The Impact of Information Sharing in a Two-Level Supply Chain with Multiple Retailers

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Information sharing is an important component of cooperation in supply chain management. This paper presents a study to evaluate the impact of information sharing on inventory and expected cost in a two-level supply chain with multiple retailers. Three levels of information sharing are given and the optimal inventory policy under each level is derived. We show that both the inventory level and expected cost of the manufacturer decrease with an increase in the level of information sharing.

Keywords: Supply chain; Information sharing; Inventory

Introduction

In the past businesses used to adopt the traditional mode of determining their inventory — they decide their inventory only according to the ordering quantities of their downstream organizations in the supply chain. It has been shown that this approach of inventory management suffers from many deficiencies, one of which is the so-called "bullwhip effect" (see Refs. 1, 2). The bullwhip effect is essentially the phenomenon of demand variability amplification along a supply chain, which can create problems for suppliers, such as grossly inaccurate demand forecasts, low capacity utilization, excessive inventory, and poor customer service. With recent advances of information technologies, Electronic Data Interchange (EDI) is widely used in retailing systems to obtain real-time information about the system, which provides an enabling means for information sharing. By information sharing, suppliers can decide their inventory according to customers' demands observed, so the harmful effects of demand distortion can be mitigated.

In general there are three levels of information sharing in a two-level supply chain. Level 1 is the traditional ordering process where the manufacturer and the retailer belong to different organizations and they operate in a decentralized fashion. At level 2, the manufacturer and the retailer decide their inventory policies under coordinated control and the manufacturer has access to the customers' demand information, in addition to the ordering information from the retailer. At level 3, the manufacturer and the retailer cooperate under centralized control. By EDI, the manufacturer establishes its inventory policy based on the customers' demand information directly. Meanwhile, the manufacturer helps the retailer to make its replenishment decision. This is the so-called vendor-managed inventory (VMI) practice.

The most celebrated implementation of demand information sharing is Wal-Mart's Retail Link program, which provides an on-line summary of point-of-sales data to suppliers such as Johnson and Johnson, and Lever Brothers (see Ref. 3). Another successful application of EDI was made by P&S Company, one of the two largest supermarket chains in Hong Kong, and its important supplier J&J, a famous local beauty care product distributor (see Ref. 4). With real-time information about its product inventory at all retail stores of P&S, J&J's distribution centre can make joint replenishments for P&S's retailing network periodically. J&J can therefore arrange its inventory and delivery planning at its distribution centre. It is found that both its inventory cost and delivering cost have been greatly reduced. Information sharing has led P&S and J&J to form a stable partnership. Both firms can improve their performance and obtain economic benefits for the long run.

Lee *et al.* (Ref. 5) and Yu *et al.* (Ref. 4) discuss the benefits of information sharing in a two-level supply chain consisting of one manufacturer and one retailer. Both papers propose that the analogous problem of a two-level supply chain with multiple retailers is worthy of further study. This paper aims to study the impact of information sharing on inventory and expected cost in a two-level supply chain with multiple retailers. Moreover, we extend the restriction in Ref. 4 that the coefficient of correlation of orders ρ satisfies $0 \leqslant \rho \leqslant 1$ to $-1 \leqslant \rho \leqslant 1$. The paper is organized as follows. In section 2, we discuss three levels of information sharing. The optimal inventory policy under each of the three levels of information sharing is derived in Section 3. Then the impact of information sharing on inventory and expected cost is analyzed in Section 4. The paper ends with a conclusion in Section 5.

The Modelling Framework

Consider a two-level supply chain that consists of one manufacturer (which can be the distribution center of a distributor, a wholesaler or a warehouse of a manufacturer) and multiple retailers. An order-up-to periodic review procedure is adopted for inventory management and each party reviews its inventory level and replenishes its inventory from the upstream party. Excess demand is backlogged. We also assume that the manufacturer can make an expedited delivery from an outside source to fulfill the retailers' replenishing requirements when stock-outs occur. The additional cost incurred is treated as the penalty cost for shortfalls in inventory. In this section we introduce the three levels of information sharing.

Level 1. At this level, there is neither information sharing nor any ordering coordination between the retailers and the manufacturer. We assume that the system comprises n retailers and the retailer is indexed by i, $i = 1, 2, \cdots, n$. At time period t, $t = 1, 2, \cdots$, the demand d_t^i has been realized for retailer i, $i = 1, 2, \cdots, n$. Then the retailer reviews its inventory level and places an order to replenish its inventory. By l we denote the replenishment lead time from the manufacturer to the retailers. So the retailers will receive their orders at time period $t + 1 + l$. The size of all the orders that the n retailers placed D_t is viewed as the demand to the manufacturer. The manufacturer also reviews its inventory level upon realization of the demand D_t. If there is not enough stock, the manufacturer places an order to replenish its inventory from the outside source. We assume that the shortfall cost is borne solely by the manufacturer. Also by L we denote the lead time from the outside source to the manufacturer. Thus, the manufacturer will receive its order at time period $t + 1 + L$. A diagrammatic representation of such an ordering process is shown in Fig. 1.

Figure 1 about here.

Level 2. There is no change in the way the n retailers place their orders according to their forecasts at this level. But the manufacturer can share the customer ordering information with the n retailers. That is, the manufacturer places its order to the outside source not only according to the size of all the orders from the n retailers but also the customers' demand information. We diagrammatically present this process in Fig. 2.

Figure 2 about here.

Level 3. Electronic Data Interchange (EDI) is used at this level to capture real-time information about the retailing system. The retailers and the manufacturer master the customers' demand information in a synchronized manner. Hence, the manufacturer does not depend on the size of all the orders from the retailers but on the customers' demand information directly to place its own order to the outside source. At the same time, the manufacturer makes inventory replenishment decisions for the retailers proactively. This is called the vendor managed inventory (VMI) strategy. Such an ordering process is diagrammatically shown in Fig. 3.

Figure 3 about here.

Optimal Inventory Policies under Three Different Information Sharing Levels

As considered in Ref. 5, external demand occurring at each retailer is assumed to be a simple autocorrelated AR(1) process. Let d_t^i, $t = 1, 2, \cdots,$ $i = 1, 2, \cdots, n$ be the AR(1) demand process at retailer i at time period t, i.e.,

$$d_t^i = d + \rho d_{t-1}^i + \varepsilon_t^i, \tag{3.1}$$

where $d > 0$ and $-1 < \rho < 1$ are constant, and ε_t^i is independent and identically (*i.i.d.*) normally distributed with mean 0 and variance σ_i^2. Like Ref. 5, it is further assumed that σ_i is significantly smaller than d so that the probability of a negative demand is negligible.

Remark 3.1. (see Ref. 4) The demand model (3.1) is adopted on the assumptions that the retailers face nonstationary demand over time and demand forecasts are updated based on observed demand, which is less restrictive than the assumption of independently and identically distributed (*i.i.d.*) demand. Examples of such demand models can be found in previous research by Lee *et al.* (Ref. 2, 5), Kahn (Ref. 6) and Urban (Ref. 7). The demand process parameters (d, ε and σ) are known. Similar assumptions were also made in the work by Lee *et al.* (Ref. 2, 5). However, in practical situations, these parameters need to be estimated. The assumption with known parameters can technically be considered as the large-sample case, in which the parameters estimation can be made on a one-time basis from an initial set of data. In many practical situations, a large set of observed data at one time is not available, i.e., the small-sample case. To obtain more accurate estimates, the data set should be updated regularly by some pre-defined recording procedures. The estimates of our demand model (3.1), for example, can be updated through time as new demand information is received.

Throughout this paper, parameters expressed in upper case and lower case are used to designate the manufacturer and the retailers, respectively. It is assumed that no fixed ordering cost is incurred, and all other costs such as unit holding cost and unit shortage cost are constant. We use the following notation:

s_t^i=retailer i's order-up-to level at time period t, $t = 1, 2, \cdots,\ i = 1, 2, \cdots, n$;

S_t=the manufacturer's order-up-to level at time period t, $t = 1, 2, \cdots$;

c=unit ordering cost for the retailers;

C=unit ordering cost for the manufacturer;

p=unit shortage cost for the retailers;

h=unit holding cost for the retailers;

P=unit shortage cost for the manufacturer;

H=unit holding cost for the manufacturer.

Since there is no change in the way the n retailers place their orders at the three levels of information sharing, we will investigate the optimal inventory policy of the retailers identically.

Remark 3.2. (see Ref. 4) We see that the retailers' order-up-to levels remain the same and so the retailers will not gain any improvement in their inventory levels and expected costs under the three proposed information sharing scenarios. There are two reasons for this outcome. One is that we assume that the retailers have perfect information about the customers' demand and there is no further customers' demand information that the retailers can obtain when they share the information with the manufacturer. The other is that we assume the retailers' lead time l is fixed, which is subject to the manufacturer's reliability. Without any information sharing between the supply chain members, the retailers' lead time is an estimate of the manufacturer's time for order processing, manufacturing and delivering. Supply chain partnerships allow the manufacturer share lead time information with the retailers, or even make the manufacturer shorten the retailers' lead time. The retailers can also obtain more accurate lead time information with information sharing-based partnerships. Therefore, the manufacturer should take the initiative to establish information sharing-based partnerships and also give the retailers some incentives (such as sharing the logistical costs with the retailers or guaranteeing supply reliability) to induce the retailers' cooperation.

For retailer i at time period t, the customers' demand d_t^i has been realized. This retailer reviews its inventory level and places an order y_t^i to the manufacturer to replenish its inventory; the order will arrive at time period $t+1+l$. We know

$$y_t^i = d_t^i + (s_t^i - s_{t-1}^i). \tag{3.2}$$

What we need to do is to decide the optimal order-up-to level ${s^*}_t^i$ that minimizes the total expected holding and shortage costs at period $t+1+l$.

By $\sum_{j=1}^{l+1} d_{t+j}^i$, we denote the total demands during the lead time for retailer i. From (3.1), we have

$$\sum_{j=1}^{l+1} d_{t+j}^i = d\sum_{j=1}^{l+1}\frac{1-\rho^j}{1-\rho} + \frac{\rho(1-\rho^{l+1})}{1-\rho}d_t^i + \frac{1}{1-\rho}\sum_{j=1}^{l+1}(1-\rho^j)\varepsilon_{t+l+2-j}^i.$$

Let $m_t^i = E(\sum_{j=1}^{l+1} d_{t+j}^i | d_t^i)$ and $v_t^i = Var(\sum_{j=1}^{l+1} d_{t+j}^i | d_t^i)$ be the conditional expectation and conditional variance of $\sum_{j=1}^{l+1} d_{t+j}^i$, respectively. Then

$$m_t^i = d\sum_{j=1}^{l+1}\frac{1-\rho^j}{1-\rho} + \frac{\rho(1-\rho^{l+1})}{1-\rho}d_t^i \tag{3.3}$$

and

$$v_t^i = v\sigma_i^2,$$

where $v = \frac{1}{(1-\rho)^2}(\sum_{j=1}^{l+1}(1-\rho^j)^2)$.

By Ref. 5, the optimal order-up-to level ${s^*}_t^i$ of retailer i at time period t is

$$ {s^*}_t^i = m_t^i + k\sigma_i\sqrt{v}, \tag{3.4}$$

where $k = \Phi^{-1}[p/p+h]$, and Φ^{-1} is the inverse function of the standard normal distribution function Φ.

Hence, we have derived the optimal order-up-to level for each retailer. Next, we will investigate the optimal inventory policy of the manufacturer at each of the three levels of information sharing.

The size of all the retailers' orders at time period t is the demand to the manufacturer. When the retailers place their orders to the manufacturer, the manufacturer reviews its inventory. If there is not enough stock, it will replenish its inventory from the outside source and it will receive its order at time period $t + L + 1$. By D_t we denote the size of all the retailers' orders. Then, from (3.1), (3.2), (3.3) and (3.4), we have

$$\begin{aligned} D_t &= \sum_{i=1}^{n} y_t^i \\ &= \sum_{i=1}^{n} [d_t^i + ({s^*}_t^i - {s^*}_{t-1}^i)] \\ &= \sum_{i=1}^{n} [d_t^i + (m_t^i - m_{t-1}^i)] \\ &= \sum_{i=1}^{n} [d_t^i + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_t^i - d_{t-1}^i)]. \end{aligned} \tag{3.5}$$

For the sake of convenience, we consider D_{t+1}. Again, from (3.1), (3.2), (3.3) and (3.4), we obtain

$$
\begin{aligned}
D_{t+1} =& \sum_{i=1}^{n} y_{t+1}^{i} \\
=& \sum_{i=1}^{n} [d_{t+1}^{i} + ({s^*}_{t+1}^{i} - {s^*}_{t}^{i})] \\
=& \sum_{i=1}^{n} [d_{t+1}^{i} + (m_{t+1}^{i} - m_{t}^{i})] \\
=& \sum_{i=1}^{n} [d_{t+1}^{i} + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_{t+1}^{i} - d_{t}^{i})] \\
=& \sum_{i=1}^{n} [d + \rho d_{t}^{i} + \varepsilon_{t+1}^{i} + \frac{\rho(1-\rho^{l+1})}{1-\rho}(\rho(d_{t}^{i} - d_{t-1}^{i}) + (\varepsilon_{t+1}^{i} - \varepsilon_{t}^{i}))] \\
=& nd + \rho \sum_{i=1}^{n} [d_{t}^{i} + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_{t}^{i} - d_{t-1}^{i})] \\
& + \sum_{i=1}^{n} [\frac{\rho(1-\rho^{l+1}) + (1-\rho)}{1-\rho}\varepsilon_{t+1}^{i} - \frac{\rho(1-\rho^{l+1})}{1-\rho}\varepsilon_{t}^{i}] \\
=& nd + \rho D_t + \sum_{i=1}^{n} [\frac{1-\rho^{l+2}}{1-\rho}\varepsilon_{t+1}^{i} - \frac{\rho(1-\rho^{l+1})}{1-\rho}\varepsilon_{t}^{i}]. \qquad (3.6)
\end{aligned}
$$

Applying the above formula repeatedly, we obtain

$$
\begin{aligned}
D_{t+j} =& \frac{1-\rho^{j}}{1-\rho} nd + \rho^{j} D_t + \frac{1-\rho^{l+2}}{1-\rho}(\sum_{i=1}^{n} \varepsilon_{t+j}^{i}) + \sum_{r=1}^{j-1} \rho^{l+1+r}(\sum_{i=1}^{n} \varepsilon_{t+j-r}^{i}) \\
& - \frac{\rho^{j}(1-\rho^{l+1})}{1-\rho}(\sum_{i=1}^{n} \varepsilon_{t}^{i}), \qquad j = 1, 2, \cdots.
\end{aligned}
$$

Hence, the total shipment quantity over the lead time L from the manufacturer to all the retailers is

$$
\begin{aligned}
\sum_{j=1}^{L+1} D_{t+j} =& [L + 1 - \frac{\rho(1-\rho^{L+1})}{1-\rho}]\frac{nd}{1-\rho} + \frac{\rho(1-\rho^{L+1})}{1-\rho} D_t + \frac{1-\rho^{l+2}}{1-\rho}(\sum_{i=1}^{n} \varepsilon_{t+L+1}^{i}) \\
& + \frac{1}{1-\rho} \sum_{j=1}^{L} (1 - \rho^{L+l+3-j})(\sum_{i=1}^{n} \varepsilon_{t+j}^{i}) - \frac{\rho(1-\rho^{L+1})(1-\rho^{l+1})}{(1-\rho)^2}(\sum_{i=1}^{n} \varepsilon_{t}^{i}).
\end{aligned}
$$

Level 1. In this case, the manufacturer determines its optimal order-up-to level ${S^*}_t|_1$ that minimizes the total expected holding and shortage costs over the lead time L. Since it knows nothing except the shipment quantity D_t, D_t is regarded as a known variable and ε_{t+j}^{i} $(i = 1, 2, \cdots, n,\ j = 0, 1, \cdots, L+1)$ is considered as a stochastic variable. Then the manufacturer deals with $\sum_{j=1}^{L+1} D_{t+j}$ as a normal distribution with mean $M_t|_1$ and variance $V_t|_1 \cdot (\sum_{i=1}^{n} \sigma_i^2)$, where

$$M_t|_1 = \frac{nd}{1-\rho}[L+1-\frac{\rho(1-\rho^{L+1})}{1-\rho}] + \frac{\rho(1-\rho^{L+1})}{1-\rho} D_t$$

and

$$\begin{aligned} V_t|_1 =& \frac{1}{(1-\rho)^2}[(1-\rho^{l+2})^2 + \sum_{j=1}^{L}(1-\rho^{L+l+3-j})^2 \\ &+ \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}]. \end{aligned}$$

Obviously, $V_t|_1$ is independent of t. So, we denote it by $V|_1$. Also by Ref. 5, the optimal order-up-to level $S^*{}_t|_1$ of the manufacturer at level 1 of information sharing is

$$S^*{}_t|_1 = M_t|_1 + K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_1}, \qquad t = 1, 2, \cdots,$$

where $K = \Phi^{-1}[P/(P+H)]$.

Level 2. In this situation, the manufacturer knows not only the total size of the retailers' orders, but also the customers' demands. That is, the manufacturer masters D_t and $\varepsilon_t^i,\ i = 1, 2, \cdots, n$. Thus, D_t and ε_t^i, $(i = 1, 2, \cdots, n)$ are known variables and ε_{t+j}^i $(i = 1, 2, \cdots, n,\ j = 1, \cdots, L+1)$ is stochastic. Then, the manufacturer treats $\sum_{j=1}^{L+1} D_{t+j}$ as another normal distribution, whose mean $M_t|_2$ and variance $V_t|_2 \cdot (\sum_{i=1}^{n} \sigma_i^2)$ are respectively

$$\begin{aligned} M_t|_2 =& \frac{nd}{1-\rho}[L+1-\frac{\rho(1-\rho^{L+1})}{1-\rho}] + \frac{\rho(1-\rho^{L+1})}{1-\rho} D_t \\ &- \frac{\rho(1-\rho^{L+1})(1-\rho^{l+1})}{(1-\rho)^2}(\sum_{i=1}^{n} \varepsilon_t^i) \end{aligned}$$

and

$$V_t|_2 = \frac{1}{(1-\rho)^2}[(1-\rho^{l+2})^2 + \sum_{j=1}^{L}(1-\rho^{L+l+3-j})^2].$$

We denote $V_t|_2$, which is independent of t, by $V|_2$. Hence, the optimal order-up-to level $S^*{}_t|_2$ of the manufacturer at level 2 of information sharing is

$$S^*{}_t|_2 = M_t|_2 + K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_2}, \qquad t = 1, 2, \cdots,$$

where K is defined as above.

Level 3. With EDI, the manufacturer can obtain the customers' demand information directly. The demand the manufacturer faces is the total shipment quantity the retailers replenish. The manufacturer needs to deliver D_t units of the item to replenish the retailers' inventory at period t. However, the size D_t should satisfy the demand d_t^i from all the customers, not the orders from the retailers. So, we need to deduce the relation between $\sum_{j=1}^{L+1} D_{t+j}$ and d_t^i, not the relation between $\sum_{j=1}^{L+1} D_{t+j}$ and D_t as at level 1 and level 2. Thus, by (3.5), the total units demanded over the lead time L is

$$\begin{aligned}\sum_{j=1}^{L+1} D_{t+j} =& \sum_{j=1}^{L+1}[\sum_{i=1}^{n}[d_{t+j}^i + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_{t+j}^i - d_{t+j-1}^i)]] \\ =& \sum_{i=1}^{n}[\sum_{j=1}^{L+1}[d_{t+j}^i + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_{t+j}^i - d_{t+j-1}^i)]] \\ =& \sum_{i=1}^{n}[\sum_{j=1}^{L+1} d_{t+j}^i + \frac{\rho(1-\rho^{l+1})}{1-\rho}(d_{t+L+1}^i - d_t^i)] \\ =& nd[\sum_{j=1}^{L+1}\frac{1-\rho^j}{1-\rho} + \frac{\rho(1-\rho^{l+1})(1-\rho^{L+1})}{(1-\rho)^2}] \\ &+ \sum_{i=1}^{n}[\frac{\rho^{l+2}(1-\rho^{L+1})}{1-\rho}d_t^i + \frac{1}{1-\rho}\sum_{j=1}^{L+1}(1-\rho^{l+1+j})\varepsilon_{t+L+2-j}^i].\end{aligned}$$

At this level, d_t^i $(i = 1, 2, \cdots, n)$ is a known variable and ε_{t+j}^i $(i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, L+1)$ is stochastic. Then the mean $M_t|_3$ and variance $V_t|_3 \cdot (\sum_{i=1}^{n} \sigma_i^2)$ of the normal distribution $\sum_{j=1}^{L+1} D_{t+j}$ are respectively

$$\begin{aligned}M_t|_3 =& nd[\sum_{j=1}^{L+1}\frac{1-\rho^j}{1-\rho} + \frac{\rho(1-\rho^{l+1})(1-\rho^{L+1})}{(1-\rho)^2}] \\ &+ \sum_{i=1}^{n}\frac{\rho^{l+2}(1-\rho^{L+1})}{1-\rho}d_t^i\end{aligned}$$

and

$$V_t|_3 = \frac{1}{(1-\rho)^2}(\sum_{j=1}^{L+1}(1-\rho^{l+1+j})^2).$$

By $V|_3$, we denote $V_t|_3$, which is independent of t. Then the optimal order-up-to level

${S^*}_t|_3$ is

$$ {S^*}_t|_3 = M_t|_3 + K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_3}, \qquad t = 1, 2, \cdots, $$

where K is defined as above.

The Impact of Information Sharing

We see in Section 3 that information sharing has no impact on the retailers (see Remark 3.2). Hence, we only consider the effect of the three different levels of information sharing on the manufacturer in this section. We will show that information sharing will result in reductions in both inventory and expected cost directly for the manufacturer.

Inventory Reduction

As discussed in Ref. 8, for any order-up-to system with S_t being the order-up-to level, D_t the "demand" at period t, and $\sum_{j=1}^{L+1} D_{t+j}$ the total "demand" from period $t+1$ to period $t+L+1$, the average (on-hand) inventory level can be approximated by

$$ \begin{aligned} I_t =& S_t - E(\sum_{j=1}^{L+1} D_{t+j}) + \frac{E(D_t)}{2} \\ =& S_t - M_t + \frac{E(D_t)}{2}, \end{aligned} $$

where M_t is the mean of $\sum_{j=1}^{L+1} D_{t+j}$.

From Section 3, we see that $S_t - M_t$ is always $K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V_t}$, where $V_t \cdot (\sum_{i=1}^{n} \sigma_i^2)$ is the variance of $\sum_{j=1}^{L+1} D_{t+j}$ at all three information sharing levels. Since V_t is independent of t, we denote it by V. Combining (3.6) and the fact that $E(\varepsilon_t^i) = 0,\ t = 1, 2, \cdots,\ i = 1, 2, \cdots, n$, we obtain $\lim_{t \to +\infty} E(D_t) = \frac{nd}{1-\rho}$. Thus, we derive the approximated average (on-hand) inventory level as

$$ I = K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V} + \frac{nd}{2(1-\rho)}. $$

By $I_1,\ I_2,\ I_3$, we denote the approximated average (on-hand) inventory level at the three levels of information sharing, respectively. Then

$$ I_1 = K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_1} + \frac{nd}{2(1-\rho)}, $$

$$I_2 = K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_2} + \frac{nd}{2(1-\rho)},$$

$$I_3 = K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V|_3} + \frac{nd}{2(1-\rho)}.$$

Proposition 1. For any $-1 \leqslant \rho \leqslant 1$, $V|_1 \geqslant V|_2 = V|_3$.

The proof is given in the appendix.

From Proposition 1, we can see that $I|_1 \geqslant I|_2 = I|_3$. It shows that the inventory level of the manufacturer decreases with an increase in the level of information sharing.

In addition, from the proof of Proposition 1, we can see that

$$\begin{aligned} I_1 - I_2 =& K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} (\sqrt{V|_1} - \sqrt{V|_2}) \\ =& K \frac{1}{(1-\rho)} [\sqrt{(1-\rho^{l+2})^2 + \sum_{j=1}^{L} (1-\rho^{L+l+3-j})^2 + \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}} \\ & - \sqrt{(1-\rho^{l+2})^2 + \sum_{j=1}^{L} (1-\rho^{L+l+3-j})^2} \,][\sqrt{\sum_{i=1}^{n} \sigma_i^2} \,]. \end{aligned}$$

It is easy to see that the first term of the above equation is independent of n. So $I_1 - I_2$ is increasing in n. In other words, $I_2 - I_1$ is decreasing in n. That is, the larger the number of the retailers is, the greater is the inventory reduction for the manufacturer due to information sharing.

Expected Cost Reduction

Let $L(x)$ be the right loss function for the standard normal distribution, where

$$L(x) = \int_x^{\infty} (z - x) d\Phi(z),$$

and $\Phi(z)$ is the standard normal probability distribution.

Assume that $S_t = M_t + K \cdot \sqrt{\sum_{i=1}^{n} \sigma_i^2} \sqrt{V}$ is an order-up-to level of the manufacturer, where $V \cdot (\sum_{i=1}^{n} \sigma_i^2)$ is the variance of $\sum_{j=1}^{L+1} D_{t+j}$, and F_t is a normal distribution function with mean M_t and variance $(\sum_{i=1}^{n} \sigma_i^2)V$. From Ref. 5, we know that the manufacturer's expected holding and shortage costs at period $t+L+1$ is

$$
\begin{aligned}
C_t =& E(P\int_{S_t}^{\infty}(x-S_t)dF_t(x)+H\int_{-\infty}^{S_t}(S_t-x)dF_t(x)) \\
=& \sqrt{\sum_{i=1}^{n}\sigma_i^2\sqrt{V}}[(H+P)L(K)+HK].
\end{aligned}
$$

Obviously, C_t is independent of t. We denote it by C. By C_1, C_2, C_3, we denote the manufacturer's expected holding and shortage costs at the three different levels of information sharing, respectively. Then

$$C_1 = \sqrt{V|_1}[(H+P)L(K)+HK]\sqrt{\sum_{i=1}^{n}\sigma_i^2},$$

$$C_2 = \sqrt{V|_2}[(H+P)L(K)+HK]\sqrt{\sum_{i=1}^{n}\sigma_i^2},$$

$$C_3 = \sqrt{V|_3}[(H+P)L(K)+HK]\sqrt{\sum_{i=1}^{n}\sigma_i^2}.$$

From Proposition 1, we can see that $C|_1 \geqslant C|_2 = C|_3$. Hence, the manufacturer can also achieve a reduction in expected cost with an increase in the level of information sharing.

Analogous to Section 4.1, we know that

$$
\begin{aligned}
C_1 - C_2 =& (\sqrt{V|_1}-\sqrt{V|_2})[(H+P)L(K)+HK]\sqrt{\sum_{i=1}^{n}\sigma_i^2} \\
=& \frac{1}{(1-\rho)}[\sqrt{(1-\rho^{l+2})^2+\sum_{j=1}^{L}(1-\rho^{L+l+3-j})^2+\frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}} \\
& -\sqrt{(1-\rho^{l+2})^2+\sum_{j=1}^{L}(1-\rho^{L+l+3-j})^2}\ ][(H+P)L(K)+HK][\sqrt{\sum_{i=1}^{n}\sigma_i^2}\ ].
\end{aligned}
$$

Since the first term of the above equation is independent of n, $C_1 - C_2$ is increasing in n. That is, the larger the number of the retailers is, the more the manufacturer's expected cost will reduce due to information sharing.

Discussion

From Section 4, we see that there is no difference in inventory level and expected cost of the manufacturer between information sharing at level 2 and level 3. Mathematically, we know that both the inventory level I and expected cost C depend only on the variance $V(\sum_{i=1}^{n} \sigma_i^2)$ of $\sum_{j=1}^{L+1} D_{t+j}$. In fact, it is easy to see from the proof in the appendix that V at level 2 is equal to that at level 3. Hence, the inventory level and expected cost at level 2 and level 3 are equal. Thus, the management implication is that businesses do not need to consider the practice of VMI if there are only two echelons in a supply chain. However, this is not the case if there are more than two echelons in a supply chain. In Ref. 9, we consider the impact of information sharing in a three-echelon supply chain and show that both the inventory level and expected cost of the manufacturer at level 3 are less than those at level 2.

Conclusions

In this paper we studied the problem proposed in Lee *et al.* (Ref. 5) and Yu *et al.* (Ref. 4). That is, we considered the impact of information sharing on inventory and expected cost in a two-level supply chain with multiple retailers. First, we introduced the three different levels of information sharing. Then the optimal inventory policy under each of them was derived. Finally, we showed that both the inventory level and expected cost of the manufacturer decrease with an increase in the level of information sharing.

This paper is a follow-up study to previous work with the purpose of generalizing existing results through theoretical analysis of a model based on some strong assumptions. As the theory becomes mature, the focus of research should move on to the relevance of the findings to real problems faced by organizations. It is therefore suggested that further studies consider simulation of the proposed model using real data, gradually relaxing the assumptions, to assess the robustness of the model.

Appendix

Proof of Proposition 1. From Section 3, we know

$$V|_1 - V|_2 = \frac{1}{(1-\rho)^2} \cdot \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}.$$

Obviously, $V|_1 - V|_2 \geqslant 0$. So, $V|_1 \geqslant V|_2$.

By the equation of $V|_2$, we get

$$\begin{aligned} V|_2 =& \frac{1}{(1-\rho)^2}[(1-\rho^{l+2})^2 + \sum_{j=1}^{L}(1-\rho^{L+l+3-j})^2] \\ =& \frac{1}{(1-\rho)^2}[\sum_{j=1}^{L+1}(1-\rho^{L+l+3-j})^2]. \end{aligned}$$

Let $\bar{j} = L + 2 - j$. Since $j = 1, 2, \cdots, L + 1$, $\bar{j} = L + 1, L, \cdots, 1$. The above equation becomes

$$V|_2 = \frac{1}{(1-\rho)^2}[\sum_{\bar{j}=1}^{L+1}(1 - \rho^{l+1+\bar{j}})^2].$$

That is, it is exactly $V|_3$. Therefore, $V|_1 \geqslant V|_2 = V|_3$.

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References

1. Lee HL, et al. (1997). Information distortion in a supply chain: the bullwhip effect. *Management Science* 43: 546-558.

2. Lee HL, et al. (1997). Bullwhip effect in a supply chain. *Sloan Management Review* 38: 93-102.

3. Gill P and Abend J (1997). Wal-Mart: The supply chain heavyweight champ. *Supply Chain Management Review* 1: 8-16.

4. Yu Z, et al. (2002). Modelling the benefits of information sharing-based partnerships in a two-level supply chain. *Journal of the Operational Research Society* 53: 436-446.

5. Lee HL, et al. (2000). The value of information sharing in a two-level supply chain. *Management Science* 46: 626-643.

6. Kahn JA (1987). Inventories and the volatility of production. *America Economics Review* 77: 667-679.

7. Urban TL (2000). Reorder level determination with serially-correlated demand. *Journal of the Operational Research Society* 51: 762-768.

8. Silver E and Petersen R (1995). *Decision Systems for Inventory Management and Production Planning*, 2nd edition. John Wiley and Sons, New York.

9. Cheng TCE and Wu YN (2003). The impact of information sharing in a multiple-echelon supply chain. Working Paper, Faculty of Business, The Hong Kong Polytechnic University.

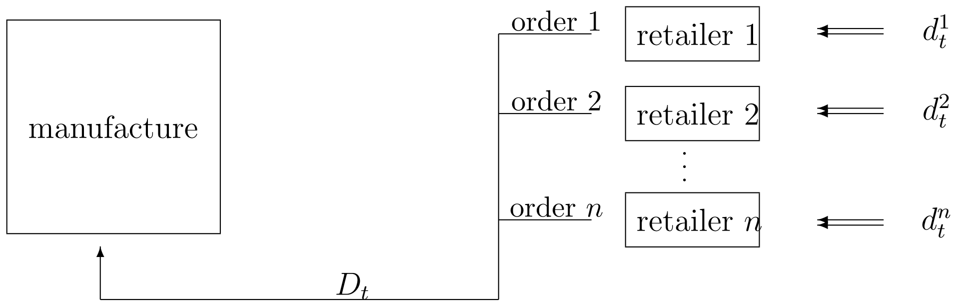

Fig. 1. An ordering process based on information sharing level 1

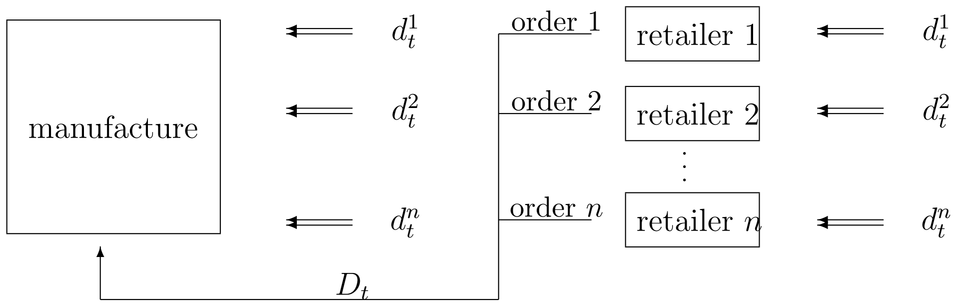

Fig. 2. An ordering process based on information sharing level 2

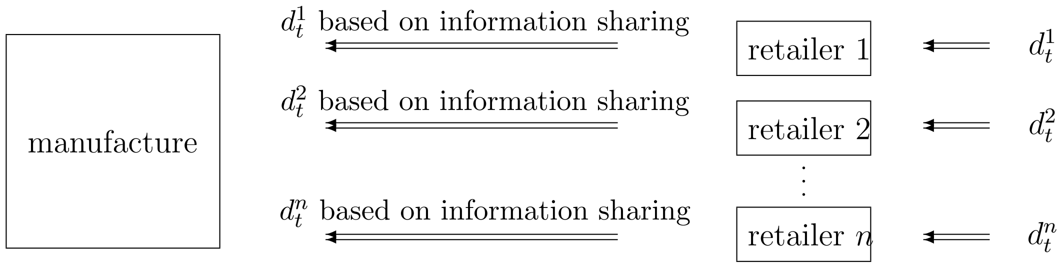

Fig. 3. An ordering process based on information sharing level 3