Directional Independent Component Analysis with Tensor Representation

Lei Zhang[1], Quanxue Gao[1,2] and David Zhang[1]

[1]Biometric Research Center, The Hong Kong Polytechnic University, Hong Kong, China
[2]School of Telecommunications Engineering, XIDIAN University, Xi'an, China
cslzhang@comp.polyu.edu.hk

Abstract

Conventional independent component analysis (ICA) learns the statistical independencies of 2D variables from the training images that are unfolded to vectors. The unfolded vectors, however, make the ICA suffer from the small sample size (SSS) problem that leads to the dimensionality dilemma. This paper presents a novel directional multilinear ICA method to solve those problems by encoding the input image or high dimensional data array as a general tensor. In addition, the mode-k matrix of the tensor is re-sampled and re-arranged to form a mode-k directional image to better exploit the directional information in training. An algorithm called mode-k directional ICA is then presented for feature extraction. Compared with the conventional ICA and other subspace analysis algorithms, the proposed method can greatly alleviate the SSS problem, reduce the computational cost in the learning stage by representing the data in lower dimension, and simultaneously exploit the directional information in the high dimensional dataset. Experimental results on well-known face and palmprint databases show that the proposed method has higher recognition accuracy than many existing ICA, PCA and even supervised FLD schemes while using a low dimension of features.

1. Introduction

How to find a suitable representation of the data is a key problem in pattern analysis, such as face recognition. Many subspace analysis methods (SAM) [1-4] have been proposed to represent the high-dimensional data into a compact low-dimensional space to extract faithfully the meaningful and unique structures embedded in the data. The most representative unsupervised SAM technique may be the principal component analysis (PCA) [1-2]. PCA exploits the second-order correlation of the training datasets but it ignores the higher-order statistical dependencies, which may contain more structural information of the 2D or higher dimensional data for the subsequent feature classification [3, 4].

Independent component analysis (ICA), as an extension of PCA, extracts a set of statistically independent components by analyzing the higher-order statistics in the training dataset [5]. Many schemes have been reported recently by using ICA for face representation and recognition [6-9]. These works can be generally classified into two groups: one is to study how to evaluate the performance of ICA [6-7] and the other is to study how to improve the performance of ICA in feature extraction and classification [8-9]. In those algorithms, linear algebra is used to extract the feature of independent components (IC). Thus they are hard to distinguish the statistic features arise from different factors, or modes, inherent to image formation, such as viewpoint, illumination, etc., [4]. To overcome this problem of different imaging factors, recently Vasilescu *et al* [4] used multilinear algebra to represent the datasets and extract ICs and they obtained better performance.

However, all the above algorithms stretch the input image into a vector for IC extraction. The unfolded vector may lose some structural information embedded in image and will lead to a very high dimensionality of the data for the subsequent analysis. The available number of training samples is usually much smaller than the dimensionality of unfolded vector in practical applications. This is the so called small sample size (SSS) problem in SAM based pattern recognition. Some recent works have been taking the image directly as a two dimensional matrix or a high-order tensor for statistical learning and have obtained good results in biometric authentication [10-13]. However, few works have been reported for two dimensional matrix or tensor based ICA. Although a tensor representation is used in [4], the input image is still unfolded to a vector before applying ICA.

In this paper, we investigate how to implement ICA by encoding the image as a 2^{nd} (or higher order) tensor, and propose a framework for IC extraction by using directional tensor image representation. The proposed method uses multiple interrelated subspaces corresponding to different tensor dimensions rather than one subspace as in traditional ICA for IC extraction. An efficient learning procedure is presented via a novel tensor analysis, called mode-*k*

978-1-4244-2243-2/08/$25.00 ©2008 IEEE

directional ICA. Different from traditional tensor analysis that directly learns mode-k subspace from the mode-k tensor images, the proposed mode-k directional ICA learns the low-dimensional subspace from the mode-k directional images, which are formed by re-sampling and re-arranging the mode-k matrix of the original tensor. The mode-k directional images are viewed as the new subjects to be analyzed in the k^{th} subspace.

Compared with conventional ICA algorithms, the proposed method alleviates greatly the SSS problem and hence the dimensionality dilemma. In the proposed mode-k directional ICA, the dimensionality of variables is reduced to the k^{th} dimension of the tensor image, while the sample size is increased by a large factor. On the other hand, more useful structural information embedded in training images is preserved and the directional information can also be embedded in the tensor representation. Experiments on UMIST and AR face databases and the palmprint database show that the proposed method achieves higher recognition accuracy while using a lower dimension of features.

The remainder of this paper is organized as follows. Background and notations of ICA and multilinear algebra are presented in section 2. The multilinear directional ICA algorithm is presented in Section 3. Section 4 presents extensive experiments and Section 5 concludes the paper.

2. Background and Notations

2.1. Independent component analysis (ICA)

In [3], Bartlett *et al* proposed two architectures for ICA. Here we use the architecture I. Denote by $\boldsymbol{x}$ a p-dimensional image vector, the ICA of $\boldsymbol{x}$ seeks for a sequence of projection vectors $\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_q$ ($q<p$) to maximize the statistical independence of the projected data. It can be expressed as follows:

$$\boldsymbol{s} = \boldsymbol{W}^T \boldsymbol{x} \tag{1}$$

where $\boldsymbol{s}$ denotes the ICs of $\boldsymbol{x}$ and $\boldsymbol{W} = \left[\boldsymbol{w}_1, \cdots, \boldsymbol{w}_q\right]$ is called the projection matrix.

Various criteria, such as those based on mutual information, negentropy and higher-order cumulants, have been proposed for computing $\boldsymbol{W}$ [5]. Among them the FastICA algorithm has been widely used in pattern recognition [5, 9]. Usually, PCA is implemented to whiten the data and reduce the dimensionality before applying ICA.

Natural images are usually represented in the form of matrices (2^{nd} order tensor) or higher-order tensors. Therefore it is not well suited to represent natural images using one-dimensional vectors. The image-to-vector transformation also leads to the SSS problem and the dimensionality dilemma. To address these problems in conventional ICA, we will propose a novel multilinear directional ICA scheme in Section 3.

2.2. Multilinear algebra

This section briefly introduces the concepts and notations of multilinear algebra [4, 14] that will be used in the following development.

A tensor is a higher order generalization of a vector (1^{st} order tensor) and a matrix (2^{nd} order tensor) and it is a multilinear mapping over a set of vector spaces. Denote by $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$ a tensor of order K. The size of the k^{th} dimension of $\mathcal{A}$ is I_k. An element of $\mathcal{A}$ is denoted as $\mathcal{A}_{i_1 \cdots i_K}$ or $a_{i_1 \cdots i_K}$, where $1 \le i_k \le I_k$. In tensor terminology, matrix column and row vectors are referred to as mode-1 and mode-2 vectors, respectively. For higher-order tensors, we have the following definitions.

Definition 1. [14] (Mode-k matrixizing or matrix unfolding) The mode-k matrixizing or matrix unfolding of a K^{th} order tensor $\mathcal{A}$ is a matrix $\boldsymbol{D}_{(k)} \in \mathbb{R}^{I_k \times I_{\bar{k}}}$, $I_{\bar{k}} = \left(\prod_{j \ne k} I_j\right)$. $\boldsymbol{D}_{(k)}$ is the ensemble of vectors in $\mathbb{R}^{I_k}$ obtained by keeping index i_k fixed and varying the other indices.

Definition 2. [14] (Mode-k product) The mode-k product $\mathcal{A} \times_k \boldsymbol{U}$ of a tensor $\mathcal{A}$ and a matrix $\boldsymbol{U} \in \mathbb{R}^{I_k \times I_k'}$ is a $I_1 \times I_2 \times \ldots \times I_{k-1} \times I_k' \times I_{k+1} \times \ldots \times I_K$ tensor defined by

$$\left(\mathcal{A} \times_k \boldsymbol{U}\right)_{i_1 \times i_2 \times \ldots \times i_{k-1} \times j \times i_{k+1} \times \ldots \times i_K} = \sum_{i_k} \left(\mathcal{A}_{i_1 \times i_2 \times \ldots \times i_{k-1} \times i_k \times i_{k+1} \ldots \times i_K} \boldsymbol{U}_{j \times i_k}\right) \tag{2}$$

for all index values. The mode-k product is a type of contraction.

Definition 3. [4, 14] (Mode-k vectors) The mode-k vectors of a K^{th} order tensor $\mathcal{A}$ are the I_k-dimensional vectors obtained from $\mathcal{A}$ by varying index i_k while keeping the other indices fixed. The mode-k vectors are the column vectors of matrix $\boldsymbol{D}_{(k)} \in \mathbb{R}^{I_k \times (I_1 \cdots I_{k-1} I_{k+1} \cdots I_K)}$ resulted by mode-k matrixizing the tensor $\mathcal{A}$.

Definition 4. [14] (k-rank) The k-rank of tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$, denoted by R_k, is defined as the dimension of the vector space generalized by the mode-k vectors.

Definition 5. [4] The distance between tensors $\mathcal{A}$ and $\mathcal{B}$ is defined as

$$d\left(\mathcal{A}, \mathcal{B}\right) = \sqrt{\left\langle \mathcal{A} - \mathcal{B}, \mathcal{A} - \mathcal{B} \right\rangle} \tag{3}$$

where $\left\langle \mathcal{A}, \mathcal{B} \right\rangle = \sum_{i_1=1, \cdots, i_K=1}^{I_1, \cdots, I_K} \mathcal{A}_{i_1 \cdots i_K} \mathcal{B}_{i_1 \cdots i_K}$ denotes the inner product of tensors $\mathcal{A}$ and $\mathcal{B}$ with the same dimension.

3. Directional Tensor ICA

3.1. Model of ICA with tensor representation

Most existing ICA algorithms consider an image as a vector and thus have a very high dimension of feature space. As a result, these methods suffer from the SSS problem due to the dimensionality dilemma. Images can be more naturally represented as 2nd or higher order tensors. In this section, we study how to perform ICA with a general tensor representation of images.

Given an arbitrary K^{th} order tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$, it can be expressed as follows [14]

$$\mathcal{S} = \mathcal{A} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \cdots \times_K \boldsymbol{U}_K \quad (4)$$

where tensor $\mathcal{S}$, called the core tensor, governs the interaction between the matrices $\boldsymbol{U}_k \in \mathbb{R}^{L_k \times I_k}$ ($L_k < I_k$, k=1,2,…,K), which are called lower-dimensional independent subspaces. Matrix $\boldsymbol{U}_k$ contains the orthogonal vectors spanning the column space of the matrix $\boldsymbol{D}_{(k)}$ that resulted from the mode-*k* matrixizing of $\mathcal{A}$. Our goal is to find *K* transformation matrices $\boldsymbol{U}_k$ such that the elements of $\mathcal{S}$ are as independent as possible.

The proposed model may seem similar to the model proposed by Vasilescu [4]. However, they are very different in essence. In model (4), $\mathcal{A}$ denotes a high dimensional dataset (e.g. an image) that is represented in a tensor form rather than a vector as in [4]. It can thus alleviate significantly the SSS problem in SAM. The matrices $\boldsymbol{U}_k$ in the proposed model can be used to represent the mode-*k* dimension of tensor images with factors such as illumination, pose, etc.

In this paper, we will propose a framework to estimate multiple subspaces $\boldsymbol{U}_k$ in (4) by obtaining the mode-*k* directional images from tensor $\mathcal{A}$ and then viewing the mode-*k* vectors of the directional images as training samples. The method will be described detailedly in the following sub-sections.

3.2. Mode-*k* directional image

In subspace analysis with a tensor representation, the k^{th} subspace, containing the k^{th} dimension structural information of the tensor, can be directly calculated from the mode-*k* matrix obtained by unfolding the tensor using Definition 1. However, the mode-*k* matrixizing ignores the relationship between the current dimension and other dimensions, which is related to the image formation process and may be useful for classification. To solve this problem and improve the classification accuracy, we propose to use mode-*k* directional images to estimate the k^{th} subspace. The mode-*k* directional image is defined and obtained from the original tensor as follows.

Definition 6. (Mode-*k* directional image) Given a K^{th} order tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$, the mode-*k* ($k$=1,2,…,$K$) directional image is obtained as follows:

i.) Obtain the mode-*k* matrix $\boldsymbol{D}_{(k)} \in \mathbb{R}^{I_k \times I_{\bar{k}}}$ from tensor $\mathcal{A}$ via Definition 1.

ii.) Re-sample and re-arrange the mode-*k* matrix $\boldsymbol{D}_{(k)}$ along the $I_{\bar{k}}$-dimensional direction to generate the mode-*k* directional image $\boldsymbol{B}_{(k)}$, as shown in Fig. 1. The integer parameter *l* controls the resulted directional image.

Definition 7. (Mode-*k* directional vectors) The mode-*k* directional vectors of a K^{th} order tensor $\mathcal{A}$ are the I_k-dimensional vectors of directional image $\boldsymbol{B}_{(k)}$, i.e., mode-2 vectors of $\boldsymbol{B}_{(k)}$.

When *l* is equal to $I_{\bar{k}}$, the mode-*k* directional image $\boldsymbol{B}_{(k)}$ is the mode-*k* matrix $\boldsymbol{D}_{(k)}$ and the re-sampling direction is of zero degree. When *l* is 1, the directional image $\boldsymbol{B}_{(k)}$ will be the diagonal image of $\boldsymbol{D}_{(k)}$. In Fig. 2, we show an examples when *l* is 2 and 4 for the 2nd order tensor, i.e. two dimensional image. The mode-1 and mode-2 directional images of the original image (Fig. 2 (a)) are shown in Figs. 2 (b), (c), (d) and (e), respectively, where the value of *l* is 2 in (b) and (c), and is 4 in (d) and (e).

By constructing the mode-*k* directional image, more directional information of the original tensor can be embedded. Compared with the mode-*k* matrix $\boldsymbol{D}_{(k)}$, mode-*k* directional image $\boldsymbol{B}_{(k)}$ will be able to employ the pixels along the $I_{\bar{k}}$-dimensional direction for training and feature extraction. Next in section 3.3 we introduce the mode-*k* directional ICA and then in section 3.4 we present the directional tensor ICA algorithm.

3.3. Mode-*k* directional ICA

Before introducing the directional tensor ICA, we first summarize the mode-*k* directional ICA of image $\mathcal{A}$ as follows:

i.) For k=1, 2,…, K, compute the mode-*k* matrixizing matrix $\boldsymbol{D}_{(k)}$ according to Definition 1.

ii.) Form the mode-*k* directional image $\boldsymbol{B}_{(k)}$ according to Definition 6.

iii.) Take the mode-*k* directional vectors (refer to Definition 7) of $\mathcal{A}$ as training samples, and compute the matrix $\boldsymbol{U}_k$ in (4) by using the FastICA algorithm.

Dimensionality reduction in the linear case does not have a trivial multilinear counterpart. According to [4, 14], a useful generalization to tensor involves an optimal rank approximation which iteratively optimizes each of the modes of the given tensor. Each optimization step will involve a best reduced-rank approximation of a positive semi-definite symmetric matrix. This is a high-order extension of the orthogonal iteration for matrices. The proposed mode-*k* directional ICA algorithm avoids the iterative step in training.

3.4. Directional tensor ICA algorithm

With the above development, the multilinear directional ICA algorithm can be summarized as follows:

i.) Input the original training dataset $\mathcal{A}_i \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$, i=1,2,…, N, where N is the number of training samples and K is the order of tensor $\mathcal{A}_i$. Set the dimensionality of the output tensor $\mathcal{S}_i \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$.

ii.) Training stage

For k=1, 2,…, K

Calculate $\boldsymbol{D}^i_{(k)} \leftarrow \mathcal{A}_i$ using Definition 1;

Form the directional image $\boldsymbol{B}^i_{(k)}$ for $\boldsymbol{D}^i_{(k)}$ by using Definition 6;

Calculate the matrix $\boldsymbol{U}_k$ for $\boldsymbol{B}^i_{(k)}$ using the mode-*k* ICA algorithm in section 3.3.

End

iii.) Extract the ICs as follows:

$$\mathcal{S}_i = \mathcal{A}_i \times_1 \boldsymbol{U}_1 \cdots \times_K \boldsymbol{U}_K, \quad i = 1,2,\cdots,N$$

iv.) Extract ICs of the probe image $\mathcal{A}^*$:

$$\mathcal{S}^* = \mathcal{A}^* \times_1 \boldsymbol{U}_1 \cdots \times_K \boldsymbol{U}_K$$

v.) Classification based on the distance (refer to Definition 5) between $\mathcal{S}_i$ and $\mathcal{S}^*$.

Note that the discriminability of each column of $\boldsymbol{U}_k$ is unknown in prior. In the experiments, we used the method proposed in [3] to rearrange the column of $\boldsymbol{U}_k$ to improve the classification accuracy and reduce the dimensionality of features.

3.5. Discussions

In the whitening stage of conventional ICA, the size of the covariance matrix will be $\prod_{k=1}^{K} I_k \times \prod_{k=1}^{K} I_k$ if we unfold a tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_K}$ to a vector. Usually the training sample size $N \ll \prod_{k=1}^{K} I_k$ in most practical applications. It is hard to calculate accurately and robustly the statistics of the vector variable because the training sample size is much smaller than the dimensionality of the vector variable. In the proposed method, however, the size of the step-wise covariance matrix is $I_k \times I_k$, which is much smaller than that of ICA. On the other hand, as described in section 3.3, the training samples are the mode-*k* directional vectors of $\mathcal{A}$ and the number of them is $\prod_{j \neq k} I_j \times N$, which is much larger than I_k. Therefore, the dimensionality dilemma is significantly alleviated.

For a K^{th} order tensor, there are K projection matrices calculated in the proposed algorithm, which contain the structural information embedded in different tensor dimensions. The proposed multilinear ICA can conduct dimensionality reduction from different directions and extract effectively the directional features. In the training process, we form the directional image for the mode-*k* matrix to embed the directional information along the direction with $I_{\bar{k}}$ -dimension. As we can see in the experiments, this novel processing can achieve higher recognition accuracy.

4. Experimental Results

In this section, we verify the performance of the proposed method on a palmprint database and two benchmark face databases, UMIST [15] and AR [16]. The proposed method is compared with both unsupervised methods, including PCA (Eigenfaces) [2], ICA [3], 2DPCA [10] and B-2DPCA [17], and supervised methods, including FLD [18], 2DFLD [19] and tensor-FLD [11]. In all the experiments, we consider the image as a 2$^{\text{nd}}$ order tensor (i.e. K=2) and used the nearest neighborhood classifier for classification.

4.1. Palmprint database

The used palmprint database (http://www.comp.polyu.edu.hk/~biometrics/) was collected from 50 people at different times. The palmprints from right-hand and left-hand of each person are treated as palmprints from different people. The resolution of the original palmprint images is 384×284. After preprocessing, the central part of the image, whose size is 128×128, is cropped for feature extraction and matching. Fig. 3 shows an example of the preprocessing result.

In the experiment, we selected palmprint images from 100 different palms for gallery with each palm having 6 samples taken in two sessions. The samples from the first session were used for training, and the samples from the second session were used for testing. Thus, the total number of training samples and test images are both 300. Table 1 shows the top recognition accuracy of different schemes with the corresponding dimensionality of features. It can be seen that the proposed method is obviously superior to all the other unsupervised algorithms (PCA, 2DPCA, B-2DPCA and ICA) and even the supervised methods (FLD 2DFLD and Tensor-FLD) in recognition accuracy. However, the proposed method may need more features than conventional ICA. It can also be seen that with a suitable selection of parameter *l* (such as *l*=2, 4, 8) in the formation of the mode-*k* directional image, the recognition accuracy will be higher than that with the original mode-*k* image, i.e. when *l*=128. Empirically we found that in most experiments on palmprint and face databases, the highest classification accuracy can be achieved when *l* is 2.

4.2. UMIST face database

The UMIST database [15] is a multi-view face database, consisting of 575 images from 20 people and covering a wide range of poses from profile to frontal views. Fig. 4 shows some images of one subject. Each image is of size 112×92. In the experiments, the first nineteen samples of each person were used. Then we used the first *p*=1, 3, 6, 9 images for training and used the remaining images for testing. Table 2 lists the top classification accuracies of different algorithms and the associated number of features. We see clearly that the proposed method achieves much higher accuracy than the unsupervised methods PCA, ICA and 2DPCA, and even higher accuracy than the supervised methods FLD, 2DFLD and Tensor-FLD. The proposed method has the best classification accuracy when *l* is 2.

4.3. AR face database

In the AR database [16], the images of 120 individuals (65 men and 55 women) were taken in two sessions (separated by two weeks) and each session contains 13 images. In our experiments, the facial portion of each image is manually cropped and then normalized to a size of 50×40. The images from the first session with (a) neutral expression, (b) smile, (c) anger, (d) scream, (e) left light on, (f) right light on, and (g) both side light on were selected for gallery. Thus we have 840 images from 120 individuals. Fig. 5 shows some sample images of one subject.

Two experiments were performed. In the first experiment, the four sample images per person with (a) neural expression, (b) smile, (c) anger and (d) scream in the first session were selected for training, and the other three images for testing. The second experiment exchanges the training and testing images. Table 3 lists the top classification accuracies of different algorithms and the associated number of features. We can have the same conclusion as in the previous experiments.

Fig. 6 plots the recognition accuracy of the proposed method under different number of features in the first experiment. It can be seen that recognition accuracy of the proposed method will increase when the number of features increase, when the number of features is 86, it has the best accuracy (99.72%).

5. Conclusion

A general framework for independent feature extraction with tensor representation was proposed in this paper. The proposed method learns multiple low-dimension subspaces to extract independent features. A novel mode-*k* directional image formation was used in training to better exploit the directional information in the mode-*k* matrix of the tensor. Then the mode-*k* ICA was presented to learn the subspaces via mode-*k* directional images and finally the multilinear directional ICA algorithm was presented to extract the IC features, which is also in a tensor form. Compared with the traditional ICA algorithms, the proposed method alleviates significantly the small sample size problem and can preserve better the structural information embedded in the tensor datasets. From the experiments on one palmprint database and two face (UMIST and AR) databases, it can be concluded that the proposed algorithm has higher classification accuracy than many existing unsupervised algorithms such as PCA, ICA and 2DPCA, and even supervised algorithms such as FLD, 2DFLD and Tensor-FLD.

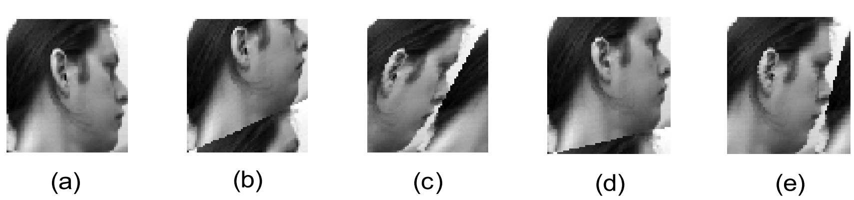

Figure 2. Original image and transformed directional images. (a) Original 2nd order tensor image; (b) mode-1 directional image with *l*=2; (c) mode-2 directional image with *l*=2; (d) mode-1 directional image with *l*=4; (e) mode-2 directional image with *l*=4.

Table 1. Top recognition accuracies (%) and the associated dimensionalities on the Palmprint database by different schemes.

Method	PCA	ICA	FLD	2DPCA	B-2DPCA	Tensor-FLD	2DFLD	Proposed method				
								l=2	l=1	l=4	l=8	l=128
Accuracy	88.00	92.00	94.33	94.00	94.33	99.00	78.00	99.33	95.33	99.00	99.00	98.67
Dimension	109	39	96	2432	264	224	768	112	126	98	91	90

Table 2. The recognition accuracies (%) of different schemes on the UMIST database. The values in parentheses are the corresponding number of features.

Training number	PCA	ICA	2DPCA	B-2DPCA	FLD	Tensor-FLD	2DFLD	Proposed method		
								l=2	l=4	l=92
1	57.50 (18)	52.78 (17)	59.72 (672)	61.94 (60)	--	--	--	66.11 (27)	70.28 (20)	63.89 (36)
3	58.75 (16)	47.81 (18)	62.19 (336)	63.13 (30)	66.56 (8)	65.63 (140)	67.81 (224)	80.00 (16)	64.38 (27)	74.38 (33)
6	58.85 (46)	57.69 (50)	66.92 (336)	70.00 (42)	79.23 (10)	76.15 (72)	76.54 (448)	80.00 (14)	76.54 (18)	78.01 (22)
9	65.00 (48)	70.00 (25)	76.50 (336)	81.00 (14)	89.50 (9)	87.00 (48)	82.00 (224)	91.50 (20)	84.00 (22)	86.50 (24)

Table 3. The recognition accuracies (%) of different schemes on the AR database. The values in parentheses are the corresponding number of features.

Method	PCA	ICA	2DPCA	B-2DPCA	Tensor-FLD	FLD	2DFLD	Proposed method			
								l=2	l=5	l=10	l=40
First experiment	68.89 (418)	97.22 (99)	97.78 (1300)	96.67 (375)	98.61 (270)	92.78 (113)	98.06 (1150)	99.17 (108)	99.44 (91)	99.72 (96)	98.89 (91)
Second experiment	79.79 (110)	95.00 (96)	90.21 (1000)	91.04 (360)	95.21 (60)	96.46 (79)	95.63 (250)	95.83 (72)	95.21 (99)	94.38 (88)	94.38 (88)

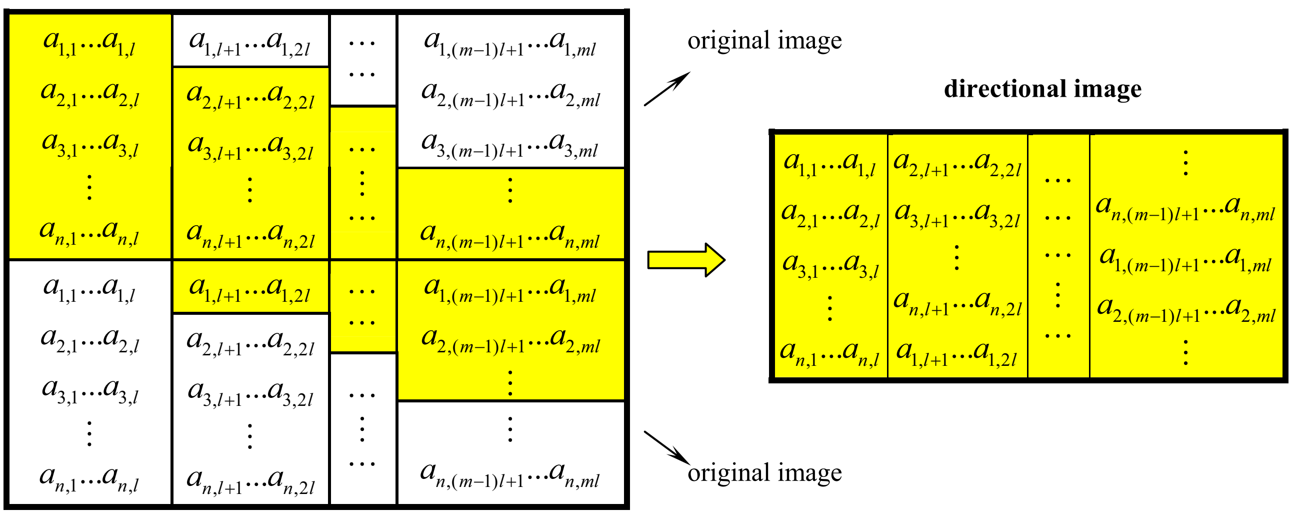

Figure 1. Mode-*k* directional image formation.

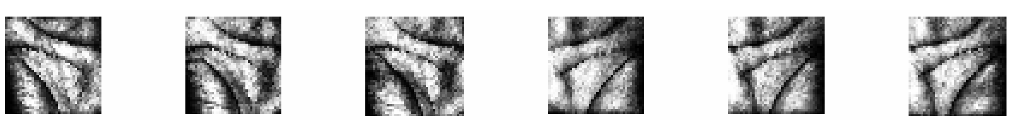

Figure 3. Some preprocessed images (128×128) in the palmprint database.

Figure 4. Some sample images of one subject in the UMIST database.

Figure 5. Some sample images of one subject in the AR database.

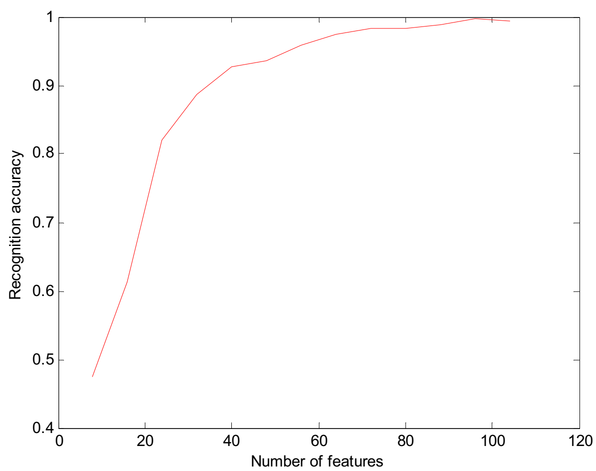

Figure 6. Recognition accuracy vs. number of features by the proposed method (*l*=10) on AR database.

References

[1] S. Yan, D. Xu, B. Zhang, H. Zhang, Q. Yang and S. Lin, Graph Embedding and Extensions: A General Framework for Dimensionality Reduction, IEEE Trans. Pattern Anal. Mach. Intell, 29(1):40-51, 2007.

[2] M. Turk and A. Pentland. Eigenfaces for recognition. Journal of Cognitive Neuroscience, 3(1):72-86, 1991.

[3] M. S. Bartlett, J. R. Movellan and T. J. Sejnowski. Face recognition by independent component analysis. IEEE Trans. on Neural Networks, 13(6):1450-1464, 2002.

[4] M. Alex O. Vasilescu and D. Terzopoulos. Multilinear Independent Component Analysis. In Proc. Of IEEE Conference on Computer Vision and Pattern Recognition (CVPR05), 2005.

[5] A. Hyvärinen, J. Karhunen, and E. Oja. Independent component analysis. Wiley, New York, 2001.

[6] B. Moghaddam. Principal manifolds and probabilistic subspaces for visual recognition. IEEE Trans. IEEE Trans. Pattern Anal. Mach. Intell, 24(6):780–788, 2002.

[7] M. A. Vicente, P. O. Hoyer, and A. Hyvärinen. Equivalence of some common linear feature extraction techniques for appearance-based object recognition tasks. IEEE Trans. Pattern Anal. Mach. Intell, 29(5):896-900, 2007.

[8] M. Bressan and J. Vitrià. On the selection and classification of independent features. IEEE Trans. Pattern Anal. Mach. Intell, 25(10): 1312-1317, 2003.

[9] J. Kim, J. M. Choi, J. Yi, M. Turk. Effective representation using ICA for face recognition robust to local distortion and partial occlusion. IEEE Trans. Pattern Anal. Mach. Intell. 27(12):1977-1981, 2005.

[10] J. Yang, D. Zhang, A. F. Frangi, and J. Y. Yang. Two-dimensional PCA: a new approach to appearance-based face representation and recognition. IEEE Trans. Pattern Anal. Mach. Intell, 26(1): 131-137, 2004.

[11] D. Tao, X. Li, X. Wu. General Tensor Discriminant Analysis and Gabor Features for Gait Recognition. IEEE Trans. Pattern Anal. Mach. Intell. 29(10): 1700-1715, 2007.

[12] S. Yan, D. X, Q, Yang, L. Zhang, X. Tang and H. Zhang. Discriminant analysis with tensor representation. In proc. Of IEEE conference on Computer Vision and Pattern Recognition (CVPR), 2005.

[13] T. Kim, S. Wong, and R. Cipolla. Tensor canonical correlation analysis for action classification. In proc. Of IEEE conference on Computer Vision and Pattern Recognition (CVPR), 2007.

[14] L. Lathauwer, B. Moor, and J. Vandewalle. A multilinear singular value decomposition. SIAM J. Matrix Anal. Appl., 21(4): 1253–1278, 2000.

[15] UMIST Face Database, Daniel Graham, http://images.ee.umist.ac.uk/danny/database.html, 2005.

[16] The AR Face Database, http://rvl1.ecn.purdue.edu/~aleix/aleix _face_DB.html.

[17] J.Ye. Generalized low rank approximations of matrices. Machine Learning, 61:167-191, 2005.

[18] P. N. Belhumeur, J. P. Hespanha, and D. J. Kriegman. Eigenfaces vs. Fisherfaces: recognition using class specific linear projection. IEEE Trans. Pattern Anal. Mach. Intell, 19(7): 711–720, 1997.

[19] K. Liu, Y. Cheng and J. Yang. Algebraic feature extraction for image recognition based on an optimal discriminant criterion. Pattern Recognition, 26(6): 903-911, 1993.