Precise Analytical Modelling Magnetic Characteristics of Switched Reluctance Motor Drives Using Two-Dimensional Least Squares

X. D. Xue, K. W. E. Cheng, S. L. Ho and D.Sutanto
Power Electronics Research Centre
Department of Electrical Engineering, the Hong Kong Polytechnic University
Hong Hum, Kowloon, Hong Kong, P. R. of China
http://perc.polyu.edu.hk

Abstract The precise analytical modeling magnetic characteristics in the switched reluctance motors is presented in this study, based on the two-dimensional least squares. The proposed precise modelling is the analytical expression with respect to both the rotor position and the current. The coefficients in the expression are determined by the two-dimensional least squares method. The derivations of the modelling are described in detail. Furthermore, the effect of the degree number of the fitting polynomials on the fitting errors is discussed. The fitting curves from the proposed modelling are fully in agreement with ones from the experiment. It validates the proposed modelling. This study is very helpful for accurate prediction of performance, simulation, torque control, as well as sensorless control of the switched reluctance motor drives.

I. INTRODUCTION

Accurate descriptions of nonlinear magnetic characteristics in the switched reluctance motor drives are crucial when performing performance predictions, simulations, computer-aided designs, as well as sensorless control of the switched reluctance motor drives. Nonlinear magnetic characteristics in the switched reluctance motor drives can be described accurately by use of the nonlinear interpolation methods [1] [2]. To implement rapid simulation and real-time computations, however, establishing precise analytical models of nonlinear magnetic characteristics is a challenging area for researchers.

In general, nonlinear magnetic characteristics in the switched reluctance motor drives are obtained from measurements on existing motor or from numerical computations such as finite element (FE) analysis. It is desirable to model analytically and accurately the nonlinear magnetic characteristics. Such analytical models should provide all of the magnetic information for arbitrary rotor position and arbitrary current, and match the experimental or numerical data. Some publications offered their contributions to analytical modeling of nonlinear magnetic characteristics in the switched reluctance motor drives [3]-[7].

In [3], Torrey and Lang present an analytical expression for the flux linkage/current/position data. The equation can provide all of the magnetic information. But the expression is a complicated function that includes the cosine function and exponential function, in which the coefficients are needed to be computed by Fourier cosine series. On the basis of [3], Torrey, Niu, and Unkauf developed the piece-wise linear model [4]. The nonlinear magnetization characteristics of the switched reluctance motor drive are modeled analytically by the piecewise first- or second-order functions of the flux linkage against the rotor position, with the current as an undermined parameter [5]. In the method of [5], the rotor position region is divided into the three sub-regions and the current region is divided into the two sub-regions. Chan and Weldon developed a nonlinear switched reluctance motor model, in which the multiple shape functions are used to fit magnetic data. The shape function is a complicated function that is composed of the exponential function and the logarithm function [6]. In [7], the nonlinear model of the switched reluctance motor uses the equivalent magnetic circuit of the motor as a set of reluctances linked in series and in parallel.

Least squares are widely applicable to science and engineering and are very efficient tools to precisely fit discrete data [8]-[10]. The two-dimensional fitting function is directly regarded as the model of nonlinear magnetic characteristics in the switched reluctance motor drives, because the flux linkage in the switched reluctance motor drives is the function of both the rotor position and the phase current. This study just utilizes the two-dimensional least squares method to model nonlinear magnetic characteristics in the switched reluctance motor drives. The polynomials, in which the rotor position and the phase current are directly defined as the variables, are chosen as the fitting function. The coefficients in the fitting function are determined by the two-dimensional least squares. The detailed derivations are described in this paper. Furthermore, the effect of degree number in the fitting polynomials on the model errors is discussed. The comparisons between the fitting curves/surface and the experimental curves/surface verify the proposed modelling of nonlinear magnetic characteristics in the switched reluctance motor drive. Because the proposed modelling is based on the two-dimensional least squares and obtained through the strict mathematical derivations, this modelling is more accurate than the previous reported models

0-7803-7754-0/03/$17.00 ©2003 IEEE

of magnetic characteristics in the switched reluctance motor drives. Very good agreements of the fitting results with the experimental results can also testify it. All the coefficients in the proposed analytical modelling can be computed in advance. One only needs to determine the few data and can obtain flux linkage values at arbitrary rotor position and at arbitrary current using the proposed precise analytical modelling. This will enhance the rapidity and accuracy of the computation markedly. Thus, this study will be very useful for accurate prediction of performance, simulation, torque control, as well as sensorless control of switched reluctance motor drives.

II. Establishing Model and Determination of the Coefficients

A. Modelling Nonlinear Magnetic Characteristics

Assuming that the $n\times m$ flux linkage values ψ_{kj} with respect to the rotor position θ_k and the phase current i_j are obtained through the measurement on existing motor or through the numerical computation (k=0, 1, ... , n-1; j=0, 1, ... , m-1), the model based on the fitting polynomials of the two-dimensional least squares is given by (1).

$$\psi(\theta,i)=\sum_{k=0}^{p-1}\sum_{j=0}^{q-1}a_{kj}\theta^k i^j \tag{1}$$

where $p\leq n$ and $q\leq m$.

Equation (1) shows that the proposed model is the two-dimensional polynomials, in which the highest degree of the rotor position θ is (p-1) and the highest degree of the phase current i is (q-1).

B. Determination of Coefficients a_{kj}

1) *Construction of the fitting polynomial with respect to the rotor position*: For the rotor position θ, the m fitting polynomials based on the least squares are constructed, as shown in (2).

$$g_j(\theta)=\sum_{u=0}^{p-1}\lambda_{uj}\gamma_u(\theta), j=0,1,\cdots,m-1 \tag{2}$$

where the total $\gamma_u(\theta)$ (u=0,1,..., p-1) are the orthogonal polynomials and are determined by the following (3).

$$\begin{aligned}&\gamma_0(\theta)=1\\&\gamma_1(\theta)=\theta-\alpha_0\\&\gamma_{u+1}(\theta)=(\theta-\alpha_u)\gamma_u(\theta)-\beta_u\gamma_{u-1}(\theta), u=1,2,\cdots,p-1\end{aligned} \tag{3}$$

If letting,

$$d_u=\sum_{k=0}^{n-1}\gamma_u^2(\theta_k), u=0,1,\cdots,p-1 \tag{4}$$

then one can obtain

$$\begin{aligned}&\alpha_u=\sum_{k=0}^{n-1}\theta_k\gamma_u^2(\theta_k)/d_u, u=0,1,\cdots,p-1\\&\beta_u=d_u/d_{u-1}, u=1,2,\cdots,p-1\end{aligned} \tag{5}$$

From the least squares, (6) can be yielded.

$$\lambda_{uj}=\sum_{k=0}^{n-1}\psi_{kj}\gamma_u(\theta_k)/d_u \quad \begin{aligned}&j=0,1,\cdots,m-1;\\&u=0,1,\cdots,p-1\end{aligned} \tag{6}$$

2) *Construction of the fitting polynomial with respect to the current*: For the current i, the fitting polynomial based on the least squares is determined by (7).

$$h_u(i)=\sum_{v=0}^{q-1}\mu_{uv}\eta_v(i), u=0,1,\cdots,p-1 \tag{7}$$

where the total $\eta_v(i)$ (v=0,1,...,q-1) are the orthogonal polynomials and are computed by the following (8).

$$\begin{aligned}&\eta_0(i)=1\\&\eta_1(i)=i-\alpha_0'\\&\eta_{v+1}(i)=(i-\alpha_v')\eta_v(i)-\beta_v'\eta_{v-1}(i), v=1,2,\cdots,q-1\end{aligned} \tag{8}$$

If letting,

$$\delta_v=\sum_{j=0}^{m-1}\eta_v^2(i_j), v=0,1,\cdots,q-1 \tag{9}$$

then (10) can be obtained.

$$\begin{aligned}&\alpha_v'=\sum_{j=0}^{m-1}i_j\eta_v^2(i_j)/\delta_v, v=0,1,\cdots,q-1\\&\beta_v'=\delta_v/\delta_{v-1}, v=1,2,\cdots,q-1\end{aligned} \tag{10}$$

From the least squares, one can yield (11).

$$\mu_{uv}=\sum_{j=0}^{m-1}\lambda_{uj}\eta_v(i_j)/\delta_v \quad \begin{aligned}&u=0,1,\cdots,p-1\\&v=0,1,\cdots,q-1\end{aligned} \tag{11}$$

C. Two-Dimensional Fitting Polynomials

From the above derivations, the two-dimensional fitting polynomials based on the least squares can be obtained as shown in (12).

$$\psi(\theta,i)=\sum_{u=0}^{p-1}\sum_{v=0}^{q-1}\mu_{uv}\gamma_u(\theta)\eta_v(i) \tag{12}$$

It is seen that (12) can be changed into the standard form of (1). To avoid the computation overflow, the following modelling should be employed instead of (1) [8].

$$\psi(\theta,i)=\sum_{k=0}^{p-1}\sum_{j=0}^{q-1}a_{kj}(\theta-\overline{\theta})^k(i-\overline{i})^j \tag{13}$$

where

$$\begin{aligned}\overline{\theta}&=\sum_{k=0}^{n-1}\theta_k/n\\ \overline{i}&=\sum_{j=0}^{m-1}i_j/m\end{aligned} \tag{14}$$

It is clear from (13) that the presented analytical modelling is the two-dimensional polynomials respect to the rotor position and the current. The number of the coefficients in the modelling depends on the number of the degrees. Eqn (13) can be utilized to compute the flux linkage values at arbitrary rotor position and at arbitrary current. Thus, rapider computation is obtained and less computer memory is taken up, using the presented analytical modelling than interpolation method.

III. APPLICATIONS

For the prototype of the switched reluctance motor drive in this study, the main data are written as follows: Number of the phases = 4 stator poles = 8 rotor poles = 6 phase resistance = 0.687 Ω, the phase flux linkage when the stator pole is aligned with the rotor pole = 0.4164 Wb at the phase current = 12 A, the phase flux linkage when the stator pole is unaligned with the rotor pole = 0.0839 Wb at the phase current = 12 A, the phase flux linkage when the stator pole is aligned with the rotor pole = 0.1676 Wb at the phase current = 2 A, and the phase flux linkage when the stator pole is unaligned with the rotor pole = 0.01264 Wb at the phase current = 2 A.

For the prototype of the four-phase switched reluctance motor drive, the period is equal to 60 mechanical degree. Because the flux linkage values are symmetrical about the rotor position of 30 degree within a period, it is enough to study fitting the magnetic data within half a period. Within half a period, the 13 rotor position data from 0 degree to 30 degree, the 7 current data from 0 A to 12 A, and the corresponding 13×7 flux linkage data are obtained through the experiment (i.e., m=13 and n=7). Thus, it is selected that the maximum degree of the fitting polynomials with respect to the rotor position is equal to 7 and the maximum degree of the fitting polynomial with respect to the current is equal to 6 (i.e., p=8 and q=7).

$\overline{\theta}$ and $\overline{i}$ in (14) is equal to 15 degree and 6.0 A, respectively. From the above derivations in Section II, the computation results of the coefficients a_{kj} (k = 0, 1, ..., 7 and j = 0, 1, ..., 6) in (14) are shown in Table I.

Fig. 1 illustrates the changes of the phase flux linkage with the phase current from both the experiment and the proposed modelling. Whereas the changes of the phase flux linkage with the rotor position angle from the experiment and the proposed modelling can be seen in Fig. 2. Table II shows the errors between the presented modelling and the experimental data.

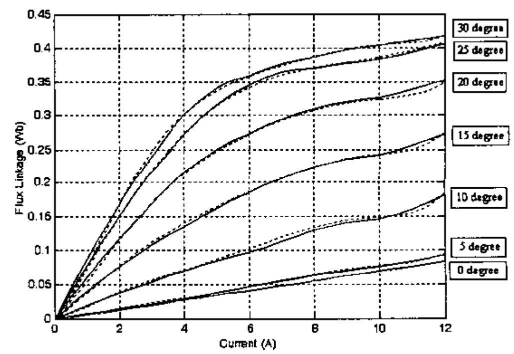

Fig. 1. Comparisons between the experiment and the presented modelling (the solid curves from the experiment and the dotted curves from the presented modelling)

TABLE I
COEFFICIENTS IN THE PRESENTED MODELLING

k↓ j→	0	1	2	3	4	5	6
0	0.185506E+00	0.212722E-01	-0.681549E-03	-0.100343E-03	-0.892657E-04	0.403938E-05	0.197376E-05
1	0.172323E-01	0.464162E-03	-0.266467E-03	0.662989E-04	-0.811327E-06	-0.120180E-05	0.251275E-07
2	0.495901E-04	-0.580348E-04	-0.231760E-04	0.220016E-05	0.666872E-06	-0.465463E-07	-0.673947E-08
3	-0.138488E-05	0.821745E-05	-0.316703E-05	-0.659221E-06	0.236683E-06	0.157412E-07	-0.347280E-08
4	0.599126E-06	-0.401359E-07	0.400549E-07	0.195339E-07	0.578146E-08	-0.329661E-09	-0.174014E-09
5	-0.319811E-06	-0.799320E-07	0.418121E-07	0.410335E-08	-0.262243E-08	-0.100879E-09	0.393419E-10
6	-0.244672E-08	0.531650E-09	0.230158E-09	-0.786641E-10	-0.392510E-10	0.142795E-11	0.907334E-12
7	0.864888E-09	0.182501E-09	-0.108192E-09	-0.817040E-11	0.658349E-11	0.202721E-12	-0.984998E-13

TABLE II
ERRORS BETWEEN THE PRESENTED MODELLING AND THE EXPERIMENT

Errors	*SSE*	*SAVE*	*MVAE*	*MRE*
Values	0.3637939E-03	0.1317916E+00	0.5833498E-02	0.9933754E-01

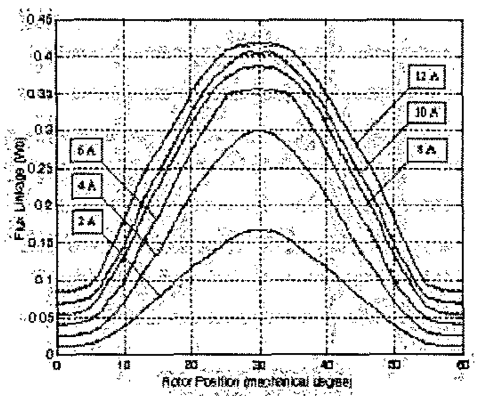

Fig. 2. Comparisons between the experiment and the presented modelling (the solid curves from the experiment and the dotted curves from the presented modelling)

It is clear from Fig. 1 and Fig. 2 that the experimental curves of the flux linkage agree very well with the fitting curves from the proposed modelling. This can be also observed from Table II, which shows the computation results of the four errors between the experimental and fitting values.

In Table II, *SSE* represents the sum of the squares errors, which is determined by (15); *SAVE* represents the sum of the absolute values of the errors, which is computed from (16); *MAVE* denotes the maximum value of the absolute errors, which is determined by (17); and *MRE* denotes the maximum relative error, which is computed from (18).

$$SSE = \sum_{k=0}^{n-1} \sum_{j=0}^{m-1} [f(\theta_k, i_j) - \psi_{kj}]^2 \tag{15}$$

$$SAVE = \sum_{k=0}^{n-1} \sum_{j=0}^{m-1} \left| f(\theta_k, i_j) - \psi_{kj} \right| \tag{16}$$

$$MAVE = \max_{\substack{0 \le k \le n-1 \\ 0 \le j \le m-1}} \left| f(\theta_k, i_j) - \psi_{kj} \right| \tag{17}$$

$$MRE = \frac{MAVE}{\psi_{kjm}} \tag{18}$$

where $f(\theta_k, i_j)$ represents the fitted flux linkage values from the presented modelling, ψ_{kj} represents the flux linkage values from the experiment, and ψ_{kjm} is the flux linkage value with respect to *MAVE*.

It can be seen from Table II that the four errors are all quite small. Therefore, Fig. 1, Fig. 2, and Table II verify that the proposed analytical modelling based on the two-dimensional least squares can precisely model nonlinear magnetic characteristics in the switched reluctance motor drives. This indicates that the presented analytical modelling is more accurate than the previously reported analytical models because this modelling is based on the two-dimensional least squares and the strict mathematical derivations.

Fig. 3 depicts the effect diagram from the experiment, and Fig. 4 depicts the one from the proposed analytical modelling. It can be seen from Fig. 3 and Fig. 4 that flux linkage values at arbitrary rotor position and at arbitrary current can be computed accurately from the analytical modelling presented in this study and based on the few given data.

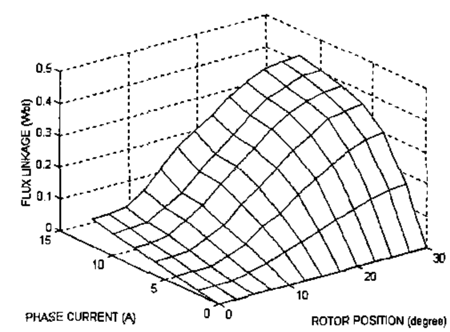

Fig. 3. Flux linkage with respect to the rotor position and the current from the experiment within half a period

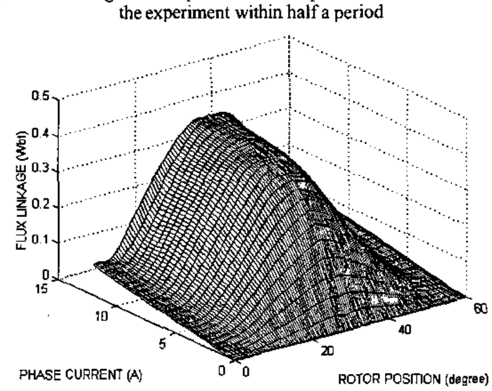

Fig. 4. Flux linkage with respect to the rotor position and the current from the proposed analytical modelling within a period

IV. Discussions of the Accuracy of the Modelling

The accuracy of the presented analytical modelling can be verified by the fitting errors of the modelling, which includes *SSE*, *SAVE*, *MAVE*, and *MRE* in this study and which are defined in the last section. In general, the fitting errors of the presented analytical modelling depend on the maximum degrees of the fitting polynomials in the modelling. However, that does not mean that the higher degree in the modelling results in the smaller fitting errors [8]. Furthermore, the higher degrees in the modelling are not propitious to the rapid computation. Hence, the determination of the maximum degrees of both the rotor position and the current in the modelling should take account of the number of the given rotor position angles, the number of the given currents, the fitting errors of the modelling, and the rapidity of the computation. The results from Fig. 5 to Fig. 8 show the changes of the four fitting errors with the maximum degrees in the modelling. In the figures, the numbers of the abscissa represent the index of the maximum degree groups and also determine the values of p and q, which are shown in Table III.

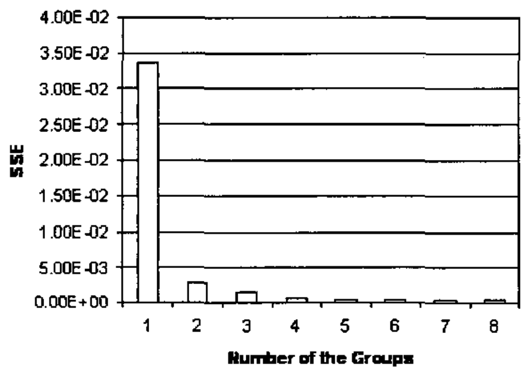

Fig. 5. Sums of the square errors

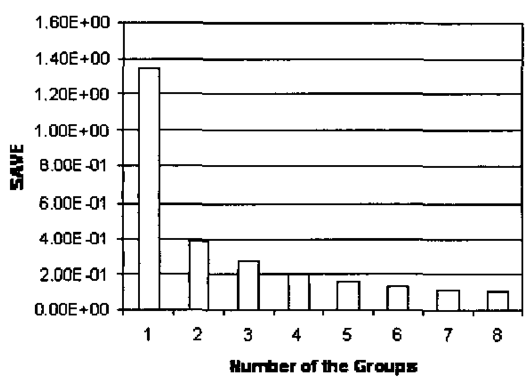

Fig. 6. Sums of the absolute values of the errors

TABLE III
Maximum Degrees of the Rotor Position and the Current

Groups	1	2	3	4
P	3	4	5	6
Q	3	4	5	6
Groups	5	6	7	8
P	7	8	9	10
q	7	7	7	7

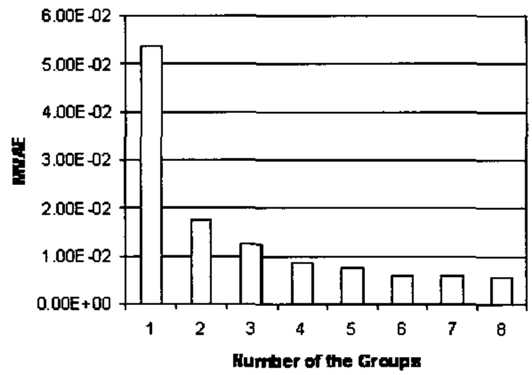

Fig. 7. Maximum values of the absolute errors

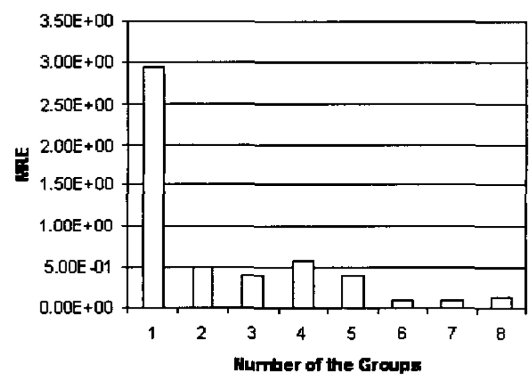

Fig. 8. Maximum relative errors

It is seen from Fig. 5 to Fig. 8 that the degrees in the modelling have a great effect on the fitting errors. If the degrees in the modelling are selected to the small values, such as the degrees of Group 1 in Table III, the four errors values are fairly large and thus the accuracy of the modelling is poor. When the degrees go up from small to large, the four errors values decrease fleetly. However, the fitting errors values almost do not vary if the degrees are selected to the larger values, such as the values more than those in Group 6. This study suggests that *MRE* is used to determine the maximum degrees of the fitting polynomials. In this study, the maximum degrees of the fitting polynomials must satisfy $MRE \leq 0.1$. Under this constraint, it is determined that p=8 and q=7.

V. CONCLUSIONS

- This paper presents a novel precise analytical modelling nonlinear magnetic characteristics in the switched reluctance motor drives, based on the two-dimensional least squares. The modelling is the two-dimensional polynomials with respect to both the rotor position and the phase current. The few coefficients in the modelling can be determined in advance from the few given data. The modelling can be used to compute flux linkage values at arbitrary rotor position and at arbitrary current.
- The experimental and computational results show that the fitting values from the proposed modelling agree very well with the given values from the experiment and the fitting errors are very small. Thus, the proposed modelling in this paper can precisely describe nonlinear magnetic characteristics in the switched reluctance motor drives.
- The proposed analytical modelling can enhance the rapidity and accuracy of the computation markedly. This study is very helpful for accurate prediction of performance, simulation, torque control, as well as sensorless control of the switched reluctance motor drives.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the financial support of the RGC of Hong Kong (Project code: PolyU5085/98E) and Research Committee of the Hong Kong Polytechnic University (Project code: G-W081).

REFERENCES

[1] X. D. Xue, K. W. E. Cheng, and S. L. Ho, "Simulation of Switched Reluctance Motor Drives Using Two-dimensional Bicubic Spline", *IEEE Transactions on Energy Conversion,* Vol. 17, No. 4, December 2002, pp471-477.

[2] D.W.J. Pulle, "New data base for switched reluctance drive simulation", *IEE PROCEEDINGS-B,* Vol. 138, No. 6, November 1991, pp331-337.

[3] D.A. Torrey and J.H. Lang, "Modelling a nonlinear variable-reluctance motor drive", *IEE PROCEEDINGS,* Vol. 137, Pt. B, No. 5, September 1990, pp314-326.

[4] D.A. Torrey, X.-M. Niu, and E.J. Unkauf, "Analytical modelling of variable-reluctance machine magnetisation characteristics", *IEE Proc.-Electric Appl.,* Vol. 142, No.1, January 1995, pp14-22.

[5] T.J.E. Miller and M. McGlip, "Nonlinear theory of the switched reluctance motor for rapid computer-aided design", *IEE PROCEEDINGS,* Vol. 137, Pt. B, No. 6, November 1990, pp337-347.

[6] W.M. Chan and W.F. Weldon, "Development of a Simple Nonlinear Switched Reluctnace Motor Model using Measured Flux Linkage Data and Curve Fit", *Thirty-Second IAS Annual Meeting, IAS'97,* Vol. 1, 1997, pp318-325.

[7] V. Vujicic and S. N. Vukosavic, "A Simple Nonlinear Model of the Switched reluctance Motor", *IEEE TRANSACTIONS ON ENERGY CONVERSION,* Vol. 15, No. 4, December 2000, pp395-400.

[8] Shiliang Xu, *The Frequently-used Algorithms and Programs Using C,* Tsinghua University Publisher, Beijing, China, 1994.

[9] Charles L. Lawson and Richard J. Hanson, *Solving Least Squares Problems,* Prentice-Hall, Inc., Englewood Clifes, New Jersey, 1974.

[10] Samuel S. M. Wong, *Computational Methods in Physics and Engineering,* World Scientific, 1997.