A Simple Large-Signal Non-linear Model for Fast Simulation of Zero-Current-Switch Quasi-Resonant Converters[1]

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Abstract **- A nonlinear model for zero-current-switch quasi-resonant-converters is proposed which can be derived easily using simple analytical techniques. It is readily accepted by MATLAB for analysis and simulation in fast speed. Simulations have shown its accuracy even for large-signal transient dynamics. The condition for zero-current switching is also identified from this model.**

I. INTRODUCTION

The properties and characteristics of zero-current-switch (ZCS) quasi-resonant converters (QRCs) can be found in [1-3]. In general, a ZCS is formed by adding a pair of resonant inductor and capacitor to the electronic switch so that the value of current flowing through the electronic switch is zero when it is turned on and off. The switching loss of the converter can thus be reduced and the operating frequency can be increased, making the power density to be high. However, the design of regulated switch mode power supplies based on ZCS QRCs are difficult to realize without a good model of the open-loop converter. Still, due to its non-linearities and complicated operating characteristics, ZCS QRCs are hard to model.

Steady-state analyses of QRCs were carried out and relationships between their static voltage conversion ratios and the switching frequencies have been reported [3]. This information is useful in getting an understanding of the operation of the QRC in steady-state, but cannot be used to predict the transient behavior. A dynamic model of ZCS QRC based on a circuit-oriented approach was reported in [4, 5] . On the other hand, Chan and Chau [7] used finite element method to obtain a dynamic model for QRCs, but it is just a numerical method which cannot be easily applied to the problem of design and analysis of the closed-loop regulated power supply.

In this paper, a model for ZCS QRCs is proposed through which the converter model can be derived by simple analytical techniques. The result is a nonlinear differential equation model readily absorbed by some analytical tools, such as MATLAB, for analysis and simulation. Inside the MATLAB environment with the Control Systems Toolbox, the open-loop converter can be simulated easily. The resulting converter model is found to be accurate even for large-signal transient responses. In particular, the models of the three basic topologies of buck, boost, and buck-boost converters are to be investigated. All these details will be given in Section II and Section III. It was reported in [6] that zero-current switching is governed by a given condition. This condition is verified on applying the proposed modelling approach, and will be explained in Section IV. Simulation results for the three basic topologies are to be given in Section V as application examples.

II. THE ZERO-CURRENT SWITCH

The circuit diagrams of a full-wave ZCS and a half-wave ZCS are shown in Fig. 1(a) and Fig. 1(b) respectively. It consists of an electronic switch S, diodes D_1, D_2 (for full-wave only), the resonant inductor L_r and the resonant capacitor C_r. The nodes on the two sides of the ZCS are denoted by A and K respectively. When the ZCS is off, the voltage across the switch is V_{AK} and the current from A to K is zero. When the ZCS is on, the current from A to K is I. The value of I is obtained based on the QRC topology and is assumed to be constant *within one switching cycle.* This assumption is valid because QRCs are operating at high frequency such that the period of one switching cycle is negligibly small with respect to the dynamics of the converter filter.

From the operation principle of the ZCS, a switching cycle is divided into four stages [1-3]. They are: 1) inductor charging stage, 2) resonant stage, 3) capacitor charging stage, and 4) free-wheeling stage. The time duration of stage 1) to stage 4) are T_1, T_2, T_3 and T_4 respectively. They are stated as follows:

1) Inductor charging stage

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$$T_1 = \frac{L_r I}{V_{AK}} \tag{1}$$

2) Resonant stage

$$T_2 = \frac{\theta}{\omega} \tag{2}$$

where $\theta = \sin^{-1}(\frac{-Z_n I}{V_{AK}})$, $Z_n = \sqrt{\frac{L_r}{C_r}}$ and $\omega = \frac{1}{\sqrt{L_r C_r}}$

and $\pi < \theta < 3\pi/2$ for half-wave mode of operation,
$3\pi/2 < \theta < 2\pi$ for full-wave mode of operation.

3) Capacitor charging stage

$$T_3 = \frac{C_r V_{AK}(1 - \cos\theta)}{I} \tag{3}$$

4) Free-wheeling stage

$$T_4 = T_s - T_1 - T_2 - T_3 \tag{4}$$

where T_s is the period of one switching cycle.

III. THE QUASI-RESONANT CONVERTER MODEL

The idea of the proposed modeling approach is to reduce the operation of the ZCS from four stages to two stages, viz. on-stage and off-stage. When the current from A to K is zero, the ZCS is regarded as off. When the current from A to K is I, the ZCS is regarded as fully on. From this point of view, a ZCS is very similar to an ideal switch. The derivation of the model is based on the fact that the ZCS is turned fully on in Stage II and Stage III, during which the current from A to K is I. In Stage IV, the current flowing through the ZCS is zero, so that it is fully turned off. In Stage I, both the ZCS and the associated passive diode switch supply or sink current I to the filter inductor. The current flowing through the ZCS is linearly raised from zero to I during T_1 [3]. As I is the summation of the current of the ZCS and that of the passive diode switch, on average, it is equivalent to say that the ZCS and the associated diode switch each take half of T_1 to supply or sink the current I. Let t_{on} be the time duration in one switching period that the ZCS is on, then

$$t_{on} = 0.5\, T_1 + T_2 + T_3 \tag{5}$$

and the turn off time is

$$t_{off} = 0.5\, T_1 + T_4 \tag{6}$$

The state equations of the converter for the on-stage and the off-stage can then be found. Since the period of each switching cycle is relatively short, time-averaging within one switching cycle can be applied so that the two state equations are combined to reach a nonlinear state equation model relating the output voltage to the switching frequency.

For a quasi-resonant buck converter, let L, C, and R be the inductance, capacitance, and load resistance of the output filter respectively. The state equations of the converter operating during t_{on} are given by:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s \tag{7}$$

where I_L is the current flowing through the filter inductor (I_L=I), V_o is the output voltage and V_s is the input voltage. The state equations of the quasi-resonant buck converter operating during t_{off} are given by

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_s \tag{8}$$

By using time-averaging, the weighted average of equ. (7) and equ. (8) with respect to t_{on} and t_{off} respectively is given by:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ t_{on}/T_s L \end{bmatrix} V_s \tag{9}$$

where $T_s = t_{on} + t_{off}$.

From (1), (2), (3) and (5), it can be shown that t_{on} is a function of I_L only,

$$t_{on} = 0.5\frac{L_r I_L}{V_s} + \sqrt{L_r C_r}\ \sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}} I_L}{V_s}\right) + \frac{C_r V_s\left(1 - \cos\left(\sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}} I_L}{V_s}\right)\right)\right)}{I_L} \tag{10}$$

Hence, equ. (9) and equ. (10) constitute a nonlinear differential equation model for the ZCS quasi-resonant buck converter relating its output voltage V_o to the switching frequency $1/T_s$. Although the expressions look fairly complicated, such a model can easily be handled by some software tools, such as MATLAB and its Toolboxes, for analysis and simulation. In addition, this model can be used as a basis for the design of the compensating network for optimal closed-loop behavior. On using this model, all simulations can be done in much faster speed than those based on circuit simulators.

For a ZCS quasi-resonant boost converter, the state equations corresponding to t_{on} and t_{off} are:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s \qquad (11)$$

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & -1/C \\ -1/L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s \qquad (12)$$

Similarly, by applying time-averaging, the nonlinear model of the converter can be represented as follows:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & -(1-t_{on})/T_s C \\ -(1-t_{on})/T_s L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s \qquad (13)$$

$$t_{on} = 0.5\frac{L_r I_L}{V_o} + \sqrt{L_r C_r}\,\sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}}I_L}{V_o}\right) + \frac{C_r V_o\left(1-\cos\left(\sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}}I_L}{V_o}\right)\right)\right)}{I_L} \qquad (14)$$

For a ZCS quasi-resonant buck-boost converters, the state matrix is the same as that of the boost converter. The state equations corresponding to t_{on} and t_{off} are:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_s \qquad (15)$$

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & -1/C \\ 1/L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_s \qquad (16)$$

By applying time-averaging, the nonlinear model of the converter can be represented as follows:

$$\frac{d}{dt}\begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} -1/RC & -(1-t_{on})/T_s C \\ (1-t_{on})/T_s L & 0 \end{bmatrix}\begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ t_{on}/L \end{bmatrix} V_s \qquad (17)$$

$$t_{on} = 0.5\frac{L_r I_L}{V_s - V_o} + \sqrt{L_r C_r}\,\sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}}I_L}{V_s - V_o}\right) + \frac{C_r (V_s - V_o)\left[1-\cos\left(\sin^{-1}\left(\frac{-\sqrt{\frac{L_r}{C_r}}I_L}{V_s - V_o}\right)\right)\right]}{I_L} \qquad (18)$$

IV. Zero Current Switching Condition

It can be seen from (2) that for a real θ, the following condition must be satisfied:

$$\left|\frac{-Z_n I}{V_{AK}}\right| < 1 \qquad (19)$$

Physically, it represents a condition for zero-current switching. If (19) fails, the time T_2 is a complex number which means that the ending of the resonant stage does not exist. It is because the current of the switch does not return to zero so that the switch cannot be turned off at zero current. The cause may be either due to a large current I or a low voltage V_{AK}. This will usually happen during the transient state, especially when large disturbances are involved, even though the zero current switching condition is satisfied in the steady-state. Another possible cause is a poor choice of Z_n, that is the value of L_r and C_r.

V. Simulation Result

The ZCS quasi-resonant buck, boost and buck-boost converters in full-wave mode of operation are used as illustrative examples. The schematic diagrams of the three topologies are shown in Fig. 2 to Fig. 4 respectively. The start-up transient responses associated with these three converters obtained by MATLAB based on the proposed modelling approach and a circuit simulator are shown in Fig. 5 to Fig. 7 respectively. The component values of each example circuit are listed in Table 1. The switching frequency is set at 300 kHz for all topologies. The electronic switch and the diodes are assumed to be ideal. It can be seen that the responses obtained based on the proposed modelling approach match well with those from a circuit simulator, even though large-signal start-up transient dynamics are involved. Still, the results offered by the proposed models can be obtained in much faster speed when compared with those provided by a circuit simulator.

	ZCS Buck	ZCS Boost	ZCS Buck Boost
V_s	15V	15V	15V
L_r	1.6μH	0.16μH	1.6μH
C_r	0.064μF	0.64μF	0.064μF
L	100μH	100μH	100μH
C	1μF	10μF	3.3μF
Load	10Ω	20Ω	20Ω

Table 1. Component value of test circuits

VI. Conclusion

A modeling approach for ZCS QRCs is proposed which can be derived using simple analytical techniques. A nonlinear differential equation model is reached which can accurately predict the transient behavior of the QRC even for large signal operation. The responses of the QRC can be easily obtained by using MATLAB and its toolbox in fast speed. This helps the design of regulated power converters significantly, especially when tuning of controller parameters and converter component values are necessary. The condition for maintaining zero-current switching is also identified from this model. Designer of QRCs should be aware of this fact and try to avoid non-zero-current switching through proper choices of circuit components and operation range.

VII. References

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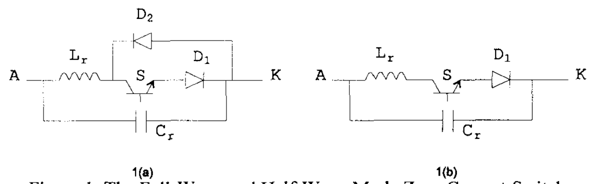

Figure 1. The Full-Wave and Half-Wave Mode Zero-Current-Switch Quasi-Resonant Switch

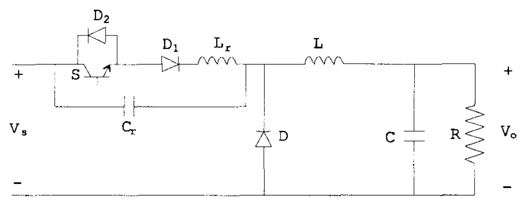
Fig. 2. A full-wave ZCS QR buck converter

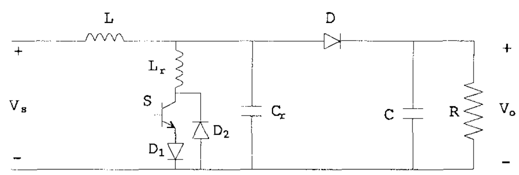
Fig. 3. A full-wave ZCS QR boost converter

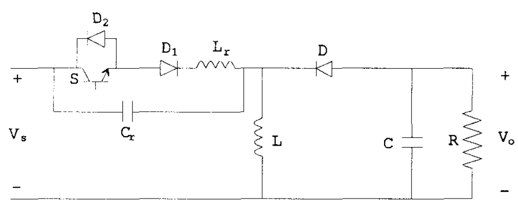
Fig. 4. A full-wave ZCS QR buck-boost converter

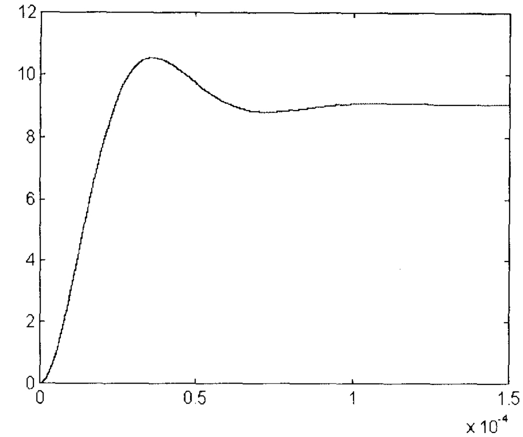

Fig. 5(a). Transient response of a full-wave ZCS buck QRC based on the proposed model

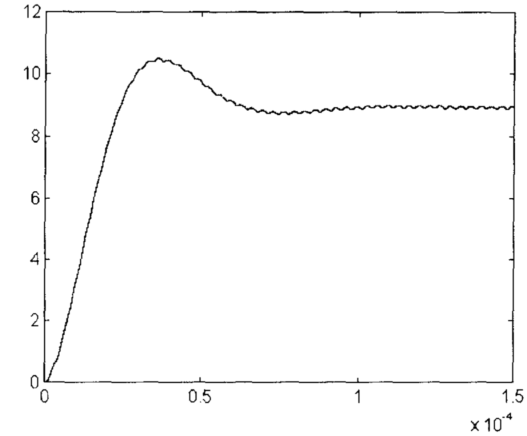

Fig. 5(b). Transient response of a full-wave ZCS buck QRC by circuit simulator

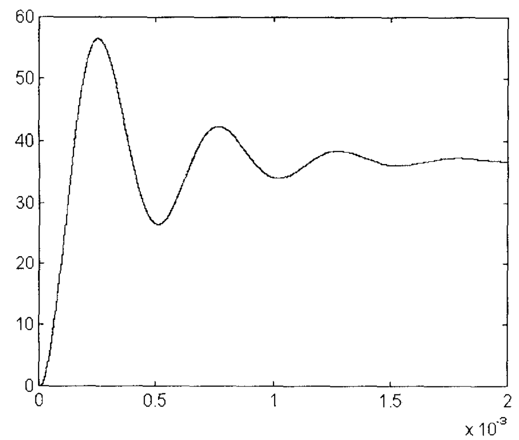

Fig. 6(a). Transient response of a full-wave ZCS boost QRC based on the proposed model

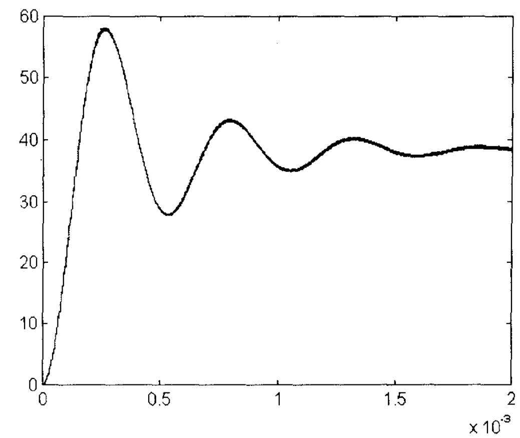

Fig. 6(b). Transient response of a full-wave ZCS boost QRC by circuit simulator

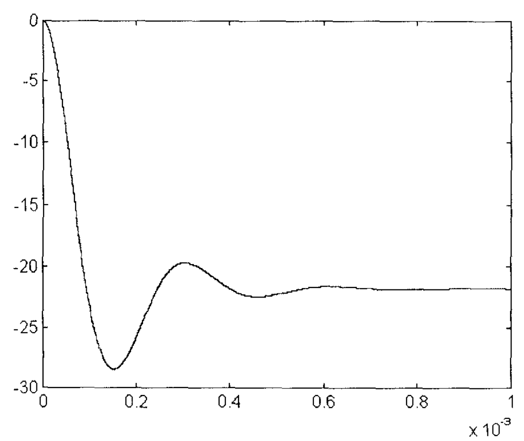

Fig. 7(a). Transient response of a full-wave ZCS buck boost QRC based on the proposed model

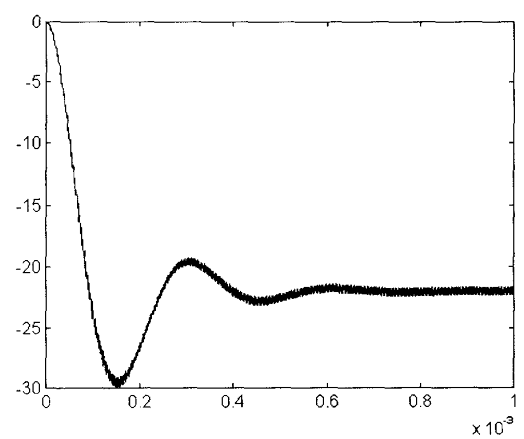

Fig. 7(b). Transient response of a full-wave ZCS buck boost QRC by circuit simulator