Combination of Sliding Mode Controller and PI Controller using Fuzzy Logic Controller

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Abstract - This paper proposes an approach to combine a sliding mode controller (SMC) and a PI controller using a fuzzy logic controller. An SMC can give good transient performance. However, the steady state performance is poor due to the presence of discontinuous control which causes chattering. On the other hand, a PI controller can offer zero steady state error. Hence, combining these two controllers by a fuzzy logic controller can combine their advantages and remove their disadvantages. Although the PI sub-system has one order higher than the SMC sub-system, the closed-loop system stability under the control of this fuzzy logic controller can still be proved by applying a new proposed method. An illustrative example shows that good transient and steady state responses can be obtained by applying the proposed controller.

1. Introduction

It is well known that sliding mode controllers (SMCs) are powerful devices for controlling non-linear systems with unknown disturbances [4-6]. They offer good robustness and transient performances even in large-signal operation. However, since a discontinuous control action is involved, chattering exists which will degrade the steady state performance. One method to alleviate this drawback is to introduce a boundary layer about the sliding plane [5, 6]. This method can give a chattering free output response, but a finite steady-state error must exist for a type-0 system. There are two methods to achieve zero steady-state error: switching and integration. It is because under these two cases, the dc gain of the closed-loop system is infinity. Then the steady-state value of the control signal is not fixed by the steady-state values of the system states, but varied so as to drive the errors of the states to zero.

The boundary layer method will give finite steady-state error because only proportional feedback (no integral feedback nor switching) is used when the states are lying within the boundary layer. To eliminate steady-state error, a PI controller should be employed. This paper proposes a fuzzy logic controller (FLC) to combine an SMC and a PI controller. As the SMC and PI controllers can give good transient and steady-state performance respectively, the role of the FLC is to schedule them under different operation conditions [3]. System stability will be proved by a newly proposed stability analysis method [1, 2]. This method requires a common Lyapunov function for different sub-systems. Here, a sub-system is the closed-loop system under the control of either the SMC or the PI controller only. One of the major difficulties in the stability analysis is that the PI sub-system is one order higher than the SMC sub-system. Consequently it is more difficult, as compared with that in [1, 3], to find a common Lyapunov function for both sub-systems. Fortunately, it can be proved that by properly designing the gain of the SMC and the location of the fuzzy levels, this problem can be solved.

The stability analysis method will be reviewed in Section 2. Then the stability analysis procedure of the proposed FLC will be developed in Section 3. Although the stability proof seems complex, the design of the FLC is as easy as to follow a few formulae. Section 4 illustrates the proposed FLC by a non-linear plant with external disturbances. Finally, a conclusion will be drawn in Section 5.

2. Review of the stability analysis method

2.1 Fuzzy logic control system

Consider a single input n-th order non-linear system with external disturbances of the following form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x})u + \boldsymbol{w} \quad (1)$$

where $\boldsymbol{x} = [x_1, x_2, ..., x_n]^T$ is the state-vector, $\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),......, f_n(\boldsymbol{x})]^T$, $\boldsymbol{b}(\boldsymbol{x}) = [b_1(\boldsymbol{x}), b_2(\boldsymbol{x}),......, b_n(\boldsymbol{x})]^T$ are functions describing the dynamics of the plant, u is the control input of which the value is determined by an FLC,

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and $w = [w_1, w_2, \ldots\ldots, w_n]^T$ is the vector describing the external disturbances. It is assumed that the values of $w_1, w_2, \ldots\ldots, w_n$ are unknown but constant and bounded by a positive constant vector $w_b = [w_{b1}, w_{b2}, \ldots\ldots, w_{bn}]^T$ such that $|w_j| < w_{bj}$ for all $j = 1, 2, \ldots\ldots, n$. The i-th IF-THEN rule in the fuzzy rule base of the FLC is of the following form:

Rule i: IF <*premise i*> THEN $u = u_i$ (2)

where <*premise i*> is the premise of rule i with a certain general input variable z; $u = u_i$ is the control output of rule i. It can be a single value or a function of the state x. The shape of the membership functions associated with the input fuzzy levels, the method of fuzzification, and the algorithm of rule inference can be arbitrary because they do not affect the stability analysis. A degree of membership $\mu_i \in [0, 1]$ is obtained for each rule i. It is assumed that for any z in the input universe of discourse $\mathbf{Z}$, there exists at least one μ_i among all rules that is non-zero. By applying the weighted sum defuzzification method, the overall output of the FLC is given by:

$$u = \frac{\sum_{i=1}^{k} \mu_i u_i}{\sum_{i=1}^{k} \mu_i} \tag{3}$$

where k is the total number of rules. Here we need to define the following terms: active/inactive fuzzy rules, and active region of a fuzzy rule.

Definition 2.1: For any input $z_o \in \mathbf{Z}$, if the degree of membership μ_i corresponding to fuzzy rule i is zero, this fuzzy rule i is called an *inactive fuzzy rule* for the input z_o; otherwise, it is called an *active fuzzy rule*. An *active region* of a fuzzy rule i is defined as a region $\mathbf{Z}_r \subset \mathbf{Z}$ such that its degree of membership μ_i is non-zero for all $z_o \in \mathbf{Z}_r$.

It should be noted that for any input z_o, an inactive fuzzy rule will not affect the controller output u. Hence, (3) can be re-written so as to consider all active fuzzy rules (where $\mu_i \neq 0$ for $z = z_o$) only,

$$u = \frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i u_i}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i} \tag{4}$$

Now, among all the control output u_i for $z = z_o$ of the sub-systems corresponding to the active fuzzy rules, there exists a maximum value u_{max} and a minimum value u_{min}. Then,

$$\frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i u_{min}}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i} \leq \frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i u_i}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i} \leq \frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i u_{max}}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i}$$

$$\Rightarrow \quad u_{min}\frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i} \leq u \leq u_{max}\frac{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i}{\sum_{i=1,\mu_i\neq 0}^{k} \mu_i}$$

$$\Rightarrow \quad u_{min} \leq u \leq u_{max}, \tag{5}$$

equality holds when $u_i = u_{min} = u_{max}$.

In conclusion, the overall FLC output is bounded by u_{max} and u_{min} among the rules if the weighted sum defuzzification method is employed to derive u for any $z_o \in \mathbf{Z}$.

2.2 Stability analysis method

The premise of the stability criterion in this paper is that on applying each rule to the plant individually, the closed-loop sub-system formed is stable in the sense of Lyapunov (ISL) in the rule's active region, and each rule shares a common quadratic Lyapunov function $V(x) = x^T P x$ such that

$$\begin{array}{l} \text{i) } V(x) \text{ is positive definite and continuously differentiable,} \\ \text{ii) } \dot{V}(x) \leq 0 \text{ in the rule's active region} \end{array} \tag{6}$$

For an input $z_o \in \mathbf{Z}$, let the maximum and minimum control signals among all active fuzzy rules be u_{max} and u_{min} respectively. From (6), we have the sub-systems formed by these two rules satisfying the following conditions

$$\dot{V}(x) \leq 0 \quad \text{for } z = z_o,\ u = u_{max} \tag{7}$$

$$\dot{V}(x) \leq 0 \quad \text{for } z = z_o,\ u = u_{min} \tag{8}$$

Lemma 2.1: If a system in the form of (1) satisfies the premise of the stability criterion of (6), for all $z_o \in \mathbf{Z}$, we have $\dot{V}(x) \leq 0$ for $u \in [u_{min}, u_{max}]$.

Proof:

$$V(x) = x^T P x \tag{9}$$

$$\Rightarrow \dot{V}(x) = \dot{x}^T P x + x^T P \dot{x} \tag{10}$$

From (1),

$$\begin{aligned} \dot{V}(x) &= (f(x) + b(x)u + w)^T P x + x^T P (f(x) + b(x)u + w) \\ &= F(x) + B(x)u \end{aligned} \tag{11}$$

where $F(x) = (f(x) + w)^T P x + x^T P (f(x) + w)$,

$B(x) = b(x)^T P x + x^T P b(x)$.

Note that both $F(x)$ and $B(x)$ are scalars. Then, two cases should be considered: $B(x)$ is positive and $B(x)$ is negative for $z = z_o$.

Case 1: $B(x)$ is positive

By using condition (7), for $z = z_o$ and $u = u_{max}$

$$\begin{aligned} &\dot{V}(x)\big|_{u=u_{max}} = F(x) + B(x)u_{max} \leq 0 \\ \Rightarrow\ &\dot{V}(x) = F(x) + B(x)u \leq 0 \quad \forall\, u \leq u_{max} \end{aligned} \tag{12}$$

$$\Rightarrow \dot{V}(x) \leq 0 \text{ for } z = z_o \text{ and } u \in [u_{min}, u_{max}]$$

Case 2: B(x) is negative

By using condition (8), for $z = z_o$ and $u = u_{min}$

$$\dot{V}(x)\Big|_{u=u_{\min}} = F(x)+B(x)u_{\min} \le 0$$
$$\Rightarrow \quad \dot{V}(x) = F(x)+B(x)u \le 0 \qquad \forall\, u \ge u_{\min} \tag{13}$$

$$\Rightarrow \dot{V}(x) \le 0 \ \text{ for } z = z_o \text{ and } u \in [u_{\min}, u_{\max}] \tag{14}$$

From (12) and (13), the lemma is proved.

QED

Theorem 2.1: Consider an FLC as described in section 2.1, if every rule of the FLC applying to the plant of (1) individually gives a stable sub-system ISL in the active region of the fuzzy rule subject to a common Lyapunov function, and the defuzzification method is realized as given by (3), the whole fuzzy logic control system is stable ISL.

Proof: It has been shown in (5) that for an arbitrary input $z_o \in Z$, the control output of an FLC is bounded by $u_{\min}$ and $u_{\max}$ if the weighted sum defuzzification method is employed. Hence, if all sub-systems satisfy $\dot{V}(x) \le 0$ in the active regions of the rules, by *Lemma 2.1*, $\dot{V}(x) \le 0$ for all $z_o \in Z$ and the closed-loop system is stable ISL under the control of the FLC.

QED

3. Combination of SMC and PI controller

A sliding mode controller (SMC) and a PI controller is combined into a single FLC to control a plant of the form of (1). The input variable of the FLC is σ which is defined as

$$\sigma = \mathbf{s}\,\mathbf{x} \tag{15}$$

where $\mathbf{s} = [s_1, s_2, \ldots\ldots, s_n]$ is a constant vector. It should be chosen such that when the system states $\mathbf{x}$ are in the sliding plane (i.e. $\sigma = 0$), they will slide along the plane to the equilibrium point. In addition, we define a state v which will be used on analysing the system with the PI controller as follows:

$$\begin{cases} v = \int \sigma dt \text{ when the PI controller is active} \\ v \text{ is constant when the PI controller is inactive} \end{cases} \tag{16}$$

Furthermore, define v_r as the reference value of v. It is a constant to cancel out the effect of the unknown disturbance $\mathbf{w}$ when the sliding plane is hit. Hence we have

$$v_r = \frac{\mathbf{s}\mathbf{w}}{k_i} \tag{17}$$

where k_i is a gain to be designed later. In practice, due to the integral action as given by (16), the state v will automatically become v_r under proper design of the controller when the sliding plane is hit. It is not needed to know the value of v_r. However, its maximum bound v_{rb} can be evaluated as follows:

$$v_{rb} = \frac{\max(\mathbf{s}\mathbf{w})}{k_i} \tag{18}$$

where $\max(\mathbf{s}\mathbf{w}) = |s_1|\, w_{b1} + |s_2|\, w_{b2} + \ldots\ldots + |s_n|\, w_{bn}$. Note that v_{rb} is positive. To carry out the stability analysis, we arbitrarily choose an upper bound for v and define an error state e_v as follows:

$$|\,v\,| < 10\, v_{rb} \tag{19}$$

$$e_v = v_r - v \tag{20}$$

Then from (16),

$$\dot{e}_v = \begin{cases} -\sigma \text{ when the PI controller is active} \\ 0 \text{ when the PI controller is inactive} \end{cases} \tag{21}$$

On the other hand, from (18), (19) and (20),

$$|\,e_v\,| < 11\, v_{rb} \tag{22}$$

To combine an SMC and a PI controller into a single FLC, the fuzzy rules of the FLC are defined as follows:

Rule 1: IF σ is SM THEN $u = u_1 = (\mathbf{s}\mathbf{b})^{-1}(-\mathbf{s}\mathbf{f} - k_i\, v - k_p\, \sigma)$

Rule 2: IF σ is LR THEN $u = u_2 = (\mathbf{s}\mathbf{b})^{-1}(-\mathbf{s}\mathbf{f} - k_d\, \mathrm{sgn}(\sigma))$

where SM and LR are membership functions as shown in Fig. 1, k_i, k_p and k_d are gains to be designed.

To guaranteed the system stability by applying the stability analysis method discussed in Section 2, we need to establish a Lyapunov function V and ensure that $\dot{V} \le 0$ in the active range of each rule.

3.1 PI sub-system

From Rule 1, (15) and (1) we have

$$\dot{\sigma} = \mathbf{s}\mathbf{f} + \mathbf{s}\mathbf{b}(\mathbf{s}\mathbf{b})^{-1}(-\mathbf{s}\mathbf{f} - k_i v - k_p\sigma) + \mathbf{s}\mathbf{w}$$
$$= k_i e_v - k_i v_r - k_p\sigma + \mathbf{s}\mathbf{w}$$

Hence, from (17),

$$\dot{\sigma} = k_i e_v - k_p\sigma$$

Also from (21), we have

$$\begin{bmatrix} \dot{e}_v \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ k_i & -k_p \end{bmatrix} \begin{bmatrix} e_v \\ \sigma \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} e_v \\ \sigma \end{bmatrix} \tag{23}$$

The closed-loop sub-system behaves like a linear system. If the real part of all eigenvalues of $\mathbf{A}_1$ are negative, we can define a symmetric positive definite matrix $\mathbf{Q}_1$ such that a unique symmetric positive definite matrix $\mathbf{P}$ can be found satisfying the following equation [6]:

$$\mathbf{A}_1^{\mathrm{T}}\,\mathbf{P} + \mathbf{P}\,\mathbf{A}_1 = -2\mathbf{Q}_1 \tag{24}$$

Hence, we can define a common Lyapunov function V such that

$$V = \frac{1}{2}\begin{bmatrix} e_v & \sigma \end{bmatrix} \mathbf{P} \begin{bmatrix} e_v \\ \sigma \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} e_v & \sigma \end{bmatrix} \begin{bmatrix} p_1 & p_2 \\ p_2 & p_4 \end{bmatrix} \begin{bmatrix} e_v \\ \sigma \end{bmatrix} \tag{25}$$

Obviously from (24) and (25), $\dot{V} \le 0$ such that this PI sub-system is stable. Moreover, since **P** is positive definite, p_1 and p_4 must both be positive because the principle minors of **P** must be positive.

3.2 SMC sub-system

From (25), we have

$$\dot{V} = p_1 e_v \dot{e}_v + p_2 \sigma \dot{e}_v + p_2 e_v \dot{\sigma} + p_4 \sigma \dot{\sigma} \tag{26}$$

Also from Rule 2, (15) and (1), we have

$$\dot{\sigma} = sw - k_d \mathrm{sgn}(\sigma) \tag{27}$$

To ensure that $\dot{V} \le 0$ in this sub-system, we divide this active region into two sub-regions. With reference to Fig. 1, the two sub-regions are defined by $|\sigma| \ge m_2$ and $m_1 < |\sigma| < m_2$.

Case 1: $|\sigma| \ge m_2$

From (21), $\dot{e}_v = 0$. Then (26) can be reduced to

$$\dot{V} = p_2 e_v \dot{\sigma} + p_4 \sigma \dot{\sigma} \tag{28}$$

It can be proved that if

$$k_d > k_i\, v_{rb} \tag{29}$$

$$m_2 > 11\, v_{rb} \tag{30}$$

$$|p_2| < p_4 \tag{31}$$

(28) is negative. Hence, $\dot{V} < 0$ can be satisfied by properly designing k_d, k_i, k_p and $\mathbf{Q}_1$. The detailed proof is given in Appendix A.

Case 2: $m_1 < |\sigma| < m_2$

From (21), $\dot{e}_v = -\sigma$. Then (26) becomes

$$\dot{V} = -p_1 e_v \sigma - p_2 \sigma^2 + p_2 e_v \dot{\sigma} + p_4 \sigma \dot{\sigma} \tag{32}$$

The conditions for (32) to be negative are

$$k_d = \frac{2k_D}{p_4} \tag{33}$$

and

$$m_1 = 11 v_{rb} |p_2| \left(\frac{k_i v_{rb}}{k_D} + \frac{2}{p_4}\right) \tag{34}$$

where $k_D = (11\, p_1\, v_{rb} + |p_2|\, m_2 + p_4\, k_i\, v_{rb})$. The derivations of (33) and (34) are given in Appendix B.

In conclusion, to ensure the system stability of the SMC sub-system under the Lyapunov function of (25), we firstly need to select k_i, k_p and $\mathbf{Q}_1$ to satisfy (31) and select k_d, m_1, and m_2 according to (29), (30), (33) and (34). Then, both the PI sub-system and the SMC sub-system are stable ISL. By *Theorem 2.1*, the closed-loop system under the control of the FLC is stable.

4. Illustrative example

Consider a non-linear system of the form (1) as follows:

$$\dot{x} = f(x) + b(x)u + w$$

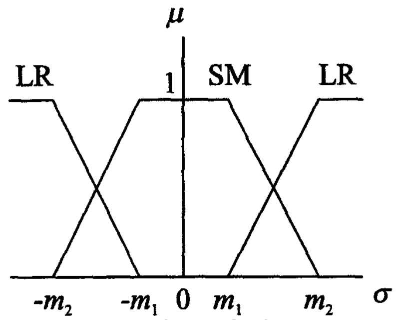

Fig. 1. Membership functions

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $f(x) = \begin{bmatrix} x_2 \\ \dfrac{9.8\sin(x_1) - 0.05x_2^2 \sin(2x_1)}{2/3 - 0.1\cos^2(x_1)} \end{bmatrix}$,

$b(x) = \begin{bmatrix} 0 \\ \dfrac{-0.1\cos(x_1)}{2/3 - 0.1\cos^2(x_1)} \end{bmatrix}$ and $w = \begin{bmatrix} 0 \\ w_2 \end{bmatrix}$.

The magnitude of w_2 is less than 1. The control objective is to regulate x to **0**. Consider the FLC as described in Section 3. Define

$$s = [1 \;\; 0.1] \tag{35}$$

It is obviously that the sliding plane $\sigma = 0$ is stable. Also we have

$$\max(s\, w) = 0.1 \tag{36}$$

Moreover, let $k_i = 2$, $k_p = 5$ and $\mathbf{Q}_1 = \begin{bmatrix} -2 & 0 \\ 0 & -200 \end{bmatrix}$, from (24) we have $p_1 = 85.4$, $p_2 = -1$ and $p_4 = 40.2$. Also from (18) and (36), $v_{rb} = 0.05$. Besides, let $m_2 = 0.6 > 11v_{rb}$ which satisfies (30). Then from (33) and (34), $k_d = 2.567$ and $m_1 = 0.0284$. Choose $w_2 = 1$, the transient responses of x_1 and x_2 are shown in Fig. 2 and Fig. 3 respectively under the control of the FLC and an SMC (i.e. only using Rule 2). The initial state $x(0) = [1 \;\; 0]^T$. Both systems are stable and have zero steady state error even when disturbances are present. From Fig. 2, it can be seen that the one using FLC has a small under-shoot and a slightly longer setteling time. However, as shown in Fig. 3, chattering exists when using the SMC alone. Hence, the steady-state performance of the system is improved by the proposed FLC

5. Conclusion

An approach to combine a sliding mode controller and a PI controller into a single fuzzy logic controller is proposed in this paper. The role of the fuzzy logic controller is to schedule different control action according to the operation conditions. Although the PI sub-system is one order higher than the SMC sub-system, the conditions

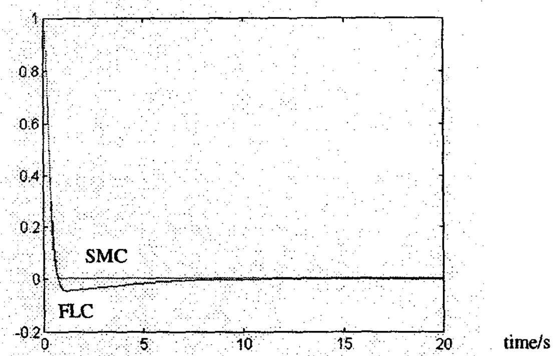

Fig. 2. Time response of x_1(rad)

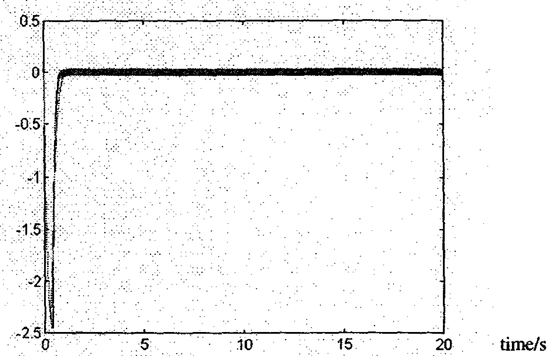

Fig. 3. Time response of x_2(rad s^{-1})

for obtaining a stable closed-loop system are derived. This combined controller is applied to a non-linear plant with disturbances to show its merits. It is found that both good transient response and zero steady-state error can be obtained by applying this controller.

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Appendix A

It will be proved in this appendix that (28) will be negative if the following three conditions are satisfied:

$$k_d > k_i\, v_{rb}$$
$$m_2 > 11\, v_{rb}$$
$$|\, p_2\,| < p_4$$

Proof: From (27),

$$\sigma\dot{\sigma} = \sigma s w - \sigma k_d \mathrm{sgn}(\sigma) \le |sw||\sigma| - k_d|\sigma|$$

From (29) and (18),

$$k_d > k_i\, v_{rb} = \max(s\, w)$$

Hence,

$$\sigma\dot{\sigma} < 0 \qquad \text{(A1)}$$

Also, since $|\,\sigma\,| \ge m_2$ for case 1. Then from (30) and (22),

$$|\,\sigma\,| > |\,e_v\,| \qquad \text{(A2)}$$

Hence, consider (28),

$$\begin{aligned} \dot{V} &= p_2 e_v \dot{\sigma} + p_4 \sigma\dot{\sigma} \\ &\le |p_2 e_v \dot{\sigma}| + p_4 \sigma\dot{\sigma} \\ &= |p_2||e_v||\dot{\sigma}| + p_4 \sigma\dot{\sigma} \\ &< |p_2||\sigma\dot{\sigma}| + p_4 \sigma\dot{\sigma} && \text{(from (A2))} \\ &= |p_2||\sigma\dot{\sigma}| - p_4|\sigma\dot{\sigma}| && \text{(from (A1))} \\ &< 0 && \text{(from (31))} \end{aligned}$$

QED

Appendix B

For $m_1 < |\,\sigma\,| < m_2$, $\dot{e}_v = -\sigma$. Then from (26) and (27),

$$\dot{V} = -p_1 e_v \sigma - p_2 \sigma^2 + p_2 e_v \dot{\sigma} + p_4 \sigma \dot{\sigma}$$
$$= -p_1 e_v \sigma - p_2 \sigma^2 + p_2 e_v \dot{\sigma} + p_4 \sigma\, s w - p_4 k_d |\sigma| \quad \text{(B1)}$$
$$\le |\sigma| \max(-p_1 e_v - p_2 \sigma + p_4\, s w) - p_4 k_d |\sigma| + p_2 e_v \dot{\sigma}$$

Then let $k_D = (11\ p_1\ v_{rb} + |p_2|\ m_2 + p_4\ k_i\ v_{rb})$ and $k_d = \dfrac{2k_D}{p_4}$

as in (33), from (22) and (18),

$$11\ p_1\ v_{rb} > p_1\ |e_v|$$
$$p_4\ k_i\ v_{rb} = \max(p_4\ s\ w)$$

Also, $$m_2 > |\sigma|$$

Then $k_D = (11\ p_1\ v_{rb} + |p_2|\ m_2 + p_4\ k_i\ v_{rb})$
$> p_1\ |e_v| + |p_2\ \sigma| + \max(p_4\ s\ w)$
$\ge \max(-p_1\ e_v - p_2\ \sigma + p_4\ s\ w)$.

Hence, (B1) becomes

$$\dot{V} \le k_D |\sigma| - 2k_D |\sigma| + p_2 e_v \dot{\sigma}$$
$$= -k_D |\sigma| + p_2 e_v \dot{\sigma}$$

A sufficient condition for $\dot{V} \le 0$ is

$$|\sigma| > \frac{\max(p_2 e_v \dot{\sigma})}{k_D} \quad \text{(B2)}$$

Since $|\sigma| > m_1$, (B2) can be satisfied by letting

$$m_1 = \frac{\max(p_2 e_v \dot{\sigma})}{k_D}$$
$$= 11 v_{rb} |p_2| \frac{\max(\dot{\sigma})}{k_D} \quad \text{(from (22))}$$
$$= 11 v_{rb} |p_2| \frac{\max(s w - \frac{2}{p_4} k_D \operatorname{sgn}(\sigma))}{k_D} \quad \text{(from (27))}$$
$$= 11 v_{rb} |p_2| (\max(\frac{s w}{k_D}) + \frac{2}{p_4}) \quad \text{(since } p_4 > 0)$$
$$= 11 v_{rb} |p_2| (\frac{k_i v_{rb}}{k_D} + \frac{2}{p_4}) \quad \text{(from (18))}$$

which gives condition (34). In conclusion, conditions (33) and (34) ensure that $\dot{V} \le 0$.