Fuzzy Control of Multivariable Nonlinear Systems Subject to Parameter Uncertainties: Model Reference Approach [1]

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Abstract - **This paper presents the control of nonlinear systems subject to parameter uncertainties under a fuzzy control approach. The nonlinear plant tackled in this paper is an *n*-order nonlinear systems with *n* inputs. A design procedure of fuzzy controller will be provided such that the states of the closed-loop system will follow those of a user-defined stable reference model despite the presence of parameter uncertainties. A numerical example will be given to show the design procedures and the merits of the proposed fuzzy controller.**

I. INTRODUCTION

Fuzzy control is a useful control technique for uncertain and ill-defined nonlinear systems. Control actions of the fuzzy controller are described by some linguistic rules. This property makes the control algorithm to be understood easily. The early design of fuzzy controllers is heuristic. It incorporates experiences or knowledge of the designer into the rules of the fuzzy controller, which is fine tuned based on trial and error. A fuzzy controller implemented by neural-fuzzy network was proposed in [3, 4]. Through the use of tuning methods, fuzzy rules can be generated automatically. These methodologies make the design simple; however, the design does not guarantee the system stability, robustness and good performance.

To facilitate the system analysis of fuzzy control systems and the design of fuzzy controllers, fuzzy plant model was proposed in [1, 10]. The fuzzy plant model represents a nonlinear system as a weighted sum of some linear systems. Based on this structure, fuzzy controllers comprising a number of sub-controllers were proposed. State feedback controllers were proposed as the sub-controllers in [4]. The closed-loop system is guaranteed to be asymptotically stable if there exists a common solution for a number of linear matrix inequalities (LMI). Other stability conditions can be found in [9-10]. For fuzzy plant models subject to parameter uncertainties, robustness analysis was carried in [5, 11]. In this paper, a design methodology will be proposed to design a fuzzy state feedback controller. An *n*-order-*n*-input nonlinear system will be tackled. This system will be represented by a fuzzy plant model. The system states of the closed-loop system will follow those of a stable user-defined reference model. A numerical example will be given to show the design procedures and the merits of the proposed fuzzy controller.

II. REFERENCE MODEL, FUZZY PLANT MODEL AND FUZZY CONTROLLER

An *n*-order-*n*-input nonlinear plant subject to parameter uncertainties will be considered. This plant is represented by a fuzzy plant model. A fuzzy controller will be designed to close the feedback loop such that the closed-loop system states will follow those of a stable reference model.

A. Reference Model

A reference model is a stable linear system given by,

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{H}_m \hat{\mathbf{x}}(t) + \mathbf{B}_m \mathbf{r}(t) \qquad (1)$$

where $\mathbf{H}_m \in \Re^{n\times n}$ is a constant stable system matrix, $\mathbf{B}_m \in \Re^{n\times m}$ is a constant input vector, $\hat{\mathbf{x}} \in \Re^{n\times 1}$ is the system state vector of this reference model and $\mathbf{r}(t) \in \Re^{m\times 1}$ is the bounded reference input.

B. Fuzzy Plant Model with Parameter Uncertainty

Let p be the number of fuzzy rules describing the uncertain nonlinear plant. The i-th rule is of the following format,

Rule i : IF $x_1(t)$ is M_1^i and … and $x_n(t)$ is M_n^i

$$\text{THEN } \dot{\mathbf{x}}(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t) \qquad (2)$$

where M_k^i is a fuzzy term of rule i corresponding to the state $x_k(t)$, $k = 1, 2, \ldots, n$, $i = 1, 2, \ldots, p$; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times n}$ are the system and input matrix respectively in phase variable canonical form; $\Delta\mathbf{A}_i \in \Re^{n\times n}$ is the parameter uncertainties of $\mathbf{A}_i$; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{n\times 1}$ is the input vector. The plant dynamics is described by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\left[(\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\right] \qquad (3)$$

$$\text{where } \sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1,\ w_i(\mathbf{x}(t)) \in [0,\ 1] \text{ for all } i \qquad (4)$$

is a known nonlinear function of $\mathbf{x}(t)$ and

$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}(x_1(t)) \times \mu_{\mathrm{M}_2^i}(x_2) \times \cdots \times \mu_{\mathrm{M}_n^i}(x_n)}{\sum_{j=1}^{p}\left(\mu_{\mathrm{M}_1^j}(x_1(t)) \times \mu_{\mathrm{M}_2^j}(x_2(t)) \times \cdots \times \mu_{\mathrm{M}_n^j}(x_n(t))\right)} \qquad (5)$$

$\mu_{\mathrm{M}_k^i}(x_k(t))$ is the grade of membership of M_k^i.

C. Fuzzy Controller

A fuzzy controller having p fuzzy rules as is to be designed for the plant. The j-th rule of the fuzzy controller is of the following format:

Rule j: IF $x_1(t)$ is M_1^i and … and $x_n(t)$ is M_n^i

$$\text{THEN } \mathbf{u}(t) = \mathbf{u}_j(t) \qquad (6)$$

where $\mathbf{u}_j(t) \in \Re^{n\times 1}$, $j = 1, 2, \ldots, n$, is the output of the j-th rule controller that will be defined in the next section. The output of the fuzzy controller is given by

$$\mathbf{u}(t) = \sum_{j=1}^{p} w_j(\mathbf{x}(t))\mathbf{u}_j(t) \qquad (7)$$

III. DESIGN OF FUZZY CONTROLLER

In this section, we shall give the design of the fuzzy controller, i.e., $\mathbf{u}_j(t)$ for $j = 1, 2, \ldots p$, such that the closed-

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loop system behaves like the stable reference model. From (3), (4), (7), writing $w_i(\mathbf{x}(t))$ as w_i, we have,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i \left[(\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}\mathbf{u}_i(t) \right] \tag{8}$$

where $\mathbf{B} = \sum_{i=1}^{p} w_i \mathbf{B}_i$ (9)

From (1) and (8), let,

$$\begin{aligned} \dot{\mathbf{e}} &= \dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t) \\ &= \sum_{i=1}^{p} w_i \left[(\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}_i \mathbf{u}_j(t) - \mathbf{H}_m \hat{\mathbf{x}}(t) - \mathbf{B}_m \mathbf{r}(t) \right] \end{aligned} \tag{10}$$

where $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$ is an error vector. We define the following Lyapunov function,

$$V = \frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t) \tag{11}$$

where $(\cdot)^{\mathrm{T}}$ denotes the transpose of a vector or matrix, $\mathbf{P} \in \Re^{n\times n}$ is a symmetric positive definite matrix. Then, by differentiating (11), we have,

$$\dot{V} = \frac{1}{2}\left(\dot{\mathbf{e}}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t) + \mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\dot{\mathbf{e}}(t) \right) \tag{12}$$

From (10) and (12), we have,

$$\begin{aligned} \dot{V} &= \frac{1}{2}\left\{ \sum_{i=1}^{p} w_i \left[(\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}\mathbf{u}_i(t) - \mathbf{H}_m \hat{\mathbf{x}}(t) - \mathbf{B}_m \mathbf{r}(t) \right] \right\}^{\mathrm{T}} \mathbf{P}\mathbf{e}(t) \\ &+ \frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\sum_{i=1}^{p} w_i \left[(\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + \mathbf{B}\mathbf{u}_i(t) - \mathbf{H}_m \hat{\mathbf{x}}(t) - \mathbf{B}_m \mathbf{r}(t) \right] \end{aligned} \tag{13}$$

We design $\mathbf{u}_i(t)$, $i = 1, 2, \ldots, p$, as follows,

$$\mathbf{u}_i = \begin{cases} \mathbf{B}^{-1}(\mathbf{H}\mathbf{e}(t) - \mathbf{A}_i\mathbf{x}(t) + \mathbf{H}_m\hat{\mathbf{x}}(t) + \mathbf{B}_m\mathbf{r}(t) \\ \quad - \dfrac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)}) & \text{if } \mathbf{e}(t) \neq \mathbf{0} \\ \mathbf{B}^{-1}\left(-\mathbf{A}_i\mathbf{x}(t) + \mathbf{H}_m\hat{\mathbf{x}}(t) + \mathbf{B}_m\mathbf{r}(t) \right) & \text{if } \mathbf{e}(t) = \mathbf{0} \end{cases} \tag{14}$$

where $\|\cdot\|$ denotes the l_2 norm for vectors and l_2 induced norm for matrices, $\|\Delta\mathbf{A}_i\| \le \|\Delta\mathbf{A}_i\|_{\max}$, $\mathbf{H} \in \Re^{n\times n}$ is a stable matrix to be designed. A block diagram of the closed-loop system is shown in Fig. 1. In (14), it is assumed that $\mathbf{B}^{-1}$ exists. In the later part of this section, we shall provide a way to check if the assumption is valid. From (13), (14) and assume that $\mathbf{e}(t) \neq \mathbf{0}$, we have,

$$\begin{aligned} \dot{V} &= \frac{1}{2}\left\{ \sum_{i=1}^{p} w_i \left[\mathbf{H}\mathbf{e}(t) + \Delta\mathbf{A}_i\mathbf{x}(t) - \frac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)} \right] \right\}^{\mathrm{T}} \mathbf{P}\mathbf{e}(t) \\ &+ \frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\sum_{i=1}^{p} w_i \left[\mathbf{H}\mathbf{e}(t) + \Delta\mathbf{A}_i\mathbf{x}(t) - \frac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)} \right] \\ &= \frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\left(\mathbf{H}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{H} \right)\mathbf{e}(t) \\ &+ \sum_{i=1}^{p} w_i \left(\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\Delta\mathbf{A}_i\mathbf{x}(t) - \frac{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)} \right) \\ &\le \frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\left(\mathbf{H}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{H} \right)\mathbf{e}(t) \\ &+ \sum_{i=1}^{p} w_i \left(\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|\|\mathbf{x}(t)\| - \frac{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{P}\mathbf{e}(t)} \right) \\ &\le -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t) + \sum_{i=1}^{p} w_i \|\mathbf{e}(t)\|\|\mathbf{P}\|\left(\|\Delta\mathbf{A}_i\| - \|\Delta\mathbf{A}_i\|_{\max} \right)\|\mathbf{x}(t)\| \end{aligned} \tag{15}$$

where $\mathbf{Q} = -\left(\mathbf{H}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{H} \right)$ is a symmetric positive definite matrix. As $\|\Delta\mathbf{A}_i\| - \|\Delta\mathbf{A}_i\|_{\max} \le 0$, from (15), we have,

$$\dot{V} \le -\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Q}\mathbf{e}(t) \le 0 \tag{16}$$

If $\mathbf{e}(t) = \mathbf{0}$, from (11) and (14), $\dot{V} = 0$. Hence, we can conclude that $\mathbf{e}(t) \to \mathbf{0}$ as $t \to \infty$. In the following, we shall derive a sufficient condition to check the existence of $\mathbf{B}^{-1}$. From (9) and considering the following dynamic system,

$$\dot{\mathbf{z}}(t) = \mathbf{B}\mathbf{z}(t) = \sum_{i=1}^{p} w_i \mathbf{B}_i \mathbf{z}(t) \tag{17}$$

If the nonlinear system of (17) is asymptotically stable, it implies that $\mathbf{B}^{-1}$ exists. To ensure the asymptotic stability, consider the following Lyapunov function,

$$V_z = \frac{1}{2}\mathbf{z}(t)^{\mathrm{T}}\mathbf{P}_z\mathbf{z}(t) \tag{18}$$

where $\mathbf{P}_z \in \Re^{n\times n}$ is a symmetric positive definite matrix. Then, by differentiating (11), we have,

$$\dot{V}_z = \frac{1}{2}\left(\dot{\mathbf{z}}(t)^{\mathrm{T}}\mathbf{P}_z\mathbf{z}(t) + \mathbf{z}(t)^{\mathrm{T}}\mathbf{P}_z\dot{\mathbf{z}}(t) \right) \tag{19}$$

From (17) and (19), we have,

$$\begin{aligned} \dot{V}_z &= \frac{1}{2}\left[\left(\sum_{i=1}^{p} w_i \mathbf{B}_i \mathbf{z}(t) \right)^{\mathrm{T}} \mathbf{P}_z\mathbf{z}(t) + \mathbf{z}(t)^{\mathrm{T}}\mathbf{P}_z \sum_{i=1}^{p} w_i \mathbf{B}_i \mathbf{z}(t) \right] \\ &= \frac{1}{2}\sum_{i=1}^{p} w_i \mathbf{z}(t)^{\mathrm{T}}\left(\mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z + \mathbf{P}_z\mathbf{B}_i^{\mathrm{T}} \right)\mathbf{z}(t) \\ &= -\frac{1}{2}\sum_{i=1}^{p} w_i \mathbf{z}(t)^{\mathrm{T}}\mathbf{Q}_i\mathbf{z}(t) \end{aligned} \tag{20}$$

where $\mathbf{Q}_i = -\left(\mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z + \mathbf{P}_z\mathbf{B}_i^{\mathrm{T}} \right)$. If $\mathbf{Q}_i < \mathbf{0}$ for all $i = 1, \ldots, p$, then, from (20), we have,

$$\dot{V} = -\frac{1}{2}\sum_{i=1}^{p} w_i \mathbf{z}(t)^{\mathrm{T}}\mathbf{Q}_i\mathbf{z}(t) \le 0 \tag{21}$$

The nonlinear system of (17) is then asymptotically stable and $\mathbf{B}^{-1}$ exists. Now, we consider $\overline{\mathbf{B}} = -\mathbf{B}$ and $\overline{\mathbf{B}}_i = -\mathbf{B}_i$. (17) becomes $\dot{\mathbf{z}}(t) = \overline{\mathbf{B}}\mathbf{z}(t) = \sum_{i=1}^{p} w_i \overline{\mathbf{B}}_i \mathbf{z}(t)$. It can be shown that $\overline{\mathbf{B}}^{-1}$ exists if there exist $\mathbf{Q}_i = \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z + \mathbf{P}_z\mathbf{B}_i^{\mathrm{T}} < \mathbf{0}$ for all i = 1, …, p. The existence of $\overline{\mathbf{B}}^{-1}$ implies the existence of $\mathbf{B}^{-1}$. The results of this section can be summarized by the following Lemma,

Lemma I. *The fuzzy control system of* (8), *subject to plant parameter uncertainties is guaranteed to be asymptotically* *stable, and its states will follow those of a stable reference model of* (1), *if there exists* $\mathbf{P}_z$ *such that,*

$-\left(\mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z+\mathbf{P}_z\mathbf{B}_i^{\mathrm{T}}\right)<\mathbf{0}$ *or* $\mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z+\mathbf{P}_z\mathbf{B}_i^{\mathrm{T}}<\mathbf{0}$

and the control laws of fuzzy controller of (7) are designed as,

$$\mathbf{u}_i=\begin{cases}\mathbf{B}^{-1}(\mathbf{He}(t)-\mathbf{A}_i\mathbf{x}(t)+\mathbf{H}_m\hat{\mathbf{x}}(t)+\mathbf{B}_m\mathbf{r}(t) \\ -\dfrac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}_i\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{Pe}(t)}) & \text{if } \mathbf{e}(t)\neq\mathbf{0} \\ \mathbf{B}^{-1}\left(-\mathbf{A}_i\mathbf{x}(t)+\mathbf{H}_m\hat{\mathbf{x}}(t)+\mathbf{B}_m\mathbf{r}(t)\right) & \text{if } \mathbf{e}(t)=\mathbf{0}\end{cases}$$

The procedure for finding the fuzzy controller can be summarized as follows.

- Step I) Obtain the mathematical model of the nonlinear plant to be controlled.
- Step II) Obtain the fuzzy plant model for the system stated in step I) by means of a fuzzy modeling method, for example, that proposed in [3, 10].
- Step III) Check if there exists $\mathbf{B}^{-1}$ by finding the $\mathbf{P}_z$ according to Lemma 1. If $\mathbf{P}_z$ can not be found, the design fails. $\mathbf{P}_z$ can be found by using some existing LMI tools
- Step IV) Choose a stable reference model.
- Step IV) Design the fuzzy controller according to Lemma 1.

IV. Numerical Example

A numerical example is given in this section to illustrate the procedure of finding the fuzzy controller. The fuzzy plant model of a nonlinear plant is assumed to be available, and we start from the step II) of the design procedure.

Step II) The nonlinear plant can be represented by a fuzzy model with the following fuzzy rules,

Rule 1: IF $x(t)$ is M_1^i AND $\dot{x}(t)$ is M_2^i

THEN $\dot{\mathbf{x}}(t)=\left(\mathbf{A}_i+\Delta\mathbf{A}_i\right)\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}(t)$, $i=1,2,3,4$ (22)

where the membership functions of M_α^i, α = 1, 2, are,

$$\mu_{\mathrm{M}_1^1}(x(t))=\mu_{\mathrm{M}_1^2}(x(t))=1-\frac{x(t)^2}{2.25},\ \mu_{\mathrm{M}_1^3}(x(t))=\mu_{\mathrm{M}_1^4}(x(t))=\frac{x(t)^2}{2.25} \quad (23)$$

$$\mu_{\mathrm{M}_2^1}(\dot{x}(t))=\mu_{\mathrm{M}_2^3}(\dot{x}(t))=1-\frac{\dot{x}(t)^2}{6.75},\ \mu_{\mathrm{M}_2^2}(x(t))=\mu_{\mathrm{M}_2^4}(x(t))=\frac{\dot{x}(t)^2}{6.75}$$

$$\mathbf{x}(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}=\begin{bmatrix}x(t)\\\dot{x}(t)\end{bmatrix},\qquad \mathbf{A}_1=\mathbf{A}_2=\begin{bmatrix}0&1\\-0.01&-1\end{bmatrix},$$

$$\mathbf{A}_3=\mathbf{A}_4=\begin{bmatrix}0&1\\-0.235&-1\end{bmatrix};\qquad \mathbf{B}_1=\mathbf{B}_3=\begin{bmatrix}0&1\\-1.4387&-2\end{bmatrix},$$

$$\mathbf{B}_2=\mathbf{B}_4=\begin{bmatrix}0&1\\-0.5613&-2\end{bmatrix};$$

$\Delta\mathbf{A}_1=\Delta\mathbf{A}_2=\Delta\mathbf{A}_3=\Delta\mathbf{A}_4=\begin{bmatrix}0&0\\d_1(t)&d_2(t)\end{bmatrix}$ where $d_1(t)$ and $d_2(t)$ are the parameter uncertainties. Practically, they are unknown values within given bounds. In this example, they are defined as time-varying functions to illustrate the robustness of the controller.

$d_1(t)=\dfrac{d_3^U+d_1^L}{2}+(c_1^L-\dfrac{d_1^U+d_1^L}{2})\cos(t)$ so that $d_1(t)\in[d_1^L,d_1^U]$ (24)

$d_2(t)=\dfrac{d_2^U+d_2^L}{2}+(c_2^L-\dfrac{d_2^U+d_2^L}{2})\cos(t)$ so that $d_2(t)\in[d_2^L,d_2^U]$ (25)

$d_1^L=-0.5$, $d_1^U=0.5$, $d_2^L=-0.1$, $d_2^U=0.1$.

Step III) We choose $\mathbf{P}_z=\begin{bmatrix}39.7945&12.6915\\12.6915&14.9997\end{bmatrix}$ such that $\mathbf{Q}_i=-\left(\mathbf{B}_i^{\mathrm{T}}\mathbf{P}_z+\mathbf{P}_z\mathbf{B}_i^{\mathrm{T}}\right)<\mathbf{0}$ for i = 1, 2, … , 4. Hence, we can guarantee the existing of $\mathbf{B}^{-1}$.

Step IV) The stable reference model is chosen as follows,

$$\dot{\hat{\mathbf{x}}}(t)=\mathbf{H}_m\mathbf{x}(t)+\mathbf{B}_m r(t) \quad (26)$$

where $\mathbf{H}_m=\begin{bmatrix}0&1\\-1&-1\end{bmatrix}$, $\mathbf{B}_m=\begin{bmatrix}0\\1\end{bmatrix}$ (27)

Step V) The rules of the fuzzy controller are designed as follows,

Rule$j$: IF $x(t)$ is M_1^j AND $\dot{x}(t)$ is M_2^j

THEN $\mathbf{u}(t)=\mathbf{u}_j$, j = 1, 2, 3, 4 (28)

where,

$$\mathbf{u}_j=\begin{cases}\mathbf{B}^{-1}(\mathbf{He}(t)-\mathbf{A}_i\mathbf{x}(t)+\mathbf{H}_m\hat{\mathbf{x}}(t)+\mathbf{B}_m\mathbf{r}(t) \\ -\dfrac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{Pe}(t)}) & \text{if } \mathbf{e}(t)\neq\mathbf{0} \\ \mathbf{B}^{-1}\left(-\mathbf{A}_i\mathbf{x}(t)-\mathbf{H}_m\hat{\mathbf{x}}(t)-\mathbf{B}_m\mathbf{r}(t)\right) & \text{if } \mathbf{e}(t)=\mathbf{0}\end{cases}$$

for j = 1, 2, 3, 4. We choose $\mathbf{H}=\begin{bmatrix}0&1\\-4&-4\end{bmatrix}$ which is a stable matrix, $\mathbf{P}=\begin{bmatrix}1.5000&0.5000\\0.5000&1.000\end{bmatrix}$ and $\|\Delta\mathbf{A}\|_{\max}=0.5099$ (from (24) and (25)).

Fig. 2 and 7 show the system responses of the fuzzy control system. It can be seen from the simulation results that the system states of the nonlinear system follow those of the reference model. The simulation results show that the response of the fuzzy control system with parameter uncertainties is better than that of the fuzzy control system without parameter uncertainties. This is because an additional control signal, i.e. $\dfrac{\mathbf{e}(t)\|\mathbf{e}(t)\|\|\mathbf{P}\|\|\Delta\mathbf{A}\|_{\max}\|\mathbf{x}(t)\|}{\mathbf{e}(t)^{\mathrm{T}}\mathbf{Pe}(t)}$, is used. It can be revealed from (15), i.e., $\dot{V}\le-\frac{1}{2}\mathbf{e}(t)^{\mathrm{T}}\mathbf{Qe}(t)+\frac{1}{2}\sum_{i=1}^{p}w_i\|\mathbf{e}(t)\|\|\mathbf{P}\|\left(\|\Delta\mathbf{A}\|-\|\Delta\mathbf{A}\|_{\max}\right)\|\mathbf{x}(t)\|$.

The term $\frac{1}{2}\sum_{i=1}^{p}w_i\|\mathbf{e}(t)\|\|\mathbf{P}\|\left(\|\Delta\mathbf{A}\|-\|\Delta\mathbf{A}\|_{\max}\right)\|\mathbf{x}(t)\|$ makes e(t) to decrease to zero in a faster rate.

V. CONCLUSION

Model reference fuzzy control of n-order n-input nonlinear systems subject to parameter uncertainties has been discussed. A design procedure of the fuzzy controller has been presented. The closed-loop system will behave like a user-defined reference model. A numerical example has been given to show the design procedures and the merits of the designed fuzzy controller.

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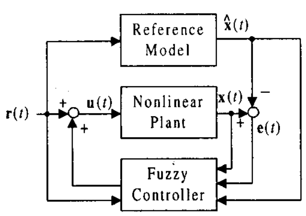

Fig. 1. A block diagram of the closed-loop system.

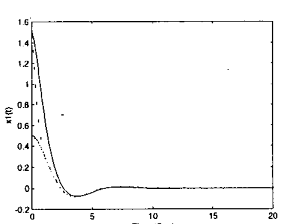

Fig. 2. Responses of $x_1(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = 0$, $\mathbf{x}(0) = [1.5 \quad 0]^T$ and $\hat{\mathbf{x}}(0) = [0.5 \quad 0]^T$.

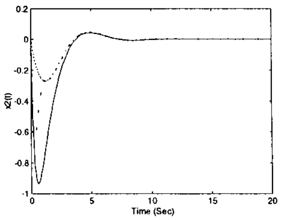

Fig. 3. Responses of $x_2(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = 0$, $\mathbf{x}(0) = [1.5 \quad 0]^T$ and $\hat{\mathbf{x}}(0) = [0.5 \quad 0]^T$.

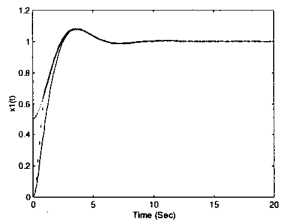

Fig. 4. Responses of $x_1(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = 1$, $\mathbf{x}(0) = [1.5 \quad 0]^T$ and $\hat{\mathbf{x}}(0) = [0.5 \quad 0]^T$.

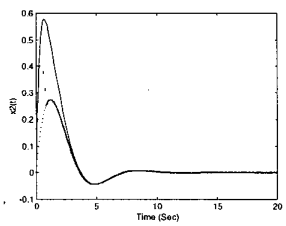

Fig. 5. Responses of $x_2(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = 1$, $\mathbf{x}(0) = [1.5 \quad 0]^T$ and $\hat{\mathbf{x}}(0) = [0.5 \quad 0]^T$.

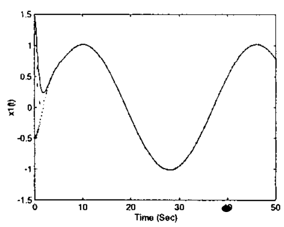

Fig. 6. Responses of $x_1(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = \sin(10t)$, $\mathbf{x}(0) = [1.5 \quad 0]^T$ and $\hat{\mathbf{x}}(0) = [0.5 \quad 0]^T$.

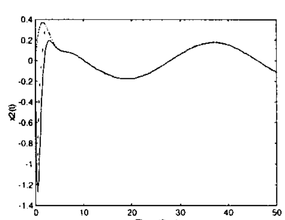

Fig. 7. Responses of $x_2(t)$ of the fuzzy control system without (solid line) and with parameter uncertainties (dash line), and the refence model (dotted line) under $r(t) = \sin(10t)$, $\mathbf{x}(0) = [1.5 \quad 0]^T$