An Adaptive Interpolating MLS Based Response Surface Model Applied to Design Optimizations of Electromagnetic Devices

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A response surface model (RSM) based on a combination of interpolating moving least squares and multistep method is proposed. The proposed RSM can automatically adjust the supports of its weight functions according to the distribution of the sampling points when it is used to reconstruct a computationally heavy design problem. Numerical examples are given to demonstrate the feasibility and efficiency of the proposed method for solving inverse problems.

***Index Terms*—Hierarchical interpolation, interpolating moving least squares approximation, response surface model.**

I. Introduction

NOWADAYS stochastic optimal methods have been playing an increasingly important role in the study of global optimizations of engineering design problems. However, the excessive demand for computer resources with these algorithms often renders these optimal methods inefficient or impractical for some practical design problems that require, for example, repetitive usages of finite element (FE) solutions. To circumvent this problem, the response surface methodology (RSM) has been introduced to reduce the number of function evaluations that involve time consuming computer simulations without sacrificing the quality of the numerical solutions [1]–[6]. So far the most popular RSMs are those based on globally supported radial basic functions (RBF) because of their good interpolating power in dealing with both grid and scattered data. However, globally supported RBFs have the inherent drawback of having the need to manipulate full interpolation matrices. Consequently, the sample points of globally supported RBF based RSMs cannot exceed an upper limit [2]. In this regard, a compactly supported RSM would be much sought after.

However, one would face a dilemma in setting a reasonable support of the compactly supported RSMs, in the construction of a function, when one attempts to strike a balance between quality and solution speed of the interpolation. In order to address those issues, a compactly supported RSM, based on a combination of the Interpolating moving least squares (IMLS) approximation and the multistep method together with a tabu search method, is proposed in the development of an efficient global optimal design tool for multimodal functions. To ensure that the sampling points will have an optimal distribution in the feasible space, a simulated annealing (SA) algorithm is suggested to be run firstly on the original time consuming optimal problem to generate the sampling points.

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II. A Multi-Step IMLS Based RSM

A. Brief Introduction of IMLS

Although the IMLS method is well documented in the literature of related disciplines, it is necessary to give it a brief introduction for the convenience of fellow researchers.

To reconstruct a function $f(x) : D \to R$ on the basis of its values f_i at a set of sample points $x_i \in D$ $(i = 1, 2, \cdots, N)$ in terms of some basis $b = \{b^{(i)}\}_{i=1}^{n}$ $(n \le N)$ in IMLS, a local approximation $L_x f$ of it at each point $x \in \overline{D} \subset D$ is firstly defined as

$$L_x f := \sum_{i=1}^{n} a_i(x) b^{(i)}. \tag{1}$$

The basis functions b satisfy the following conditions

(1) $b^{(1)} \equiv 1$.
(2) $b^{(i)} \in C^m(D)$ $(i = 1, 2, \cdots, n)$.
(3) $\{b^{(i)}\}_{i=1}^{n}$ are independent over some sets of n points of the given N points in $\overline{D}$.

One then defines a global projector Gf, such that for any point $x \in \overline{D}$

$$Gf(x) = L_x f(x) = \sum_{i=1}^{n} a_i(x) b^{(i)}(x). \tag{2}$$

To determine the coefficient $a(x)$ in (1), one employs a discrete L^2-norm by the x-dependant inner product $(u, v)_x$ of vectors u and v which is defined by

$$(u, v)_x = u^T w(x) v \tag{3}$$
$$\|u\|_x = (u, u)_x^{1/2} \tag{4}$$

where $z = (z(x_1)\ z(x_2)\ \cdots\ z(x_N))^T$ $(z = u, v)$; $w(x)$ is a $N \times N$ diagonal matrix with $w^{(i)}(x)$ as its i^{th} element, $w^{(i)}(x)$ is called the weight function of the IMLS.

A characteristic of the IMLS is that the weight function $w^{(i)}(x)$ is a compactly supported one centred at each sampling point. This feature renders the IMLS a local approximation of the function. The generalized form of the weight function of the proposed IMLS, $w^{(i)}(x)$, is

$$w^{(i)}(\bullet) = \frac{w_f^i\left(\|x - x_i\|\right)}{\|x - x_i\|^\alpha} \quad (5)$$

where α is a positive even integer, $\| \bullet \|$ is the Euclidean norm, $w_f^i(\|x - x_i\|)$ is a compactly supported function.

Since Gf is the best approximation of f in the least squares sense, it means that

$$\left(f - L_x f, b^{(i)}\right)_x = 0 \quad (i = 1, 2, \cdots, n) \quad (6)$$

which yields the following matrix equation

$$A(x)a(x) = B(x)f \quad (7)$$

$$A(x) = \sum_{i=1}^{N} w^{(i)}(x) b(x_i) b^T(x_i) \quad (8)$$

$$B(x) = \left[w^1(x)b(x_1)\ w^2(x)b(x_2)\ \cdots\ w^N(x)b(x_N)\right] \quad (9)$$

where $f = [f_1\ f_2\ \cdots\ f_N]^T$.

It follows from (7) that

$$a(x) = A(x)^{-1}B(x)f. \quad (10)$$

Obviously, this approximation procedure has no specific requirements on the point pattern. The only condition for the procedure to work is that the coefficient matrix $A(x)$ must be invertible, and this can be guaranteed by automatically adjusting the support, which in turn refers to the size of the influence domain of a point, of the weight function, to involve enough sampling points for each point whose influences are non-zero in some specific points. Correspondingly, the weight function $w^{(i)}(\bullet)$ is of the form $w^{(i)}(\bullet/\beta)$ for $\beta > 0$, and hereafter the parameter β is called the scale of the weight function.

B. A Multistep IMLS Based RSM

Ideally, the sample points of an objective function should be distributed irregularly such that the point densities are comparatively high in regions where the local optima are likely to exist in order to build a robust RSM. However this feature makes the finding of the right scale for the weight function very awkward. For example, the precision of the approximation will be low if the scale is too small. On the other hand, if the scale is too large, the interpolation will no longer be a compactly supported one. Therefore, every weight function should have the ability to adjust its support according to the point density around it. For this purpose, the multistep method as proposed in [7] is used. The use of the multistep technique is a common practice for scattered data analysis and the procedures required to facilitate the implementation of such technique are outlined in the following.

For a nested sequence of the set of sample points X

$$X^1 \subset X^2 \subset \cdots X^{M-1} \subset X^M = X \quad (11)$$

with the subset X^k defined as

$$X^k = \left\{x_1^{(k)}, x_2^{(k)}, \cdots, x_{N_k}^{(k)}\right\} \quad (1 \le k \le M) \quad (12)$$

the multistep method decomposes the interpolation problem into M substeps as described below.

Starting with $k = 1$, one will match the error function at the k^{th} step as

$$f - (s^1 + s^2 + \cdots s^{k-1}) \quad (13)$$

on X^k by computing the coefficients of the k^{th} interpolant

$$s^k(x) = \sum_{i=1}^{n} a_i(x) b^{(i)}(x) \quad (14)$$

with $a(x) = A^{-1}(x)B(x)f$, $f = [f_1\ f_2\ \cdots\ f_{N_k}]^T$, and

$$A(x) = \sum_{i=1}^{N_k} w^{(i)}(x/\beta_k) b\left(x_i^{(k)}\right) b^T\left(x_i^{(k)}\right),$$
$$B(x) = \left[w^{(1)}(x/\beta_k) b\left(x_1^{(k)}\right)\ w^{(2)}(x) b\left(x_2^{(k)}\right)\ \cdots\ w^{(N_k)}(x) b\left(x_{N_k}^{(k)}\right)\right]$$

after the value of β_k of the weight function has been chosen.

It follows naturally that

$$f\,\big|_x = \left(s^1 + s^2 + \cdots + s^M\right)\big|_X. \quad (15)$$

This approach allows one to choose a relatively large scale at the lowest level to capture the overall behavior of the function, and by decreasing it during the process of the procedure, finer and finer details of the function is obtained step by step, providing a hierarchical construction procedure with a reasonable computing time. The scales of the weight functions at a sub-step are determined in such a way that their influences will cover at least 30 sample points.

C. Projection Algorithm

To decompose the set of sample points, X, into a nested sequence, a projection algorithm is proposed as follows.

Firstly, the parameter space is divided into a discrete grid according to a user predefined precision parameter. The discrete grid is recorded as a binary string. When a sample point is generated, its location in the grid is determined by repeatedly bisecting the range of it in each direction and to identify the specific half range that contains the solution. The corresponding bit of the string is then set to a logical 1. Once the total sample points, X, are generated and the binary string is assigned, the maximum distances, $d_i^{\max}$ $(i = 1, 2, \cdots, d)$, among every two neighborhood points for each coordinate direction will be evaluated. The algorithm will then begin to decompose the sample points X into a nested sequence of subsets, $X^1 \subset X^2 \subset \cdots X^{M-1} \subset X^M = X$, by using the following algorithm.

Projection Algorithm: Generation of the first subset X^1. Use the point of the first non-zero logical 1 in the left bottom of the hyper-box as a vertex and taking $d_i^{\max}$ as the edge length of the i^{th} direction to construct the first hyper-box; Propagate from the vertexes of this hyper-box, construct, one by one, the so defined hyper-boxes until the non-zero logical 1s of the grid are covered by the vertexes of the hyper-boxes; The subset X^1 is then consisting of the vertex points of the total hyper-boxes whose values in the binary string are logical 1s; Set the bits of the binary string corresponding to points in X^1 to logical 2s:

Repeat

$X^i = X^{i-1}$;

Step 1) Halve the edge length of the hyper-boxes of the previous step to construct new hyper-boxes, find logical 1s of the vertex points, add the corresponding points to X^i and set the bit values of the binary string to logical 2s;

Step 2) Let $X^{dif} = X^i - X^{i-1}$. If the number of sample points in X^{dif} is less than a threshold value prescribed by the user, go to Step 1; Otherwise, continue the decomposing process for next level;

Until all the bits of the binary string are 2s or 0s.

III. An Efficient Global Optimizer

A. The Optimizer

For the efficient optimization of electromagnetic devices, a fast global optimizer based on a combination of the proposed multistep IMLS based RSM and a tabu search method is proposed. To make the best use of the function values of the limited number of sampling points, the sample points should be distributed in the feasible parameter space in an irregular pattern such that the point densities are higher in regions where the local optima are likely to exist. Rather than randomly or uniformly arranging the sample points, the proposed method uses SA algorithm to generate the sample points because SA has some "intelligence" in generating new states, i.e., intensifying points in regions where the local optima exist. Once the primary sampling points are generated, an optimal problem will be solved iteratively following the procedures as described below.

Repeat

(1) Reconstruct the optimal problem based on the proposed multistep IMLS based RSM and solve it by using a tabu search method, then report all the searched local/global optimal solutions;

(2) Solve the original optimal problem by using a deterministic method starting from the newly searched local optimal solutions to find the "improved" ones;

(3) Compare the optimal solutions of (1) and (2). If significant error exists for some solutions, intensify the sample points around the specified point and compute the value of the objective/constraint function and then go to (1); Otherwise, set $\mathrm{Stopcriterion} = $ "true";

Until $\mathrm{Stopcriterion} = $ "true".

B. Validation

The proposed fast global optimizer is firstly used to find the global minimum of a multimodal mathematical function to demonstrate its efficiency and global search ability. The details of this function are described as

$$f(X)=D^4-(\cos(10*D)-1))\,(D=\|X-X_0\|_2, X=(x,y)) \quad (16)$$

In the numerical implementation of the proposed optimizer, a SA algorithm is firstly run on this function to generate 455 sampling points. The optimal problem is then reconstructed using the proposed multistep IMLS based RSM, in which a cubic spline weight function (17) is used as weight function. The reconstructed function and the close form expression are depicted, respectively, in Fig. 1, and Fig. 2. The reconstructed problem is finally solved using the aforementioned iterative procedure, and it is found that only one iterative cycle is required in this case study. The final searched solution and performance of the proposed efficient optimizer, together with those obtained by running the SA algorithm directly on the mathematical function, are given in Table I. From these numerical results, it is clear that:

(1) the quality of the reconstructed function using the proposed multistep IMLS based RSM is very high, i.e., the proposed RSM reproduces exactly the function values and its stationary points;

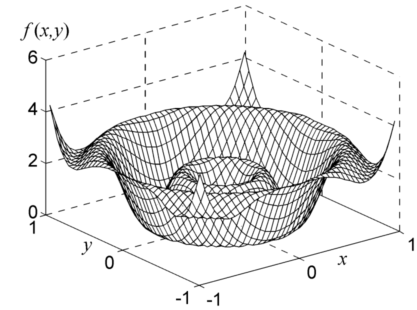

Fig. 1. The reconstructed mathematical function using the proposed multistep IMLS based RSM in terms of 455 sampling points.

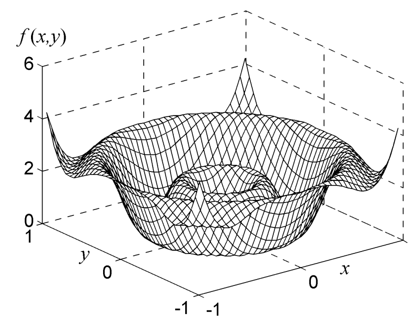

Fig. 2. The exact function expression of the mathematical function.

TABLE I
Performance Comparison of the Proposed and a SA Algorithm on Solving the Multimodal Mathematical Function

Method	x	y	f_{opt}	No. of function calls	
				on original	on constructed
Proposed	7.1×10^{-6}	8.0×10^{-6}	5.7×10^{-9}	455	2456
SA	-2.1×10^{-6}	-6.9×10^{-6}	2.6×10^{-9}	1245	/

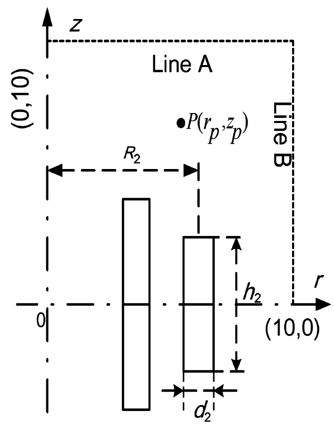

Fig. 3. The schematic diagram of the SMES.

(2) the global search ability of the proposed efficient optimizer is very strong since the final optimal solutions obtained by the proposed optimizer and the well designed SA algorithm are nearly the same.

Therefore, the efficiency and the global search ability of the proposed optimizer are positively confirmed by these primary numerical experiments on this case study

$$w(r)=\begin{cases} 2/3-4r^2+4r^3 & (r\le 1/2)\\ 4/3-4r+4r^2-4r^3/3 & (1/2<r\le 1)\\ 0 & (r>1)\end{cases}. \quad (17)$$

IV. Numerical Application

The TEAM Workshop problem 22 of a superconducting magnetic energy storage (SMES) configuration with three free parameters, as shown in Fig. 3, is selected to demonstrate the feasibility of the proposed algorithm for solving engineering design problems. The problem is formulated as

$$\begin{aligned} &\text{minimize} && f=w_1\frac{B_{stary}^2}{B_{norm}^2}+w_2\frac{|Energy-E_{ref}|}{E_{ref}}\\ &\text{subject to} && J_i\le(-6.4\,|(B_{\max})_i|+56)(A/mm_2)\ \ (i=1,2)\end{aligned} \quad (18)$$

where $Energy$ is the energy stored in the SMES device; $E_{ref} = 180$ MJ; $B_{norm} = 3\times 10^{-3}$T; w_1 and w_2 are weighting factors; J_i and $(B_{\max})_i$ $(i = 1,2)$ are, respectively, the current density and the maximum magnetic flux density in the i^{th} coil; B_{stary}^2 is a measure of the stray fields which is evaluated along 22 equidistant points of line A and line B of Fig. 3 by

$$B_{stary}^2=\sum_{i=1}^{22}(B_{stary})_i^2/22. \quad (19)$$

In the numerical implementation, 400 sampling points are firstly generated using the SA algorithm on the original problem in which the objective function is obtained through finite element simulations; These sampling points and their function values are then used to reconstruct the objective function in the case of a linear basis $\{1\ R_2\ d_2\ h_2\}$ and the same cubic spline function as defined in (17) is adopted for the proposed multistep IMLS based RSM. Two iterative cycles are required before the procedure converges to an acceptable solution, and the performance comparison of the proposed algorithm with an improved tabu search method running directly on the original problem is given in Table II. It can be seen that the proposed technique can virtually reach the same optimal solution as the traditional optimization algorithm, even though the former uses only about one quarter of the finite element analysis computations of the latter.

TABLE II
Performance Comparison of the Proposed and Traditional Approaches for Solving TEAM Workshop Problem 22

	R_2	$H_2/2$	d_2	f_{opt}	No. of FEM Computations	No. of iterations on reconstructed problem
Proposed	3.09	0.242	0.389	8.20×10^{-2}	400+52+43	3897+3946
Tabu	3.10	0.240	0.388	8.19×10^{-2}	1842	/

V. Conclusion

An efficient global optimization tool is proposed, thanks to the success in the design and development of a new response surface model using the IMLS interpolation and the multistep method in conjunction with a tabu search method. The numerical results as reported have confirmed positively the approximation quality of the proposed multistep IMLS based RSM, and the global search ability and efficiency of the proposed global optimizer. Thus the proposed method is expected to be highly promising for rapid and robust global optimizations, not only for electromagnetic design problems in which the objective/constraint functions must be determined through computationally expensive algorithms such as three dimensional finite element analysis, but also for more general engineering design problems.

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