ASSEMBLED MATRIX DISTANCE METRIC FOR 2DPCA-BASED FACE AND PALMPRINT RECOGNITION

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Abstract:

Two-dimensional Principal component analysis (2DPCA) is a novel image representation approach recently developed for image recognition. One advantage of 2DPCA is that it can extract feature matrix using a straightforward image projection technique. In this paper, we propose an assembled matrix distance metric (AMD) to measure the distance between two feature matrices. To test the efficiency of the proposed distance measure, we use two image databases, the ORL face and the PolyU palmprint. The experimental results show that the assembled matrix distance metric is very effective in 2DPCA based image recognition.

Keywords:

2DPCA; Assemble Matrix Metric; Image Recognition; Face Recognition; Palmprint Recognition

1. Introduction

Principal component analysis or PCA-based techniques have been very successful in image representation and recognition. In 1987, Sirovich and Kirby [1] used PCA to represent human faces. Subsequently, Turk and Pentland proposed a PCA-based face recognition method, Eigenfaces [2]. PCA has now been widely investigated and has been successfully applied to other image recognition tasks [3, 4].

Recently, Yang proposed a novel image representation and recognition technique, two-dimensional PCA (2DPCA) [5, 6]. 2DPCA has many advantages over classical PCA. In classical PCA, an image matrix should be mapped into a 1D vector in advance. 2DPCA, however, can directly extract feature matrix from the original image matrix. This leads to that much less time is required for training and feature extraction. Further, the recognition performance of 2DPCA is better than that of classical PCA.

Despite the great success of 2DPCA, some issues remain that deserve to be further investigated. One such arises when 2DPCA is used in image recognition. In such cases, the classification of an unknown image usually requires the calculation of the distance between feature matrices. Yet, even though previous studies of PCA and ICA have shown that distance measures greatly affect the recognition performance [7, 8, 9], with reference to the 2DPCA-base methods, distance measures have been little investigated.

In this paper, we propose an assembled matrix distance (AMD) metric which is suitable for representing the distance between two feature matrices. In order to evaluate the efficiency of the AMD metric, experiments were carried out using the ORL face database and the PolyU palmprint database. Experimental results show that, the AMD measure is effective for 2DPCA-based image recognition.

The organization of this paper is as follows: Section 2 briefly reviews 2DPCA. Section 3 proposes an assembled matrix distance metric for use in 2DPCA-based image recognition. Section 4 presents the results of experiments using the ORL face database and the PolyU palmprint database. Section 5 offers our conclusion.

2. 2DPCA and Matrix Distance Measures

In this section, we briefly reviewed the algorithm of two-dimensional PCA and presented a survey on the previous work on matrix distance measures.

2.1. Two-dimensional PCA

Given a training set $\{\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_N\}$, two-dimensional PCA first uses all training images to construct the total image covariance matrix $\mathbf{G}_t$

$$\mathbf{G}_t = \sum_{i=1}^{N} (\mathbf{X}_i - \overline{\mathbf{X}})^T (\mathbf{X}_i - \overline{\mathbf{X}}), \tag{1}$$

where $\mathbf{X}_i$ is the ith training image, $\overline{\mathbf{X}}$ is the mean image of all training samples, and N is the number of training images. Then, the projection axes of 2DPCA, $W_1, \cdots, W_d$, can be obtained by maximizing the image scatter criterion

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$$J(W) = W^T \mathbf{G}_t W , \qquad (2)$$

where W is a unitary column vector.

Actually, $W_1, \cdots, W_d$ are the eigenvectors corresponding to the first d largest eigenvalues of G_t. Finally we obtain the 2DPCA projection matrix W_{opt},

$$\mathbf{W}_{opt} = [W_1, \cdots, W_d] , \qquad (3)$$

and use

$$\mathbf{Y} = \mathbf{X}\mathbf{W}_{opt} , \qquad (4)$$

2.2. Previous work on matrix distance measures

Give two feature matrices $\mathbf{A} = (a_{ij})_{m \times d}$ and $\mathbf{B} = (b_{ij})_{m \times d}$, Yang used the Frobenius distance measure in [5]

$$d_F(\mathbf{A}, \mathbf{B}) = \left(\sum_{i=1}^{m} \sum_{j=1}^{d} (a_{ij} - b_{ij})^2 \right)^{1/2} , \qquad (5)$$

and proposed another distance measure (named hereafter as Yang distance) in [6],

$$d_Y(\mathbf{A}, \mathbf{B}) = \sum_{j=1}^{d} \left(\sum_{i=1}^{m} (a_{ij} - b_{ij})^2 \right)^{1/2} . \qquad (6)$$

Definition 1 The Frobenius norm of a matrix $\mathbf{A} = [a_{ij}]_{m \times d}$ is defined by $\|\mathbf{A}\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{d} a_{ij}^2}$ [10].

From **Definition 1**, it is easy to see that the Frobenius distance is a metric derived from the Frobenius matrix norm. Actually, both the Frobenius and the Yang distance measure are matrix metrics, and we will prove this in the next section.

3. Assembled Distance Metric

To calculate the distance between two feature matrices, we propose an assembled matrix distance (AMD) metric. Unlike PCA-based approaches to produce a feature vector, 2DPCA directly extracts a feature matrix from an original image matrix. For this reason we here present a novel assembled matrix distance metric to measure the distance between feature matrices.

Given two feature matrices $\mathbf{A} = (a_{ij})_{m \times d}$ and $\mathbf{B} = (b_{ij})_{m \times d}$, we define the assembled matrix distance $d_{AMD}(\mathbf{A}, \mathbf{B})$ as follows:

$$d_{AMD}(\mathbf{A}, \mathbf{B}) = \left(\sum_{j=1}^{d} \left(\sum_{i=1}^{m} (a_{ij} - b_{ij})^2 \right)^{\frac{1}{2}p} \right)^{1/p} , (p > 0) . \qquad (7)$$

Definition 2 A *vector norm* on $\mathbb{R}^n$ is a function $f : \mathbb{R}^n \to \mathbb{R}$ with the following properties [10]:

$$f(x) \geq 0, \qquad x \in \mathbb{R}^n (f(x) = 0 \Leftrightarrow x = 0) , \qquad (8)$$

$$f(x+y) \leq f(x) + f(y), \qquad x, y \in \mathbb{R}^n , \qquad (9)$$

$$f(\alpha x) \leq |\alpha| f(x), \qquad \alpha \in \mathbb{R}, x \in \mathbb{R}^n . \qquad (10)$$

Definition 3 A *matrix norm* on $\mathbb{R}^{m \times d}$ is a function $f : \mathbb{R}^{m \times d} \to \mathbb{R}$ with the following properties [10]:

$$f(\mathbf{A}) \geq 0, \quad \mathbf{A} \in \mathbb{R}^{m \times d} (f(\mathbf{A}) = 0 \Leftrightarrow \mathbf{A} = 0) , \qquad (11)$$

$$f(\mathbf{A} + \mathbf{B}) \leq f(\mathbf{A}) + f(\mathbf{B}), \quad \mathbf{A}, \mathbf{B} \in \mathbb{R}^{m \times d} , \qquad (12)$$

$$f(\alpha \mathbf{A}) \leq |\alpha| f(\mathbf{A}), \qquad \alpha \in \mathbb{R}, \mathbf{A} \in \mathbb{R}^{m \times d} . \qquad (13)$$

Theorem 1 Function $\|x\|_p = (\sum_{i=1}^{n} |x_i|^p)^{1/p}$ is a vector norm [10].

Theorem 2 Function $\|\mathbf{A}\|_{AMD} = \left(\sum_{j=1}^{d} \left(\sum_{i=1}^{m} (a_{ij})^2 \right)^{\frac{1}{2}p} \right)^{1/p}$, $(p > 0)$ is a matrix norm.

Proof. It can be easily shown that,

$$\|\mathbf{A}\|_{AMD} \geq 0 ,$$

$$\|\mathbf{A}\|_{AMD} = 0 \Leftrightarrow \mathbf{A} = 0 ,$$

$$\|\alpha \mathbf{A}\|_{AMD} = |\alpha| \|\mathbf{A}\|_{AMD} .$$

Next we'll prove $\|\mathbf{A} + \mathbf{B}\|_{AMD} \leq \|\mathbf{A}\|_{AMD} + \|\mathbf{B}\|_{AMD}$. Let the vector $a^{(j)} = [a_{1j}, a_{2j}, \cdots, a_{mj}]^T$ and the vector $b^{(j)} = [b_{1j}, b_{2j}, \cdots, b_{mj}]^T$, where $a^{(j)}$ and $b^{(j)}$ denote the jth column of $\mathbf{A}$ and $\mathbf{B}$. Then we can prove that,

$$\|\mathbf{A} + \mathbf{B}\|_{AMD} = \left(\sum_{j=1}^{d} \left(\sum_{i=1}^{m} (a_{ij} + b_{ij})^2 \right)^{\frac{1}{2}p} \right)^{1/p}$$

$$\leq (\sum_{j=1}^{d} (\|a^{(j)}\|_2 + \|b^{(j)}\|_2)^p)^{1/p} .$$

From Theorem 1, function $g(a) = \sum_{j=1}^{d} (\|a^{(j)}\|_2)^p)^{1/p}$, $(a = [\|a^{(1)}\|_2, \|a^{(2)}\|_2, \cdots, \|a^{(d)}\|_2]^T)$ is a vector norm. Let $b = [\|b^{(1)}\|_2, \|b^{(2)}\|_2, \cdots, \|b^{(d)}\|_2]^T$,

$$(\sum_{j=1}^{d}(\|a^{(j)}\|_2 + \|b^{(j)}\|_2)^p)^{1/p} = g(a+b)$$
$$\leq g(a) + g(b)$$
$$= (\sum_{j=1}^{d}(\|a^{(j)}\|_2)^p)^{1/p} + (\sum_{j=1}^{d}(\|b^{(j)}\|_2)^p)^{1/p}$$
$$= \left(\sum_{j=1}^{d}\left(\sum_{i=1}^{m} a_{ij}{}^2\right)^{\frac{1}{2}p}\right)^{1/p} + \left(\sum_{j=1}^{d}\left(\sum_{i=1}^{m} b_{ij}{}^2\right)^{\frac{1}{2}p}\right)^{1/p}$$
$$= \|\mathbf{A}\|_{AMD} + \|\mathbf{B}\|_{AMD}.$$

So $\|\mathbf{A}+\mathbf{B}\|_{AMD} \leq \|\mathbf{A}\|_{AMD} + \|\mathbf{B}\|_{AMD}$, and $\|\mathbf{A}\|_{AMD}$ is a matrix norm.

Definition 3 A *metric* in $\mathbb{R}^{m\times d}$ is a function $f : \mathbb{R}^{m\times d} \times \mathbb{R}^{m\times d} \to \mathbb{R}$ with the following properties [11]:

$$f(\mathbf{A},\mathbf{B}) \geq 0, \qquad \mathbf{A},\mathbf{B} \in \mathbb{R}^{m\times d}, \tag{14}$$
$$f(\mathbf{A},\mathbf{B}) = 0 \Leftrightarrow \mathbf{A} = \mathbf{B}, \tag{15}$$
$$f(\mathbf{A},\mathbf{B}) = f(\mathbf{B},\mathbf{A}), \tag{16}$$
$$f(\mathbf{A},\mathbf{B}) \leq f(\mathbf{A},\mathbf{C}) + f(\mathbf{C},\mathbf{B}), \tag{17}$$

Theorem 3 the function $d_{AMD}(\mathbf{A},\mathbf{B})$ is a distance metric.

Proof. Function $\|\mathbf{A}\|_{AMD}$ is a matrix norm, and it is simple to see that $d_{AMD}(\mathbf{A},\mathbf{B}) = \|\mathbf{A}-\mathbf{B}\|_{AMD}$ is a distance measure derived from the matrix norm $\|\mathbf{A}\|_{AMD}$. So the function $d_{AMD}(\mathbf{A},\mathbf{B})$ is a distance metric.

Corollary 1 The Frobenius distance measure $d_F(\mathbf{A},\mathbf{B}) = \left(\sum_{j=1}^{d}\sum_{i=1}^{m}(a_{ij}-b_{ij})^2\right)^{1/2}$ is a special case of AMD metric with $p = 2$.

Corollary 2 The Yang distance measure $d_Y(\mathbf{A},\mathbf{B}) = \sum_{j=1}^{d}\left(\sum_{i=1}^{m}(a_{ij}-b_{ij})^2\right)^{1/2}$ is a special case of AMD metric with $p = 1$.

4. Experimental Results and Discussions

To evaluate the efficiency of the 2DPCA image recognition method using the AMD metric (2DPCA-AMD), we used two image databases, the ORL face database and the PolyU palmprint database. For each database, we investigated the effect of AMD parameter p, and compare the recognition accuracy of different distance measures. We also compared the recognition rate obtained using 2DPCA-AMD with that obtained using other methods, such as Eigenfaces, Fisherfaces, and D-LDA.

4.1. Experimental Results on the ORL database

The ORL face database was used to test the AMD metric for 2DPCA-based face recognition [12]. The ORL database contains 400 facial images with 10 images per individual. The images vary in sampling time, light conditions, facial expressions, facial details (glasses/no glasses), scale and tilt. All the images are taken against a dark homogeneous background, with the person in an upright frontal position, with a tolerance for some tilting and rotation of up to about 20°. The size of these gray images is 112×92. Figure 1 shows ten images of one individual in the ORL database.

Figure 1. Ten images of one individual in the ORL database

For the ORL database, we randomly chose five samples per individual for training, resulting in a training set of 200 images and a testing set of 200 images with no overlap between the two sets. To reduce the variation of recognition results, the mean of recognition rates over 10 runs was calculated to derive an average recognition rate (ARR).

To construct the projection matrix $\mathbf{W}_{opt}$, we chose d = 4 eigenvectors corresponding to the first d largest eigenvalues of the total scatter matrix $\mathbf{G}_t$, and then studied the effect of AMD parameters on the performance of 2DPCA. Figure 2 depicts the average recognition rate of the AMD metric with different p values. The highest ARR, 0.9630, can be obtained when $p \leq 0.125$. The ARR decrease as parameter p increases when $p \geq 0.125$. We thus set the AMD parameter $p = 0.125$.

We also provide a comparison of the recognition performance of the Frobenius, Yang and AMD distance measures. Figure 3 compares the recognition rate of these three distance measures with different d values. Table 1 compares their highest average recognition rates. We can see that the highest recognition rate was of the AMD measure. The AMD measure can thus be regarded as the best choice for expressing the distance between different feature matrices.

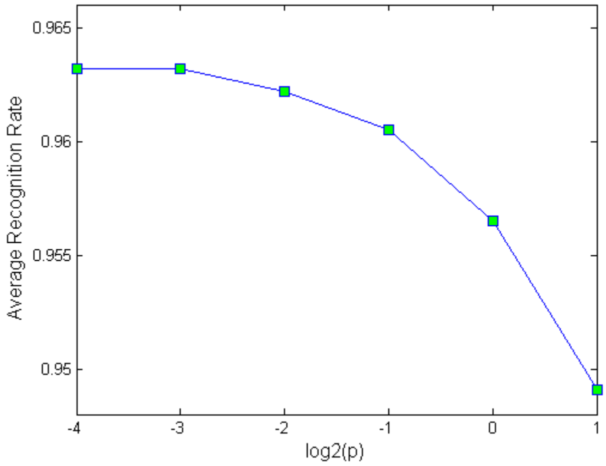

Figure 2. Average recognition rates of 2DPCA with different p values using the ORL database

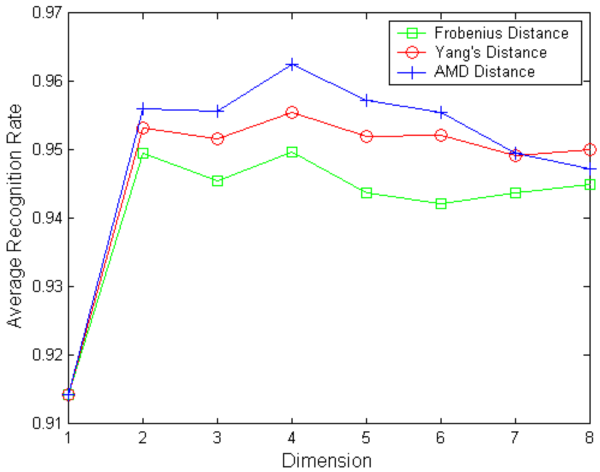

Figure 3. Comparison of recognition rates obtained with different distance measures using the ORL database

Table 2 compares the recognition rates obtained using Eigenfaces, Fisherfaces, D-LDA and 2DPCA-AMD. 2DPCA-AMD has a recognition rate of 0.9630, higher than that obtained using other image recognition methods.

4.2. Experimental Results on the PolyU Palmprint Database

Palmprint sampling is low cost, non-intrusive, and palmprint has a stable structural feature, making palmprint recognition the object of considerable recent research interest. Here we use the PolyU palmprint database to test the efficiency of the proposed AMD metric. The PolyU palmprint database contains 600 grayscale images of 100 different palms with six samples for each palm [13, 14]. Six samples from each of these palms were collected in two sessions, where the first three samples were captured in the first session and the other three in the second session. The average interval between the first and the second session was two months. In our experiments, sub-image of each original palmprint was cropped to the size of 128×128 and pre-processed by histogram equalization. Figure 4 shows six palmprint images of one palm. For the PolyU palmprint database, we choose the first 3 samples per individual for training, and thus use all the 300 images captured in the first session as training set and the images captured in the second session as testing set.

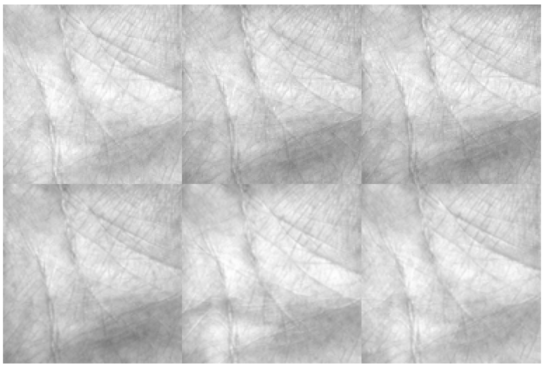

Figure 4. Six palmprint images of one palm in the PolyU palmprint database

Figure 5 shows the recognition rates obtained using different AMD parameter p values with $d = 18$. The highest recognition rate can be obtained when $p \le 0.25$. The recognition rates decrease with the increasing of p values when $p \ge 0.25$. We thus set the AMD parameter $p = 0.25$.

A detailed study is provided to compare the recognition performance of the Frobenius, Yang and AMD distance measures for 2DPCA. Figure 6 depicts the recognition rate of these three distance measures when they have different d values and Table 3 compares the highest recognition rates of these three distance measures. The AMD measure obtained the highest recognition rate for almost all d values. The highest recognition rate is 0.9767 for the AMD measure, 0.9467 for Yang distance measure, and 0.8867 for the Frobenius distance measure.

Table 4 compares the recognition rates obtained using Eigenfaces, Fisherfaces, D-LDA and 2DPCA-AMD. The recognition rate of 2DPCA-AMD is 0.9630, higher than that obtained using other image recognition methods.

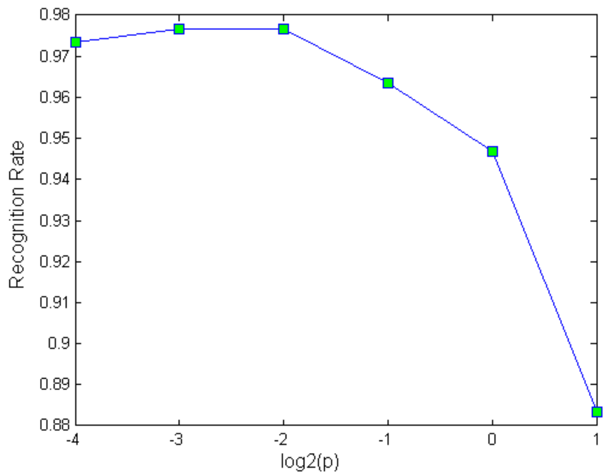

Figure 5. Recognition rates of 2DPCA with different *p* values using the PolyU palmprint database

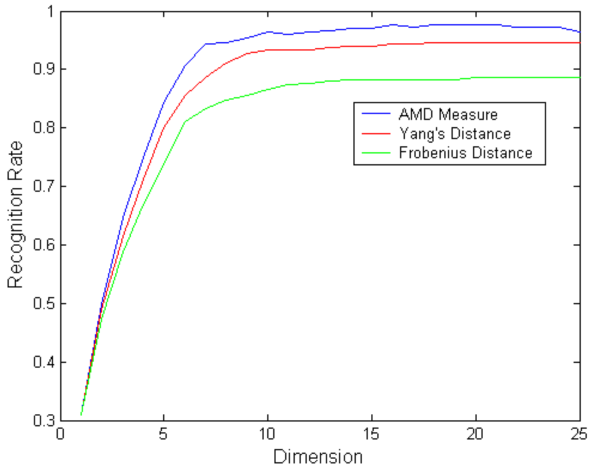

Figure 6. Comparison of recognition rates obtained with different distance measures using the PolyU palmprint database

5. Conclusions

In this paper, we proposed an assembled matrix distance (AMD) metric to calculate the distance between two feature matrices, and compared three distance measures for 2DPCA based image recognition method. We also compared the recognition accuracy of 2DPCA-AMD with that of other appearance based image recognition approaches. To test the efficiency of the AMD distance measure, a series of experiments were carried out using the ORL face database and the PolyU palmprint database. Experimental results show that the AMD metric has a better recognition rate than the Frobenius and Yang distance measures. On the ORL database with five training samples per individual, the 2DPCA-AMD method achieved an average recognition rate of 0.9670. On the PolyU palmprint database, 2DPCA-AMD achieved a recognition rate of 0.9767.

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References

[1] L. Sirovich, and M. Kirby, “Low-dimensional procedure for characterization of human faces,” *J. Optical Soc. of Am.*, vol. 4, pp. 519-524.

[2] M. Turk, and A. Pentland, “Eigenfaces for recognition,” *J. Cognitive Neuroscience*, vol. 3, no. 1, pp. 71-86, 1991.

[3] G. Lu, D. Zhang, and K. Wang, “Palmprint recognition using eigenpalms features,” *Pattern Recognition Lett.*, vol. 24, pp. 1463-1467, 2003.

[4] R. Huber, H. Ramoser, K. Mayer, H. Penz, and M. Rubik, “Classification of coins using an eigenspace approach,” *Pattern Recognition Lett.*, vol. 26, pp. 61-75, 2005.

[5] J. Yang, and J.Y. Yang, “From image vector to matrix: a straightforward image projection technique – IMPCA vs. PCA,” *Pattern Recognition*, vol. 35, pp. 1997-1999, 2002.

[6] J. Yang, D. Zhang, A.F. Frangi, J.-Y. Yang, “Two-Dimensional PCA: a new approach to appearance-based face representation and recognition”, *IEEE Trans. PAMI*, vol. 26, no. 1, pp. 131-137, 2004.

[7] H. Moon and J. Phillips. Analysis of PCA-based face recognition algorithms. In K. Boyer and J. Phillips, editors, *Empirical Evaluation Techniques in Computer Vision*. IEEE Computer Society Press, 1998.

[8] W.S. Yambor, B.A. Draper, and J.R. Beveridge, “Analyzing PCA-based face recognition algorithm: Eigenvector selection and distance measures,” In: H. Christensen, J. Phillips (Eds), *Empirical Evaluation Methods in Computer Vision*, World Scientific Press, Singapore, 2002.

[9] V. Perlibakas, “Distance measures for PCA-based face recognition,” *Pattern Recognition Lett.*, vol. 25, pp. 711-724, 2004.

[10] G.H. Golub, and C. F. Van Loan, *Matrix Computation* (3rd edition), Baltimore: The Johns Hopkins University Press, 1996.
[11] D. Chaudhuri, C.A. Murthy, and B.B. Chaudhuri, "A modified metric to compute distance," *Pattern Recognition*, vol. 25, pp.667-677, 1992.
[12] The ORL face database. AT&T (Olivetti) Research Laboratories, Cambridge, U.K.. [Online]. Available: http://www.uk.research.att.com/facedatabase.html.
[13] D. Zhang, W. Kong, J. You and M. Wong, "On-line palmprint identification", *IEEE Trans. PAMI*, vol. 25, no. 9, pp. 1041-1050, 2003.
[14] PolyU Palmprint Palmprint Database, Available: http://www.comp.polyu.edu.hk/ ~biometrics/.

Table 1. Comparison of recognition rates obtained by different distance metrics using the ORL database

Distance Measures	Frobenius	Yang	AMD
Recognition Rate	0.9490	0.9560	0.9630

Table 2. Comparison of recognition rates obtained by different methods using the ORL database

Methods	Fisherfaces	D-LDA	Eigenfaces	2DPCA-AMD
Recognition Rate	0.9250	0.9470	0.9380	0.9630

Table 3. Comparison of recognition performance of different distance metrics using the PolyU pamprint database

Distance Measures	Frobenius	Yang	AMD
Recognition Rate	0.8867	0.9467	0.9767

Table 4. Comparison of recognition rates obtained by different methods using the PolyU pamprint database

Methods	Eigenfaces	Fisherfaces	D-LDA	2DPCA-AMD
Recognition Rate	0.8867	0.9333	0.9400	0.9767