Solution of A 3-D Complex Finite Element Model of Skewed Rotor Induction Motors Using An Iterative Method

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Abstract **– One of the difficulties of the three-dimensional (3-D) eddy current finite element methods is to solve large finite element equations economically. In this paper a 3-D eddy-current finite element model using a four component formulation of complex variables to study skewed rotor induction motors is described. An iterative process among the four specific components during the solution of large algebraic equations is presented. The proposed method overcomes the non-convergence problems when the ICCG method or the shifting ICCG method is used directly. The algorithm also requires much less computer storage compared with the Gaussian elimination method.**

Index terms **– 3-D finite element methods, equation solver, iterative method, electric machines.**

I. INTRODUCTION

Finite element methods (FEM) have been playing an important role in studying the performances of induction machines. In particular, a two-dimensional (2-D) time stepping finite element method is indeed a very precise dynamic simulation tool to study the transient and steady state processes in electrical machines [1-3]. Such simulation model has also been used in studying the electromagnetic fields in the cross-sectional area of electrical machines. The external circuit equations and mechanical equation are then coupled together with the electromagnetic equations. The FEM meshes are moved according to the rotor movement. The set of equation is subsequently discretized directly by time stepping method. Such model takes into account the effects of saturation, eddy currents, slotting, rotor movements and other non-sinusoidal quantities. The algorithm can also be applied conveniently to simulate the starting operation, estimate the harmonic fields and their corresponding steady state harmonic stray losses, analyze the performance of motors fed from a PWM inverter and so on. All these investigations are difficult if one uses the analytical methods or FEM in the frequency domain. In short, the time stepping FEM method is the most complete model for induction machines and it will certainly be used in the near future for designing induction machines, including those fed from static inverters.

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However, with 2-D FEM models the inter-bar currents in the rotors are neglected and the effects of the end fields cannot be considered fully [4]. Consequently, an accurate analysis of a real machine would call upon three-dimensional (3-D) analysis. Noting the rapid developments of computer power, it is clear that future developments in FEM will move from 2-D to 3-D [5].

In order to evaluate accurately the performance of skewed rotor induction motors when rotor movements and other non-sinusoidal quantities are considered, a 3-D time stepping FEM is necessary. The aim of this paper is to develop a 3-D complex model for skewed rotor induction motors. The result of this complex 3-D model will be fed into a time stepping 3-D models for studying the steady state performance of motors [6]. The computation time to reach steady state in the time stepping process is found to be reduced significantly with such an approach [7-8].

In the complex model the total number of variables is doubled compared with that in the time stepping model. However the resulting equations with global stiffness matrices cannot be solved easily unless a very powerful computer is available. Moreover, the authors found that if the ICCG method or the shifting ICCG method [9] is applied directly to solve the complex variable model as described in the next section, the solution is not convergent. If the Gaussian elimination method is used, excessive computer storage is however required.

It is well known that 3-D FEM equations are too costly to be solved by elimination methods and efficient iterative solvers have to be developed. Ref. [10] presented an iterative method for solving four component formulas. Such method was reported to be able to reduce the scale of the equations considerably. But the method can only be used for those problems in which the reluctivities of the material are constant in the three directions.

In this paper an iterative algorithm for solving large algebraic equations of general FEM using magnetic vector potential and electric scalar potential is presented. The differences in reluctivities of the material in the three directions are modeled. A FEM equation with more than 77000 unknowns was solved successfully with 412 min. of CPU time on a personal computer. It is thus obvious that the proposed method has contributed significantly in applying FEM to study practical machine problems.

II. MACHINE MODEL TO BE STUDIED

A. Basic assumptions

The following assumptions are made:

1. There are no leakage fluxes in the outer-most surface of the machine frame, the inner-most surface of the shaft and the two endshields.
2. The stator current waveform is computed using 2-D multi-slice model [11]. The machine being studied in the 3-D model is current driven with the currents obtained from a 2-D solution along a cross-section in the motor.
3. The reluctivity of the iron material of each element is obtained from a 2-D multi-slice model. Hence the non-linear characteristics of materials do not need to be considered in the 3-D model. The CPU time is greatly reduced.
4. The 3-D finite element analysis was carried out in the frequency domain.

In other words, one is required to compute the magnetic and electric field distributions when the stator current waveform is given.

B. The Variables

Because most of the domains being studied contain eddy currents or imposed currents, a four component formulation will result. The four unknown components are the magnetic vector potential $\vec{A}$ in each of the three dimensional domains and one electric scalar potential V in the conduction domains. Since the variables are expressed in the frequency domain, complex variables are used. The derivative $\partial\vec{A}/\partial t$ can be replaced by $j\omega\vec{A}$ (where ω is the angular frequency of the imposed currents and $\vec{A}$ is a complex number).

C. The Properties of Materials

The electric conductivity σ of the conducting domains and the magnetic reluctivity ν of the materials are represented by tensor quantities in a Cartesian system of coordinates as :

$$\sigma = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \tag{1}$$

$$\nu = \begin{bmatrix} \nu_x & 0 & 0 \\ 0 & \nu_y & 0 \\ 0 & 0 & \nu_z \end{bmatrix} \tag{2}$$

The thickness of the iron core laminations is assumed to be thin compared to the characteristic skin depth [12]. The material can then be regarded as a homogeneous material whose conductivity is zero in the z direction that is normal to the plane of lamination. In other words,

$$\sigma_z = 0 \tag{3}$$

In the x and y planes of the silicon steel laminations with a stacking factor of $(1\text{-}\varepsilon)$:

$$\sigma_x = \sigma_y = (1-\varepsilon)\sigma \tag{4}$$

The reluctivity in the iron cores is [12] :

$$\frac{1}{\nu_x} = \frac{\varepsilon}{\nu_0} + \frac{1-\varepsilon}{\nu_{fx}} \tag{5}$$

$$\frac{1}{\nu_y} = \frac{\varepsilon}{\nu_0} + \frac{1-\varepsilon}{\nu_{fy}} \tag{6}$$

$$\nu_z = (1-\varepsilon)\nu_{fz} + \varepsilon\nu_0 \tag{7}$$

where ν_{fx} and ν_{fy} are the reluctivity of the steel in the x and y directions, respectively, and ν_0 is the reluctivity of air.

D. The Solution Domain

The solution domain includes the iron frame in the outer-most surface of the periphery, the shaft in the inner-most surface of the periphery and the endshields in the axial direction. Since the rotor bars are skewed, the total axial length of the machine in the axial direction should be included.

In the cross-sectional direction, because the non-sinusoidal distribution of the electromagnetic field in the space is to be considered, one pair of pole pitch in the solution domain is usually required. If the number of the stator slots and rotor slots of one pole pitch are integers, the solution domain can be reduced to one pole pitch [4].

E. Boundary Conditions

The following boundary conditions are given for the outer-most surface of the machine frame, the inner-most surface of the shaft and the two endshields:

$$\vec{A} = 0 \tag{8}$$

$$\sigma\left(j\omega\,\vec{A} + \nabla V\right)\vec{n} = 0 \tag{9}$$

where $\vec{n}$ is the normal unit vector on the boundary surface. The two sides of one pole band satisfy the semi-periodic boundary condition [13]:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}_{slave} = \cos E \begin{bmatrix} \cos M & -\sin M & 0 \\ \sin M & \cos M & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}_{master} \tag{10}$$

$$V_{slave} = (\cos E)\,V_{master} \tag{11}$$

where the variables with the subscript *master* are the unknowns and the variables with the subscript *slave* can be determined according to the boundary condition; E is the difference of the electrical angle between the *master* and the *slave* variables. For a semi-periodic boundary condition E is 180°; M is the mechanical angle between two sides. As the motor under consideration has a four-pole configuration and only one quarter of the motor is modeled, M is 90°.

III. BASIC EQUATIONS

The Maxwell's equations applied to the domains under investigation will give rise to the following equations:

$$\nabla \times (\nu\,\nabla \times \vec{A}) = -\sigma\,(\frac{\partial \vec{A}}{\partial t} + \nabla V) + \vec{J}_s \tag{12}$$

$$\nabla \cdot \sigma\,(\frac{\partial \vec{A}}{\partial t} + \nabla V) = 0 \tag{13}$$

where $\vec{J}_s$ is the imposed current density. For the steady state problems in which fields are considered as sinusoidally time-varying, (12) and (13) can be put into complex form:

$$\nabla \times (\nu\,\nabla \times \vec{A}) = -\sigma\,(j\omega\,\vec{A} + \nabla V) + \vec{J}_s \tag{14}$$

$$\nabla \cdot \sigma\,(j\omega\,\vec{A} + \nabla V) = 0 \tag{15}$$

The application of Galerkin's method to (14) and (15), gives the following integral equations [14]:

$$\iiint_\Omega \nabla \times \vec{W} \cdot \nu \nabla \times \vec{A}\,d\Omega + \iiint_\Omega j\omega\,\vec{W} \cdot \sigma \vec{A}\,d\Omega$$

$$+ \iiint_\Omega \vec{W} \cdot \sigma \nabla V d\Omega + \iint_S \vec{W} \cdot \vec{H}_t dS = \iiint_\Omega \vec{W} \cdot \vec{J}_S d\Omega \tag{16}$$

$$\iiint_\Omega \nabla W \cdot j\omega\sigma\vec{A} d\Omega + \iiint_\Omega \nabla W \cdot \sigma \nabla V d\Omega - \iint_S W \cdot J_n dS = 0 \tag{17}$$

where S is the boundary surface of Ω; and

$$\vec{H}_t = \nu\,\nabla \times \vec{A} \times \vec{n} \tag{18}$$

$$J_n = (j\omega\,\sigma\,\vec{A} + \sigma\,\nabla V) \cdot \vec{n} \tag{19}$$

and the interpolation function

$$W = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}^T \tag{20}$$

$$\vec{W} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix}^T \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} \tag{21}$$

where N_1, N_2, N_3 and N_4 are the element shape functions. Tetrahedral elements are used in discretization.

Here for simplicity, one assumes that variables are arranged as follows:

$$\begin{bmatrix} A_{x1} & A_{x2} & \cdots A_{xN} \mid A_{y1} & A_{y2} & \cdots A_{yN} \mid A_{z1} & A_{z2} & \cdots A_{zN} \mid V_1 & V_2 & \cdots V_N \end{bmatrix}^T$$

where N is the number of nodes. Making the substitution:

$$V' = V / j\omega \tag{22}$$

the global system equation with symmetric coefficient matrix can be obtained by putting element interpolation function into (16) and (17):

$$\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} & K_{xv} \\ K_{yx} & K_{yy} & K_{yz} & K_{yv} \\ K_{zx} & K_{zy} & K_{zz} & K_{zv} \\ K_{vx} & K_{vx} & K_{vz} & K_{vv} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \\ V' \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \\ P_v \end{bmatrix} \tag{23}$$

If $\vec{A}$ and V are obtained, magnetic flux density $\vec{B}$ and the eddy current $\vec{J}$ will be:

$$\vec{B} = \nabla \times \vec{A} \tag{24}$$

$$\vec{J} = -\sigma(j\omega\vec{A} + \nabla V) \tag{25}$$

III. SOLUTION OF THE SYSTEM OF EQUATIONS

Separating (22) into three sub-systems of equations as:

$$\begin{bmatrix} K_{zz} & K_{zv} \\ K_{vz} & K_{vv} \end{bmatrix}\begin{bmatrix} A_z \\ V' \end{bmatrix}=\begin{bmatrix} P_z \\ P_v \end{bmatrix}-\begin{bmatrix} K_{zx} & K_{zy} \\ K_{vx} & K_{vy} \end{bmatrix}\begin{bmatrix} A_x \\ A_y \end{bmatrix} \quad (26)$$

$$[K_{xx}][A_x] = [P_x] - [K_{xy} \quad K_{xz} \quad K_{xv}]\begin{bmatrix} A_y \\ A_z \\ V' \end{bmatrix} \quad (27)$$

$$[K_{yy}][A_y] = [P_y] - [K_{yx} \quad K_{yz} \quad K_{yv}]\begin{bmatrix} A_x \\ A_z \\ V' \end{bmatrix} \quad (28)$$

The unknown A_x, A_y, A_z and V' on the right hand sides of (26), (27) and (28) can be replaced by initial guesses and these equations can be solved separately. The values of A_x, A_y, A_z and V' on the right hand side of (26), (27) and (28) can be updated by the solutions that have just been obtained and the equations can be solved iteratively until the difference between successive solutions is less than a specific error.

Though iteration process is needed among the three groups, the bandwidth for (26) is reduced nearly to 1/4 th of its original value and that for (27) and (28) is 1/16 th of their previous values. The scale of equation for (26) is reduced nearly by 1/2 and that for (27) and (28) is reduced to 1/4 th of their original values. Another important merit is that the ICCG method shows very quick convergence in arriving at the solution of each separated group, because the complicated combinations among A_x, A_y, and A_z have now been removed.

The proposed iteration process among the three groups begins by solving the following equations:

$$\begin{bmatrix} K_{zz} & K_{zv} \\ K_{vz} & K_{vv} \end{bmatrix}\begin{bmatrix} A_z^{(1)} \\ V'^{(1)} \end{bmatrix}=\begin{bmatrix} P_z \\ P_v \end{bmatrix}-\begin{bmatrix} K_{zx} & K_{zy} \\ K_{vx} & K_{vy} \end{bmatrix}\begin{bmatrix} A_x^{(0)} \\ A_y^{(0)} \end{bmatrix} \quad (29)$$

$$[K_{xx}][A_x^{(1)}] = [P_x] - [K_{xy} \quad K_{xz} \quad K_{xv}]\begin{bmatrix} A_y^{(0)} \\ A_z^{(1)} \\ V'^{(1)} \end{bmatrix} \quad (30)$$

$$[K_{yy}][A_y^{(1)}] = [P_y] - [K_{yx} \quad K_{yz} \quad K_{yv}]\begin{bmatrix} A_x^{(0)} \\ A_z^{(1)} \\ V'^{(1)} \end{bmatrix} \quad (31)$$

The variables on the right hand side are assumed to be known at each iteration. At this first step, $A_x^{(0)}=0$ and $A_y^{(0)}=0$.

At the followed iteration steps only the increments

$$\Delta A_z^{(k)} = A_z^{(k)} - A_z^{(k-1)} \quad (32)$$

$$\Delta V'^{(k)} = V'^{(k)} - V'^{(k-1)} \quad (33)$$

$$\Delta A_x^{(k)} = A_x^{(k)} - A_x^{(k-1)} \quad (34)$$

$$\Delta A_y^{(k)} = A_y^{(k)} - A_y^{(k-1)} \quad (35)$$

are to be computed. Thus

$$\begin{bmatrix} K_{zz} & K_{zv} \\ K_{vz} & K_{vv} \end{bmatrix}\begin{bmatrix} \Delta A_z^{(k)} \\ \Delta V'^{(k)} \end{bmatrix}=-\begin{bmatrix} K_{zx} & K_{zy} \\ K_{vx} & K_{vy} \end{bmatrix}\begin{bmatrix} \Delta A_x^{(k-1)} \\ \Delta A_y^{(k-1)} \end{bmatrix} \quad (36)$$

$$[K_{xx}][\Delta A_x^{(k)}] = -[K_{xy} \quad K_{xz} \quad K_{xv}]\begin{bmatrix} \Delta A_y^{(k-1)} \\ \Delta A_z^{(k)} \\ \Delta V'^{(k)} \end{bmatrix} \quad (37)$$

$$[K_{yy}][\Delta A_y^{(k)}] = -[K_{yx} \quad K_{yz} \quad K_{yv}]\begin{bmatrix} \Delta A_x^{(k-1)} \\ \Delta A_z^{(k)} \\ \Delta V'^{(k)} \end{bmatrix} \quad (38)$$

Equations (36), (37) and (38) are solved separately by using the ICCG method. Because the number of bytes used to represent a floating data in computers is limited, and because $|\Delta A_x| \ll |A_x|$, $|\Delta A_y| \ll |A_y|$, $|\Delta A_z| \ll |A_z|$, $|\Delta V'| \ll |V'|$, one will obtain a higher precision in the solution if the values of ΔA_x, ΔA_y, ΔA_z and $\Delta V'$ rather than A_x, A_y, A_z and V' are stored in the computer. So the computation using ΔA_x, ΔA_y, ΔA_z and $\Delta V'$ in the ICCG method will yield higher accuracy than that based on A_x, A_y, A_z and V'.

In computing (29) and (36), in order to reduce the bandwidth of the coefficient matrixes, the variables A_z and V' are arranged in $[A_{z1} \quad V_1 | A_{z2} \quad V_2 | \cdots | A_{zN} \quad V_N]^T$.

In induction motors, the exciting current is mainly in the axial direction and the permeability of iron cores is higher in the cross-sectional direction compared to that in the axial direction. Usually the magnitudes of A_x and A_y are less than 15 % of that of A_z. Thus one can obtain a fairly precise solution with a few iterations.

IV. NUMERICAL RESULTS

A. A Simple Problem

A simple simulation is firstly carried out for comparing the Gaussian method and the proposed iterative method. It is a reduced scale model of a practical problem which will be described in the next subheading. The number of the total unknowns is 21986. The program runs on a Pentium / 90 MHz personal computer with 32 MB RAM. When the RAM is insufficient, the hard disk is used to exchange data with the RAM automatically.

When using the Gaussian method, the coefficient matrix is stored in a changeable bandwidth method.

In the proposed iteration procedure, only the non-zero elements in the coefficient matrix are stored. The number of large iterations among three groups is eight. For solving (36), (37) and (38), the average iteration number of the ICCG method is only 334 at each step of large iteration.

A comparison of the computer RAM and the CPU time needed between the Gaussian method and the proposed iteration method is shown in table I.

TABLE I
COMPARISON OF COMPUTER RAM AND CPU TIME

	RAM	CPU
Gaussian method (single precision)	80.6 MB	71 min
Iteration method (double precision)	1.9 MB	54 min

Because the Gaussian method requires a large amount of RAM, there will be a lot of data exchanges between the RAM and the hard disk in small computers. Therefore, if the scale of the problem is larger, the CPU time for the Gaussian method will be even longer than that of the proposed iterative method as reported in this paper.

B. A Practical Problem

The method has been applied to compute the steady state flux distribution for an 11 kW, skewed rotor, cage induction motors. Its main parameters are shown in table II.

The 3-D mesh is generated using an extrusion technique. The 2-D mesh is used as the base-plane to generate the model. The generated 3-D FEM meshes of the 11 kW skewed induction motor is shown in Fig. 1.

For this practical problem, the number of the total unknowns is 77980. It required 7.6 MB of computer RAM to run the solver that was developed by the authors for double precision computation. If the Gaussian method is used, the RAM is 881 MB even if the data is in single precision.

For solving equation (35), (36) and (37) at each step of large iteration, the average iteration number of the ICCG is 550. If one uses A_x, A_y, A_z and V instead of ΔA_x, ΔA_y, ΔA_z and ΔV in the iterations, the average iteration number of the ICCG is 654. It is shown that by using increments only in the iterative process, one can reduce the CPU time by about 16 %.

The average differences of A_x, A_y, A_z and V between adjacent steps in the process of the iteration are shown in Table III. The total CPU time needed for these 10 iterations is 412 min. It is shown that by using the developed iterative algorithm, the 3-D FEM program dealing with large number of unknowns can also be run on personal computers.

TABLE II
MAIN PARAMETERS OF THE TEST MOTOR

Rated power	(kW)	11
Rated voltage	(v)	380
Rated current	(A)	22.61
Rated frequency	(Hz)	50
Connection		Δ
Number of phases		3
Number of pole pairs		2
Number of stator slots		48
Number of rotor slots		44
Stator diameter	(mm)	240
Rotor diameter	(mm)	157
Air-gap length	(mm)	0.5
Core length	(mm)	165
Skew		1.3 stator slot pitch

The computed flux distributions at the cross-sections near the endshield and near the centre are shown in Fig. 2.

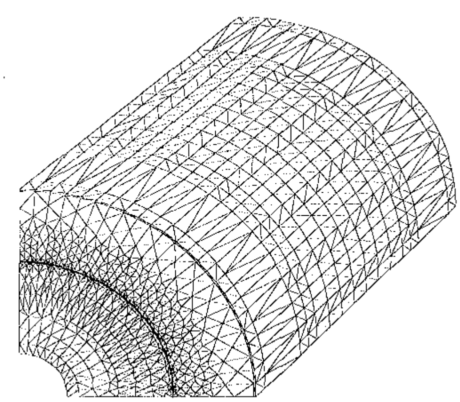
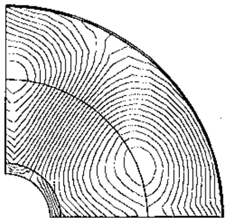


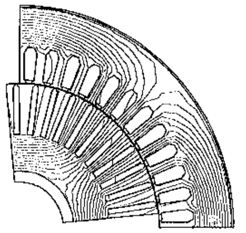
Fig. 1 A 3-D FEM mesh of an 11 kW skewed motor
(The mesh has 15232 nodes, 76635 elements)

TABLE III
THE CONVERGENCE STATUS OF THE PROPOSED ITERATIVE METHOD AT VARIOUS ITERATION NUMBER

Iteration number	$\Delta A_z^{(k)}$ 10^{-6} Wb·m^{-1}	$\Delta V^{(k)}$ 10^{-6} V	$\Delta A_x^{(k)}$ 10^{-6} Wb·m^{-1}	$\Delta A_y^{(k)}$ 10^{-6} Wb·m^{-1}
1	5906.7	898.3	286.6	260.6
2	151.6	28.1	103.8	100.7
3	60.7	10.0	103.5	91.6
4	58.9	10.4	63.2	59.4
5	36.9	5.9	69.5	62.3
6	40.0	7.2	45.8	42.7
7	26.7	4.3	53.1	48.0
8	30.5	5.6	35.3	33.4
9	20.7	3.4	43.0	40.0
10	24.5	4.6	28.7	27.5



(a) the cross-section near the endshield



(b) The cross-section near the centre

Fig. 2 The computed flux distributions

V. CONCLUSION

To deal with the 3-D complex finite element model of skewed induction motors in which the stator, rotor and ending are all involved in the solution domain and if the reluctivities of materials in the three directions are different, it is necessary to develop solvers to deal with the large scale equations efficiently and economically. The paper shows that the normal ICCG method or the shifting ICCG method are not convergent and the Gaussian elimination method requires excessive amount of computer storage. The authors find that if the system of equations is separated into three sub-systems of equations according to the characteristics of unknown variables, the solution of the original problem can be obtained very economically and quickly.

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VII. BIOGRAPHIES

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W. N. Fu was born in 1961 in Zhejiang, China. He received the B.Eng in electrical engineering from Hefei University of Technology in 1982. During 1982 to 1986, he worked in the Department of Induction Machines, Shanghai Electrical Apparatus Research Institute and got rich experiences in designing, manufacturing and testing of induction machines. He obtained the M.Eng at Shanghai University of Technology in 1989. After graduation, he joined the Automation School of Shanghai University as a lecturer. Since 1994, he has been a visiting scholar at the Hong Kong Polytechnic University. He is now a PhD student. He has published over 35 papers in journals and international conferences. His research interests include electromagnetic field computation and studies on novel machines.

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