regulated converter depends on many factors such as the choice of control law, the number of grid-points, and the operation range.

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