An Anatomy of IrisCode for Precise Phase Representation

[1,2]Adams Kong, [2]David Zhang and [1]Mohamed Kamel
[1]*Pattern Analysis and Machine Intelligence Lab, University of Waterloo, Canada*
[2]*Biometric Research Centre, The Hong Kong Polytechnic University, Hong Kong*
adamskong@ieee.org, csdzhang@comp.polyu.edu.hk, mkamel@uwaterloo.ca

Abstract

IrisCode, a widely deployed iris recognition algorithm, developed in 1993 and continuously modified by Daugman has attracted considerable attentions. IrisCode using a coarse phase representation has number of properties such as rapid matching, binomial imposter distribution and predictable false acceptance rate. Although many similar coding methods have been developed for irises and palmprints based on IrisCode, a theoretical analysis of IrisCode has not been provided. In this paper, we aim at studying (1) the nature of IrisCode, (2) the property of the phase of Gabor function, (3) the extension of bitwise hamming distance and (4) the theoretical foundation of the binomial imposter distribution and extending the coarse phase representation to a precise phase representation. Precisely, we demonstrate that IrisCode is a clustering algorithm with four prototypes; the locus of a Gabor function is a two-dimensional ellipse with respect to the phase parameter and bitwise hamming can be regarded as angular distance. Using these properties, we provide a precise phase representation for IrisCode with an effective implementation for filtering and matching. Practically, the imposter distribution of IrisCode follows binomial distribution. However, the theoretical evidence is incomplete according to our analysis.

1. Introduction

Many of the biometric systems that have been proposed and implemented for civil and commercial applications can verify, 1-to-1 match, or identify from a small database. Large databases still present a challenge in terms of accuracy and computational speed. Of existing biometric technologies, IrisCode, developed in 1993 and continuously improved by Daugman [1-2] is able to effectively identify a person in an extremely large database in real-time. This high level of has held considerable attention in the recent years. Following the key idea of IrisCode, various coding methods have been developed for iris and palmprint [3-4]. Many researchers believe that the cores of IrisCode are the operators, "≥" and "<" in coding equations and the bitwise hamming distance. Using these two operators, each feature value is encoded in one bit and two features are compared by bitwise hamming for rapid matching. However, this understanding of IrisCode is incomplete. According to our best knowledge, no one provides an analysis of IrisCode for understanding the inherent properties. In this paper, we aim at investigating (1) the nature of IrisCode, (2) the property of the phase of Gabor function, (3) the extension of bitwise hamming distance and (4) the theoretical foundation of the binomial imposter distribution. Using these properties, we present an algorithm for precise phase representation with an effective implementation.

To compute an IrisCode, 2-D Gabor functions with zero DC are applied to an iris image in dimensionless polar coordinate system, $I(\rho, \phi)$. The responses are coded according to the following inequalities:

$$h_{\mathrm{Re}} = 1\, if\ \mathrm{Re}\left(\int_\rho\int_\phi I(\rho,\phi)e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}e^{-i\omega(\theta_0-\phi)}\rho d\rho d\phi\right) \geq 0,$$

$$h_{\mathrm{Re}} = 0\, if\ \mathrm{Re}\left(\int_\rho\int_\phi I(\rho,\phi)e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}e^{-i\omega(\theta_0-\phi)}\rho d\rho d\phi\right) < 0,$$

$$h_{\mathrm{Im}} = 1\, if\ \mathrm{Im}\left(\int_\rho\int_\phi I(\rho,\phi)e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}e^{-i\omega(\theta_o-\phi)}\rho d\rho d\phi\right) \geq 0,$$

$$h_{\mathrm{Im}} = 0\, if\ \mathrm{Im}\left(\int_\rho\int_\phi I(\rho,\phi)e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}e^{-i\omega(\theta_0-\phi)}\rho d\rho d\phi\right) < 0,$$

where r_0, θ_0, ω, α and β are the parameters of the Gabor functions [2]. In this coding scheme, two bits (h_{Re}, h_{Im}) represent a sample point. Since the elements of IrisCodes are zeros and ones, hamming distance can be used to compare two IrisCodes with an effective implementation using a bitwise operator XOR. Hamming distance (*HD*) of the first version of IrisCode is defined as $HD = \sum_{i=1}^{2028} A_i \otimes B_i / 2048$, where A_i and B_i are the bits of two IrisCodes; ⊗ represents bitwise operator, XOR.

IrisCode has number of properties: 1) It is robust to local brightness and contrast variations. 2) It is a realization of cyclic code: in rotating between any

The 18th International Conference on Pattern Recognition (ICPR'06)
0-7695-2521-0/06 $20.00 © 2006 IEEE

adjacent phase quadrants, only a single bit changes. Cyclic code provides extra robustness to the genuine distribution. 3) The imposter distribution can be modeled as a binomial distribution with high degrees-of-freedom. This model provides a predictable false acceptance rate to control the decision threshold. Therefore, the false match rate of IrisCode is always zero. 4) Since the binomial imposter distribution has high degrees-of-freedom, the probability of hamming distance from two different irises being shorter than 0.333 is extremely low about 1 in 16 million. Even though two iris images from the same iris is poor degree of match say 70% agreement, the decision confidence is still very high. 5) Using the bitwise hamming distance, IrisCode can perform 1 million comparisons per second.

The rest of this paper is organized as follows. Section 2 demonstrates that IrisCode is a clustering algorithm. Section 3 presents the properties of the phase of Gabor function and an algorithm for precise phase representation. Section 4 discusses the theoretical base of the binomial imposter distribution. Section 5 offers some concluding remarks.

2. IrisCode — a clustering algorithm

A complete understanding of IrisCode is important for designing new coding methods and extending IrisCode. Let the real and imaginary parts of a Gabor function with zero DC be $M_R(\rho,\phi)$ and $M_I(\rho,\phi)$, respectively. Their definitions are given below:

$$M_R(\rho,\phi)=e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}\left(\cos(-\omega(\theta_0-\phi))-K\right) \quad (1)$$

$$M_I(\rho,\phi)=e^{-(r_0-\rho)^2/\alpha^2}e^{-(\theta_o-\phi)^2/\beta^2}\left(\sin(-\omega(\theta_0-\phi))\right), \quad (2)$$

where K is a constant to remove the DC of a Gabor funtion. For the sake of convenience, we use M_R to represent $M_R(\rho,\phi)$. For other symbols, we use the similar notations. We define a filter-generating function,

$$Z(\varphi)=\cos(\varphi)M_R+\sin(\varphi)M_I \quad (3)$$

where $\varphi\in[0, 2\pi)$. It is a continuous periodic function with respect to the parameter, φ. By substituting $5\pi/4$, $7\pi/4$, $\pi/4$, and $3\pi/4$ to φ, we can obtain four filters,

$$Z_0=Z(5\pi/4)=(-M_R-M_I)/\sqrt{2}\,, \quad (4)$$

$$Z_1=Z(7\pi/4)=(M_R-M_I)/\sqrt{2}\,, \quad (5)$$

$$Z_2=Z(\pi/4)=(M_R+M_I)/\sqrt{2}\,, \quad (6)$$

$$Z_3=Z(3\pi/4)=(-M_R+M_I)/\sqrt{2}\,, \quad (7)$$

These four filters are the cluster centers. We employ a clustering criterion defined as $j=\arg\max_i \int_\rho\int_\phi \rho I Z_i d\rho d\phi/(\|\rho I\|\|Z_i\|)$, where j is called the winning index. This criterion can be regarded as the cosine measure between Z_i and ρI. Since $\int_\rho\int_\phi M_I M_R d\rho d\phi=0$, from Eqs. 4-7, we know that the four filters have the same power, $\int_\rho\int_\phi Z_i^2 d\rho d\phi=C$, where C is a constant. We can rewrite the clustering criterion as $j=\arg\max_i\left(\int_\rho\int_\phi \rho I Z_i d\rho d\phi\right)$. To obtain a binary representation of the winning indexes, we encode the winning indexes according to the right part of Table 1. We use bitwise hamming distance to measure the difference between two encoded winning indexes as IrisCode. Comparing the binary representation of winning indexes and IrisCode in Table 1, we know that they are equivalent. In the other words, IrisCode is a clustering algorithm based on maximum cosine measure to assign an image patch to one of the prototypes, Z_i.

Table 1. Comparison of IrisCode, winning index and coded winning index

IrisCode		Winning index	Coded winning indexes	
h_{Im}	h_{Re}		Bit 2	Bit 1
0	0	0	0	0
0	1	1	0	1
1	1	2	1	1
1	0	3	1	0

3. Precise phase representation

We have demonstrated that IrisCode is a clustering algorithm. To give more precise phase representation, we firstly need more prototypes, which can be obtained by taking more sample points from the filter-generating function in Eq. 3. It should be noted that the filters generated by the filter-generating function have zero DC and the clustering criterion is independent of the contrast of the image. To embed the other inherent properties of IrisCode such as rapid matching, we need a novel coding scheme to encode the winning indexes and distance measure for matching.

3.1. Properties of the filter-generating function

Let the number of prototypes be $2N$ defined as $Z(i\pi/N)$, where i=0, 1, …, $2N$-1 and N is called the order of coding scheme. To measure the difference between two winning indexes, we need to understand the physical meaning of φ and the property of $Z(\varphi)$. One important property is that the locus of the filter-generating function is on a two dimensional ellipse with respect to φ. We discretize M_R, M_I and $Z(\varphi)$ to obtain three vectors, $\vec{M}_R$, $\vec{M}_I$ and $\vec{Z}(\varphi)$, respectively.

The 18th International Conference on Pattern Recognition (ICPR'06)
0-7695-2521-0/06 $20.00 © 2006 IEEE

We define two new vectors, $\vec{v}_R = \vec{M}_R / \|\vec{M}_R\|$ and $\vec{v}_I = \vec{M}_I / \|\vec{M}_I\|$. Since M_R and M_I are orthogonal, $\vec{v}_R$ and $\vec{v}_I$ can be regarded as orthonormal bases of a two dimensional subspace. $\vec{Z}(\varphi)$ can be rewritten as

$$\vec{Z}(\varphi) = \cos(\varphi)\|\vec{M}_R\|\vec{v}_R + \sin(\varphi)\|\vec{M}_I\|\vec{v}_I, \quad (8)$$

a linear combination of $\vec{v}_R$ and $\vec{v}_I$. Obviously, $\vec{Z}(\varphi)$ is on the two dimensional space spanned by $\vec{v}_R$ and $\vec{v}_I$. Using these bases $\vec{v}_R$ and $\vec{v}_I$ to represent $\vec{Z}(\varphi)$, the coordinate of $\vec{Z}(\varphi)$ in this two dimensional space is $(\|M_R\|\cos(\varphi), \|M_I\|\sin(\varphi))$ fulfilling the following equality,

$$(\|M_R\|\cos(\varphi))^2 / \|M_R\|^2 + (\|M_I\|\sin(\varphi))^2 / \|M_I\|^2 = 1. \quad (9)$$

It means that the locus of $\vec{Z}(\varphi)$ is an ellipse with respect to φ.

The physical meaning of φ is also important. Assuming that the patches of iris to be clustered have zero DC, we can ignore the term K in M_R and rewrite $Z(\varphi)$ as

$$Z(\varphi) = e^{-(r_0-\rho)^2/\sigma^2} e^{-(\theta_0-\phi)^2/\beta^2} \cos(-\omega(\theta_0 - \phi) - \varphi). \quad (10)$$

It is clear that that φ is the phase of Gabor function. In order words, winning indexes and IrisCode store the same information.

3.2. Angular distance

We demonstrate that $\vec{Z}(\varphi)$ is always on a two dimensional ellipse and φ is the phase. The distance between $\vec{Z}(\omega)$ and $\vec{Z}(\gamma)$ can be defined as $\min(|\omega-\gamma|, 2\pi-|\omega-\gamma|)$, the angular distance between ω and γ. If ω and γ are obtained by uniform sampling *i.e,* $\omega=2\pi p/2N$ and $\gamma=2\pi q/2N$ where p and q are two integers between 0 and $2N-1$, we can rewrite the angular distance as $\min(\pi/N|p-q|, \pi/N(2N-|p-q|))$. We define π/N as one unit distance. The angular distance can be rewritten as $\min(|p-q|, (2N-|p-q|))$. Therefore, the distance between two winning indexes, p and q, the integer presentation of the phase can also be defined as $\min(|p-q|, (2N-|p-q|))$. It should be noted that the angular distance between any two adjacent winning indexes is 1. Thus, the cyclic code property in IrisCode is embedded.

3.3. Coding scheme and bitwise matching

One important property of IrisCode is rapid matching supporting real-time large database identification. The bitwise hamming distance is the key. However, the winning indexes and the angular distance are not bitwise representation. To embed this property in the precise phase representation, we need a new coding scheme and bitwise angular distance. We provide a coding matrix $A=[a_{u,v}]$, where $1\le u\le N$; $1\le v\le 2N$ and $a_{u,v}$ defined in Fig. 1.

if $v \le N$ *and* $1 \le u < v$, *then* $a_{u,v} = 1$
else if $v > N$ *and* $v - N \le u \le N$, then $a_{u,v} = 1$
else $a_{u,v} = 0$

Figure 1. Pseudo code of the coding table.

Table 2 gives a coding table when N=3. Winning index, j is represented by the $j+1^{th}$ column of the matrix A. Each winning index is represented by N bits. Using this coding scheme, we can prove the equivalent relationship between bitwise hamming distance and the angular distance. Mathematically, we can show $\sum_{u=1}^{N} a_{u,v} \otimes a_{u,v+k} = \min(k, 2N-k)$, where $1\le v\le v+k\le 2N$. However, we do not have enough space to provide this proof.

Table 2 The coding table for N=3

Winning index	0	1	2	3	4	5
Bit 0	0	1	1	1	0	0
Bit 1	0	0	1	1	1	0
Bit 2	0	0	0	1	1	1

3.4. Effective filtering

Considering IrisCode as a clustering algorithm helps us to develop the novel coding scheme and angular distance for precisely representing and rapidly matching the phase information. However, number of filtering increases with respect to the precision of the phase. If we use N bits to represent one winning index, we need $2N$ filters. The computation cost increases dramatically. Let us consider the clustering criterion again

$$j = \arg\max_i \int_\rho \int_\phi \rho I Z(i\pi/N) d\rho d\phi / (\|\rho I\| \|Z(i\pi/N)\|), \quad (11)$$

where $\|\rho I\|$ is independent of i and $\|Z(i\pi/N)\|$ can be pre-computed. These two terms would not greatly increase the computation cost. The main problem comes from $\int_\rho \int_\phi \rho I Z(i\pi/N) d\rho d\phi$. Substituting Eq. 3 into $\int_\rho \int_\phi \rho I Z(i\pi/N) d\rho d\phi$, we have

$$\int_\rho \int_\phi \rho I (\cos(i\pi/N) M_R + \sin(i\pi/N) M_I) d\rho d\phi$$
$$= \cos(i\pi/N) C_R + \sin(i\pi/N) C_I, \quad (12)$$

where $C_R = \int_\rho \int_\phi \rho I M_R d\rho d\phi$ and $C_I = \int_\rho \int_\phi \rho I M_I d\rho d\phi$. Therefore, no matter what precision of the phase, we need only two filters, M_R and M_I. We have successfully reduced the number of filtering from $2N$ to 2.

4. Discussion of binomial imposter distribution

In the previous sections, we have analyzed IrisCode and developed an algorithm for precisely representing the phase. In this representation, the winning indexes are represented by zeros and ones and compared by their hamming distance, exactly same as IrisCode. A famous and important property of IrisCode is the binomial imposter distribution, which has been experimentally validated by large datasets. Some may expect that the imposter distributions of the precise phase representation and other coding methods [3-4] also follow binomial distribution.

Let us firstly study the theoretical base of the binomial imposter distribution of IrisCode. A random variable X following a binomial distribution should satisfy:

(1) X is defined as $\sum_{i=1}^{M} T_i$, where T_i is a Bernoulli variable;

(2) all T_i has the same probability p for success and

(3) all T_i are independent.

We refer (2) as stationary condition and (3) as independent condition. Bitwise hamming distance used in IrisCode can be regarded as the sum of Bernoulli variables. Daugman verified the stationary condition in 1993 [1]. However, IrisCode violates the independent condition since the texture features in the iris are inherently correlated. If the sum of correlated stationary Bernoulli trails would unconditionally follow binomial distribution, the imposter distribution of IrisCode had to be binomial. However, we do not have any mathematical evidence for it. Even if the correlation is the first order Markovian type, the distribution can be bimodal and trimodal shape [5]. Although the imposter distribution of IrisCode follows binomial distribution practically, theoretical evidence is not enough. Therefore, the imposter distributions of other coding methods and the precise phase representation are not guaranteed to be binomial. Some may interest why the imposter distribution of IrisCode practically follows binomial distribution. Under some conditions and making use of central limit theorem, sum of correlated Bernoulli trails can be approximated by normal distribution. Binomial distribution with high degrees-of-freedom can be approximated by normal distribution. It may be the theoretical base of the binomial imposter distribution of IrisCode. Interested readers can refer [6].

5. Conclusion

In this paper, we have provided a detailed analysis of IrisCode and made use of the properties for precise phase representation. In this paper, we demonstrate that IrisCode is a clustering algorithm; the locus of Gabor function is on a two dimensional ellipse with respect to the phase parameter and bitwise hamming distance is a special case of angular distance. Using these properties, we have presented an algorithm with effective filtering and rapid matching for precise phase representation. We have also discussed the theoretical foundation of the binomial distribution of IrisCode. In this paper, we concentrate only on the theoretical study because of the page limit. The experimental results of precise phase representation will be presented in a coming paper.

References

[1] J.G. Daugman, “High confidence visual recognition of persons by a test of statistical independence”, *IEEE Trans. on PAMI*, vol. 15, no. 11, pp. 1148-1161, 1993.

[2] J. Daugman, “How Iris Recognition Works”, *IEEE Trans. on CSVT*, vol. 14, no. 1, pp. 21-30, 2004.

[3] L. Ma, T. Tan, Y. Wang, D. Zhang, “Efficient iris recognition by characterizing key local variations”, *IEEE Trans on IP*, vol. 13, no. 6, pp. 739-750, 2004.

[4] A.W.K. Kong and D. Zhang, “Competitive coding scheme for palmprint verification”, in *Proc. of ICPR*, vol. 1, pp. 520-523, 2004.

[5] R. Viveros, K. Balasubramanian and N. Balakrishnan, “Binomial and negative binomial analogues under correlated Bernoulli trials”, *The American Statistician*, vol. 48, no. 3, pp. 243-247, 1994.

[6] W. Feller, An introduction to probability theory and its applications, John Wiley & Sons Inc, 3rd edition, vol. 1, pp. 321, 1968.