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Fast Simulation of PWM Inverters using MATLAB

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***Abstract* - This paper presents simulations of PWM inverters using MATLAB. Since MATLAB has the necessary numerical tools to solve non-linear differential equations, simulations can be carried out by developing system differential equations of the PWM inverters. More importantly, due to the CAD tools available in the MATLAB environment, a CAD package for regulated inverters can easily be developed. It is shown that the accuracy of the simulation results by using MATLAB is high as compared with that by using PSPICE. However, the simulation speed of MATLAB is much faster. Examples using resistive load, inductive load and non-linear load are shown.**

I. INTRODUCTION

PWM inverters [3] are widely used in uninterruptible power supplies (UPS) and driving motors. It converts a DC voltage into an AC sinusoidal one under various kinds of load, including resistive loads, inductive loads and non-linear loads. Simulations of PWM inverters can be carried out by using PSPICE. However, such simulations usually take long time. This will become a significant problem when many similar simulations are required to reach an optimal design by fine-tuning parameters. Moreover, PSPICE may sometimes suffer from the convergence problem. Due to these weaknesses, we propose to simulate PWM inverters using MATLAB.

There are many advantages on using MATLAB to simulate PWM inverters. First, the simulation speed can be much faster than that using PSPICE. Second, there are a lot of available tools that can be used in the MATLAB environment to design and optimize the performance of the open-loop and closed-loop PWM inverter system easily [1, 2]. In this paper, we show how to model inverters under different loads as differential equations. Then, by applying "ode23", a MATLAB function which can solve a system of ordinary non-linear differential equations using numerical method, the responses of PWM inverters can be simulated.

We present three examples to illustrate the modelling and simulation of PWM inverters under different kinds of load. Section II and III detail the simulations of PWM inverters under a resistive and an inductive load respectively. In section IV, a non-linear phase-controlled load will be used in the simulation. The results obtained by MATLAB will be compared with those obtained from PSPICE.

II. RESISTIVE LOAD

A half-bridge PWM inverter is shown in Fig. 1. It consists of an LC filter formed by an inductor L_f and a capacitor C_f, with series resistance R_{Lf} and R_{Cf} respectively. The bandwidth of this filter is designed to be much lower than the switching frequency. The load in Fig. 1 is a resistor of resistance $R_L = 10\Omega$ in this section. It is assumed that the bi-directional switches S_1 and S_2 are ideal. When S_1 is turned on, S_2 is turned off such that v_i is equal to V_s. On the other hand, when S_1 is turned off, S_2 is turned on such that v_i is equal to $-V_s$. Let t_{on} and t_{off} be the turn-on and turn-off time of S_1 respectively, we define a duty cycle d as follows:

$$d = \frac{t_{on} - t_{off}}{t_{on} + t_{off}} \tag{1}$$

where $t_{on} + t_{off}$ is the switching period which is constant. It should be noted that the value of d is ranged from -1 to 1.

Assume the switching frequency is much higher than the bandwidth of the LC filter, by applying the time-averaging technique, the value of v_i is effectively equal to

$$\begin{aligned} v_i &= \frac{t_{on}V_s + t_{off}(-V_s)}{t_{on} + t_{off}} \\ &= dV_s. \end{aligned} \tag{2}$$

Then a system differential equation with d as the control input and v_o as the output can be written as follows:

$$\begin{aligned} &\frac{d}{dt}\begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{R_{Lf}}{L_f} - \frac{R_L R_{Cf}}{L_f(R_L + R_{Cf})} & -\frac{R_L}{L_f(R_L + R_{Cf})} \\ \frac{1}{C_f} - \frac{R_{Cf}}{C_f(R_L + R_{Cf})} & -\frac{1}{C_f(R_L + R_{Cf})} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix} \\ &+ \begin{bmatrix} \frac{V_s}{L_f} \\ 0 \end{bmatrix} d, \end{aligned} \tag{3}$$

$$v_o = \begin{bmatrix} \frac{R_L R_{Cf}}{R_L + R_{Cf}} & \frac{R_L}{R_L + R_{Cf}} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \end{bmatrix}. \tag{4}$$

The parameters in (3) are listed in Table 1. A MATLAB m-file [1] describing (3) is shown in Appendix I. In this m-file, the control input is a sinusoidal wave of amplitude 0.9 and frequency 500Hz. The open-loop response of i_{Lf} and v_{Cf} can be obtained from the following command [2][†]:

```
[t, y] = ode23('invr', 0, 0.01, [0; 0]);
```

where `t` and `y` are the returned time vector and state vector respectively of the function `ode23`. The arguments of `ode23` is the file name of the m-file used, the simulation starting time and ending time, and the initial state vector respectively. The response of v_o can be obtained based on (4). To verify the accuracy of the simulation using

[†] This command is used when MATLAB version 5 is used:

```
[t, y] = ode23('invr', [0, 0.01], [0, 0]);
```

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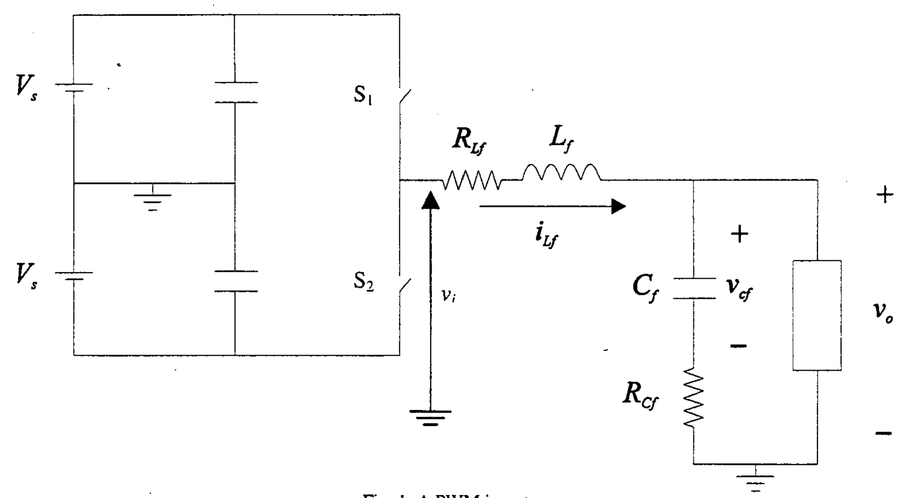

Fig. 1 A PWM inverter

MATLAB, a PWM inverter with the same set of parameters, but with the ideal switch replaced by a MOSFET IRF730 and an anti-parallel diode MUR460, is simulated using PSPICE. The switching frequency is 20kHz. The simulated responses of v_o and i_{Lf} are shown in Fig. 2 and 3 respectively. The small discrepancies in the waveforms given by MATLAB and PSPICE are due to the power loss of the non-ideal switch and diode used in PSPICE.

It should be noted that although only the open-loop responses are shown here, the closed-loop responses based on certain controller design can readily be analysed in the MATLAB environment through modifying the sinusoidal d function as another external control function. With the CAD tool available in MATLAB, a CAD package for regulated PWM inverter can easily be developed.

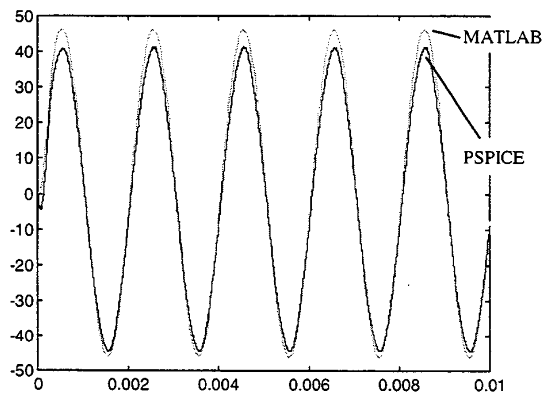

Fig. 2 Simulated responses of v_o under resistive load

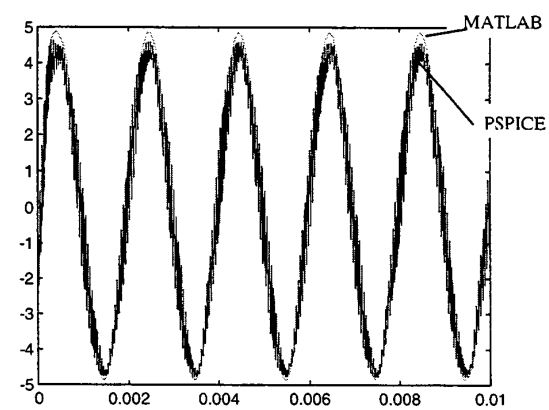

Fig. 3 Simulated responses of i_{Lf} under resistive load

III. INDUCTIVE LOAD

In this section, the load in Fig. 1 is changed to a resistor of resistance $R_L = 5\Omega$ in series with an inductor of inductance $L_L = 1000\mu H$. Since the system now is one order higher than that in the previous section, the system differential equation is modified as follows:

$$\frac{d}{dt}\begin{bmatrix} i_{Lf} \\ v_{Cf} \\ i_o \end{bmatrix} = \begin{bmatrix} -\frac{R_{Lf}+R_{Cf}}{L_f} & -\frac{1}{L_f} & \frac{R_{Cf}}{L_f} \\ \frac{1}{C_f} & 0 & -\frac{1}{C_f} \\ \frac{R_{Cf}}{L_L} & \frac{1}{L_L} & -\frac{R_L+R_{Cf}}{L_L} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \\ i_o \end{bmatrix} + \begin{bmatrix} \frac{V_s}{L_f} \\ 0 \\ 0 \end{bmatrix} d, \quad (5)$$

V_s	50V
L_f	500μH
R_{Lf}	0.1Ω
C_f	10μF
R_{Cf}	0.05Ω

Table 1. List of parameters

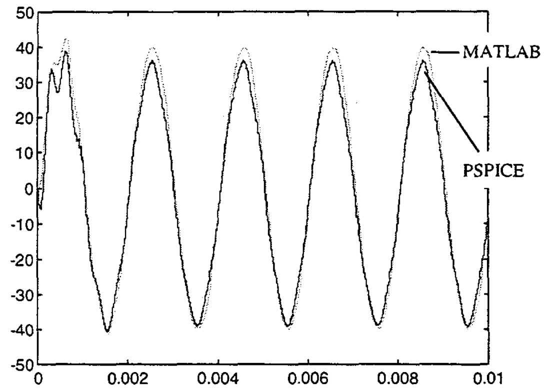

Fig. 4 Simulated responses of v_o under inductive load

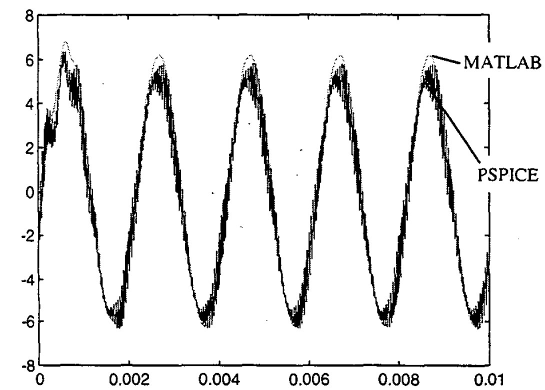

Fig. 5 Simulated responses of i_{Lf} under inductive load

$$v_o = \begin{bmatrix} R_{Cf} & 1 & -R_{Cf} \end{bmatrix} \begin{bmatrix} i_{Lf} \\ v_{Cf} \\ i_o \end{bmatrix}. \qquad (6)$$

A MATLAB m-file describing (5) is shown in Appendix II. The control input and switching frequency are the same as those in section II. The open-loop response of i_{Lf}, v_{Cf} and i_o can be obtained from the following command:

```
[t, y] = ode23('invl', 0, 0.01, [0; 0; 0]);
```

and the response of v_o can be obtained based on (6). The simulated responses of v_o and i_{Lf} are shown in Fig. 4 and 5 respectively.

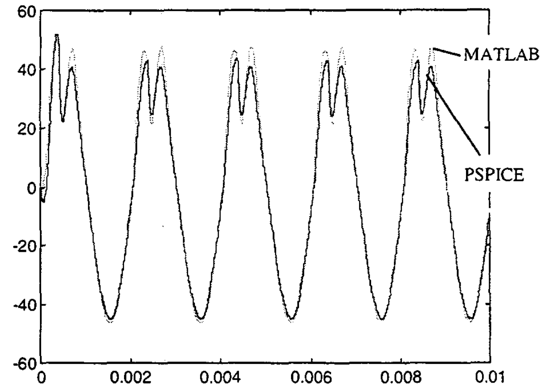

Fig. 6 Simulated responses of v_o under phase-controlled load

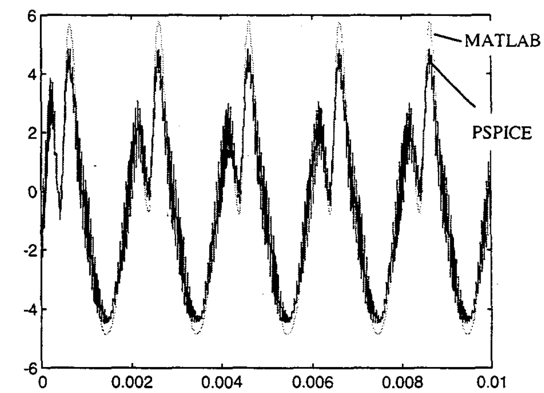

Fig. 7 Simulated responses of i_{Lf} under phase-controlled load

IV. NON-LINEAR LOAD

In this section, a non-linear phase-controlled load is used [3]. The non-linear load is effectively a resistor of 10Ω when the phase of the sine wave is from 72° to 360°. Otherwise, the load draws only a little leakage current, and is effectively a resistor of 1MΩ. The system differential equation is the same as that of (2) and (3) with the load R_L switches between 1MΩ and 10Ω with respect to the phase. The m-file of this system is shown in Appendix III. It can be seen that such a non-linear load can be easily simulated using MATLAB. The simulated responses of v_o and i_{Lf} are shown in Fig. 6 and 7 respectively.

V. CONCLUSION

Simulation of PWM inverters using MATLAB is proposed in this paper. By modelling inverters as differential equations and making use of the MATLAB function "ode23", the responses of PWM inverters can be obtained accurately and fast. Examples corresponding to a resistive load, an inductive load and a non-linear load have been shown to illustrate the simulations using MATLAB. The simulated responses obtained by PSPICE are also obtained for comparison purpose. The simulation results from MATLAB are found to be accurate. The small differences in the waveforms given by MATLAB and PSPICE are due to the power loss of the non-ideal switche and diode used in PSPICE. The simulation time of the three examples using MATLAB and PSPICE based on a Pentium II 300 system is listed in Table 2. It is found that the simulation time for PWM inverters using MATLAB is more than a hundred time shorter than that using PSPICE.

	MATLAB	PSPICE
Resistive load	0.49s	201.64s
Inductive load	0.6s	195.43s
Non-linear load	1.76s	205.75s

Table 2. Simulation time

REFERENCE

[1] *MATLAB User's Guide*, The MathWorks, Inc., August 1992.

[2] F. H. F. Leung, T. C. T. Ng, L. K. Wong, and P. K. S. Tam, "A CAD package for fast simulation of regulated dc-dc converters in large signal," in *Proc. IECON '97*, New Orleans, USA, November 1997, vol. 2, pp. 755-758.

[3] Naser M. Abdel-Rahim and John E. Quaicoe, "Analysis and design of a multiple feedback loop control strategy for single-phase voltage-source UPS inverter," *IEEE Trans. Power Electronics*, vol. 11, no. 4, July 1996.

ACKNOWLEDGEMENT

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APPENDIX I

```
function yp = InvR(t, y)

% Mathematical model for a PWM inverter
% Output load is pure resistive
% y(1) is inductor current
% y(2) is output voltage

l = 500e-6;
rl = 0.1;
c = 10e-6;
rc = 0.05;
r = 10;
vsource = 50;


d = 0.9 * sin(2 * pi * 500 * t);
d = max(-0.9, min(0.9,d));
vi = vsource * d;

rrc = r + rc;
yp = [-rl/l - r*rc/l/rrc  -r/l/rrc; 1/c - rc/c/rrc
-1/c/rrc] * y + [1/l; 0] * vi;
```

APPENDIX II

```
function yp = InvL(t, y)

% Mathematical model for a PWM inverter
% Output load is inductive
% y(1) is inductor current
% y(2) is output voltage
% y(3) is output load inductor current

l = 500e-6;
rl = 0.1;
c = 10e-6;
rc = 0.05;
r = 5;
lr = 1000e-6;
vsource = 50;


d = 0.9 * sin(2 * pi * 500 * t);
d = max(-0.9, min(0.9,d));
vi = vsource * d;

Acl = [-(rc+rl)/l   -1/l   rc/l;
       1/c   0   -1/c;
       rc/lr   1/lr   -(r+rc)/lr];
yp = Acl * y + [1/l; 0; 0] * vi;
```

APPENDIX III

```
function yp = InvNonl(t, y)

% Mathematical model for a PWM inverter
% Output load is non-linear
% y(1) is inductor current
% y(2) is output voltage

l = 500e-6;
rl = 0.1;
c = 10e-6;
rc = 0.05;
vsource = 50;


d = 0.9 * sin(2 * pi * 500 * t);
d = max(-0.9, min(0.9,d));
vi = vsource * d;

nlin = square(2 * pi * 500 * t, 20);

if nlin == 1
       r = 1e6;
else
       r = 10;
end

rrc = r + rc;
yp = [-rl/l - r*rc/l/rrc  -r/l/rrc; 1/c - rc/c/rrc
-1/c/rrc] * y + [1/l; 0] * vi;
```