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Mathematical model of water quality rehabilitation with rainwater utilization —— a case study at Haigang

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Abstract

This paper presents a mathematical model of water quality rehabilitation by utilizing rainwater, which is separated into initial polluted rainwater and later unpolluted rainwater. The planned Haigang city, belonging to Shanghai metropolis in China, is taken as a case example. An analysis is made on how to utilize rainwater and appraise its function in a water quality rehabilitation scheme. It is demonstrated that this method can significantly reduce both project investment and operation costs. Moreover, appropriate drainage plan scheme is presented to address the initial polluted rainwater problem. Thus, the conflict between drainage and rainwater utilization can be resolved appropriately.

Keywords: Mathematical model; Water quality index; Water quality rehabilitation; Rainwater utilization; Drainage planning; Wastewater treatment plant

Biographical notes

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1. Introduction

Lingang is a planned new city, which is located on the east of Nanhui district of Shanghai metropolis at latitude 30° 53′ ~31° 09′ N and longitude 121° 35′ ~121° 51′ E. The city belongs to subtropical region with average annual air temperature of 15.6℃ and average annual rainfall amount as high as 1073.8mm. The present study area, Haigang city, is part of Lingang, with East Sea and Hangzhou Bay at the east and south, respectively. The entire study area, having an area of 72 km^2, is nearly even in level with an elevation between 4.0 and 4.2m. The middle part is a circular artificial lake with an area of 4.24km^2, diameter 2.5km and average water depth 2.7m. The planned Haigang city will accommodate about 300,000 people while future water networks and project arrangement is shown in Figure 1.

Water flowing out of the total planned area is about 12.6%, and the planned water networks comprise: “outer circle”, “core lake”, “three inner circles”, and “seven harbors”. “Outer circle” is composed of north Hucheng River, Suitang River, and Luchaoyin River. “Core lake” is Dishui Lake. “Three inner circles” include circles 1, 2, and 3. “Seven harbors” refer to harbors A to G. At present, the completed water networks include Dishui Lake, circle 1, and harbors A and D. Since water in Dishui Lake originates from upper Huangpu River, the present water quality index (WQI) is Ⅵ to Ⅴ which cannot meet the recreational requirement for WQI with Ⅱ to Ⅲ. Moreover, the water networks are all over Haigang city and should be considered in the process of drainage planning. Therefore, the main objectives on water quality rehabilitation and drainage planning of Haigang city include: i) maintaining water levels of Dishui Lake between 2.5m and 2.7m and rehabilitating WQI to Ⅱ~Ⅲ; ii) controlling the maximum water level to below 3.3m when confronted with maximum 24-hour rainfall at 5% frequency.

This paper presents a mathematical model of the water quality rehabilitation and drainage planning scheme for the water system of Haigang city, with a view to reducing investment and operation costs. In this scheme, rainwater utilization is fully considered so as to create an optimal drainage scheme.

2. Scheme Architecture

In order to meet the demand to rehabilitate the present water quality and maintain future WQI at the level of Ⅱ～Ⅲ, the scheme include following studies: i) study of water quality melioration and maintenance measures, ii) study of rainwater utilization, and iii) study of drainage system.

According to the comprehensive planning scheme in Haigang city, Dishui Lake is considered as the key recreation area. Therefore initial polluted rainwater is prohibited from flowing into Dishui Lake whilst only pure rainwater at a later stage is allowed. Nevertheless, initial rainwater is allowed to flow into river networks. Figure 2 shows the scheme architecture on water quality rehabilitation and drainage system planning.

1) Water quality rehabilitation system

i) Control system on outer pollution sources

Interrupting outer pollution sources is a precondition of water quality rehabilitation. Therefore, waterlocks A, B, C, and D are designed at corresponding intersections of the outer circle and harbors A, B, C, and D, respectively. Their role is to cut the hydraulic link between inner river networks and the outer circle.

ii) Water quality purification system

It comprises wastewater treatment plant, rainwater separation system, waterlock system in river networks, and water quality monitor system. Amongst them, the wastewater treatment plant plays a key role, whose operation method can be broadly divided into inner or outer circles on the basis of the inflow water sources. In general, when the water level in Dishui Lake is below 2.5m, the required water has to be conveyed by the outer circle and purified via a wastewater treatment plant before surcharging into Dishui Lake. On the other hand, although the water level can be kept above 2.5m, the wastewater treatment plant has to be operated by the inner circle when water quality in Dishui Lake is obviously deteriorated. The route of the inner circle is from the harbors A and B or harbors A and C, through wastewater treatment plant to harbors D and E via outflow pipes, and finally to Dishui Lake.

The waterlock system in the river networks comprises eleven waterlocks whose names are generally derived from the combination of the names of the harbor and circle. For example, the term of C_2 denotes that waterlock C_2 locates at harbor C and circle 2. Furthermore, the subscripts of all waterlocks situated in circle 1 are all set as 0, i.e., A_0, B_0, C_0, D_0, E_0, F_0, and G_0. This is purposely made in order to control the hydraulic exchange between Dishui Lake and river networks since their water quality indices are often quite different. In general, waterlocks A_0, B_0,…, G_0 are kept opened during rainfall period in order to expedite drainage and facilitate pure rainwater replenishment to Dishui Lake. The role of waterlocks A_1, C_1, C_2, and D_3 is to cut connections of circles 1, 2, and 3. Together with waterlocks A_0 and B_0 or A_0 and C_0, they form a closed area for the outflow of Dishui Lake when the wastewater treatment plant is operated via the inner circle. This not only guarantees that the lake water is constantly conveyed to wastewater treatment plant, but also reduces the wastewater amount to be treated by the plant, resulting from disconnection of hydraulic exchanges among river networks.

In the planning scheme of Haigang city, stormwater and sewage pipes are designed in separate systems in order to fully utilize the precipitation. The outlet of stormwater pipes is connected directly to river networks. The process of rainwater utilization is described as follows. Prior to any rainfall, waterlocks A_0~G_0 are closed. After initial rain flows into river networks, the water quality monitor system is activated to monitor the WQI. If the WQI is higher than the stipulated water quality standard, water in rivers has to be pumped or discharged automatically into East Sea. Once the WQI fulfills the stipulated water quality standard, waterlocks A_0~G_0 are opened, which integrates river networks and Dishui Lake. Thus, both objectives on rainwater utilization and drainage conveyance are accomplished.

iii) Ecosystem

The main objective of ecologic measures is to cultivate appropriate ecosystems in Dishui Lake and river networks, attain water quality maintenance by ecologic cycle and further boost the water quality rehabilitation by itself. According to different functional requirements of Dishui Lake and river networks, aquatic creatures such as fish, shrimp, crab, and plankton, etc are cultivated in Dishui Lake whereas hydrophyte and aquatic creature, including algae, reed, and fauna, etc., are cultivated in river networks.

2) Planning of drainage system

i) Pump station planning

In the present study, drainage planning is quite peculiar due to the rainwater problem. The regulation reservoir volume for drainage is obviously reduced since initial rainwater is only allowed to flow into river networks. The water level in river networks may rise even above 3.3m before waterlocks A_0~G_0 are opened when the designed rainstorm occurs. Pump planning is thus involved to determine: whether or not a pump is required for drainage, and if required, what pump size is required and where is its location. The appropriate solutions have to be determined by drainage computation.

ii) Floodgate planning

With the interaction of inflow of later rainwater and outflow of initial rainwater by pumping, WQI in river networks will meet the demand of stipulated standard of water quality at some specific times. When this occurs, the pump will be stopped in order to reduce operation cost. Simultaneously, waterlocks A_0~G_0 are completely opened so as to enlarge regulation reservoir volume via full water exchange.

Although the reservoir volume increases obviously, water levels also could exceed 3.3m with the large later rainfall inflow. In view of that possibility, floodgates for discharge have to be considered. In floodgate planning, the main tasks are to determine the size and location of the planned floodgate by drainage computation, on the basis of the designed rainstorm and sea water level.

iii) Waterlock system in river networks

During the drainage period, the purpose of the eleven waterlocks in river networks is to control the initial runoff from flowing into Dishui Lake. When the WQI meets the stipulated water quality standard, they will expedite drainage flow at an opened status.

3. Computational models

3.1 Water quality rehabilitation model

Dishui Lake is a small and shallow urban lake whose water quality is relatively homogenous. An assumption of completely mixed condition is justified and the lake can be considered to be a reaction utensil (James, 2001). Donald and John (1970) employed a typical analytical water quality model to study the water quality of five large lakes in North America. However, in the model, the time of discharges of inflow and outflow are identical and synchronous. Zhu and Yu (2003) presented a new analytical model with major improvement on the sequential time arrangement between inflow and outflow. In this study, Zhu and Yu's model is adopted, and a brief description is presented as follows.

For a lake with reservoir volume V and average concentration of some pollutants C_0, and in every time span T from the beginning of drainage to the end of inflow of Dishui Lake, the outflow and inflow are identical with outflow first and then inflow. The average concentration of the pollutant in inflow is $\alpha_1 C_0$. The amount of a pollutant from the surroundings in one day is $\alpha_c C_0 V$. The total amount of the pollutant from the surroundings in every span T is then $\alpha C_0 V$ ($\alpha = \alpha_c T$). The volume of outflow or inflow is $(1-\beta)V$, where $1-\beta$ is called exchange coefficient. The pollutant is assumed to follow the 0 or 1 order degradation and the comprehensive degradation coefficient is K.

Based on the above conditions, the average concentration of the pollutant in a period T is expressed by the following equation:

$$\frac{dCV}{dt} = -KCV + \alpha_c C_0 V \qquad (1)$$

For simplification purpose, the degradation of the pollutant is negligible, i.e., $K=0$. The average concentration of the pollutant in nth T is shown by the following equation:

$$\overline{C_n} = \left| \beta^n + \alpha_1 (1-\beta^n) + \alpha \frac{1+\beta-2\beta^n}{2(1-\beta)} \right| C_0 \qquad (2)$$

In the present study, in view of the absence of ambient pollutants, equation (2) is further simplified as follows:

$$\overline{C_n} = \left| \beta^n + \alpha_1 (1-\beta^n) \right| C_0 \qquad (3)$$

3.2 Rainfall-runoff model

Because Haigang city is in the phase of construction, the concrete topographical condition is quite uncertain, but its river networks will be all over the entire planned area. As a simplification, the conflux process in the rainfall-runoff model will not be considered. So the process of designed net rain is viewed as the process of designed runoff. In determining the net rain, the infiltration computation is based on Green-Ampter's formula in which the total infiltration is divided into initial-period infiltration and later-period steady infiltration. In this study, the planned area of $72 km^2$ is divided into three zones, namely, water zone, construction zone, and non-construction zone, with areas $9.1\ km^2$, $47.6\ km^2$, and $15.3\ km^2$, respectively.

3.3 Drainage model

The drainage process is essentially a form of flood propagation. The relationship between the water level and the reservoir volume is shown in Table 1. The drainage computation is based on the water balance principle as follows:

$$\frac{1}{2}(Q_1+Q_2) - \frac{1}{2}(q_1-q_2) = \frac{V_1-V_2}{\Delta t} \qquad (4)$$

where Q_1 and Q_2 are inflows at the beginning and end of time step Δt, respectively (in m^3/s); q_1 and q_2 are discharges at the beginning and end of Δt, respectively (m^3/s); V_1 and V_2 are volumes at the beginning and end of Δt, respectively (m^3); and Δt is set as one hour due to limited measured data.

After V_2 is calculated via equation (4), the corresponding water level can be determined from Table 1. According to the drainage planning, the water level in river networks and Dishui Lake will drop to 2.2m prior to the advent of the designed rainstorm. The drainage computation includes two processes in the present case: discharging the initial rainwater in river networks by pumps or floodgates and discharging the later rainwater on the entire water system by floodgates. Therefore, the drainage model comprises:

i) Drainage computation for river networks

The control condition in drainage computation for river networks is that its maximum water level should not be higher than 3.3m prior to the opening of waterlocks A_0~G_0. In general, the initial precipitation entering into river networks will give rise to deterioration of water quality and upsurge of the water level. When the water level is higher than 2.2m, water in excess will be discharged by the floodgate and pump station at the seaport of harbor A. The water quality will be gradually rehabilitated when the later rainwater successively enters and river water in excess is continuously discharged. Once the water quality meets the stipulated standard, the pump will be stopped and at the same time all waterlocks will be opened. The main purpose of the drainage computation of river networks is to promptly determine the sizes of pumps and floodgates.

ii) Drainage computation for the entire water system

When the water system is fully interconnected, the reservoir volume accommodating precipitation increases and the water level also rises gradually. Under this circumstance, the floodgate is tailored for drainage, whose opening condition is such that inner water level is both higher than 2.2m and the sea water level. The discharge of the floodgate is computed by the following equation:

$$Q = \sigma_s \sigma_c m B \sqrt{2g} H_0^{3/2} \qquad (5)$$

where, Q is discharge of floodgate (m^3/s); σ_s is merging coefficient; σ_c is lateral constriction coefficient; m is discharge coefficient; B is net width of floodgate's section (m); g is gravitational acceleration (m^2/s); and H_0 is water head before the floodgate (m).

iii) Computational algorithm

The drainage computation in the entire water system is performed according to the size of the initially proposed floodgate in the drainage computation of river networks. If the water level of Dishui Lake is higher than 3.3m after the computation, the new floodgate size is adopted in the next drainage computation until the water level of Dishui Lake is below 3.3m. Then, the new size of the floodgate is adopted to further determine the size of the pump in the drainage computation of river networks. The above two steps are iterated until appropriate sizes of pump and floodgate are both determined.

4. Results and analysis

4.1 Determination of the capacity of wastewater treatment plant

i) Pure precipitation computation

According to the precipitation data from 1965 to 2003, a frequency analysis adopting the Person-Ⅲ theoretic curve is performed, with the result shown in Fig. 3. In the study case, the rainfall frequency 90% is used since the water demand for the urban environment is very high. According to the result in Fig.3, when the rainfall frequency is 90%, the corresponding rainfall amount is about 800mm, which is about the same as the value of rainfall amount in 1984. Thus, 1984 is selected as a typical hydrology year. In order to compute the designed net rainwater in 1984, various parameters in rainfall-runoff model are determined first. The initial infiltration amount in non-water zone for every storm is 20mm whilst steady infiltrations are 4mm and 0.0mm per day for construction and non-construction zones, respectively. The evaporation amount is determined by measurement.

Effective rainfall for Dishui Lake is essentially the later pure rainwater because the initial polluted rainwater has to be diluted and discharged by pump and floodgate A. The initial precipitation amount varies for every storm since rainfall is random. Therefore, for simplification, 10mm runoff is not utilized during every storm. Obviously, no rainfall can be utilized if the rainfall amount is less than 10mm.

The operation rules of complementing pure rainwater in Dishui Lake are developed first. The complement computation begins on March 1st when the initial water level of Dishui Lake is 2.5m. The lake water level will rise when the net inflow is larger than zero. However, when the lake water level rises up to 2.7m, rainwater in excess has to be discharged by floodgate A in order to keep the water level between 2.5m and 2.7m.

Based on the aforementioned operation rule, rainwater complement computation in 1984 is carried out on a daily basis. The results show that water levels in Dishui Lake will be below 2.5m for only 13 days out of 365 days. Moreover, the total net precipitation amount is $1193.51\times10^4m^3$ which can guarantee that water levels of Dishui Lake are perennially kept within 2.5~2.7m. On the other hand, the water deficit of $21.34\times10^4m^3$ is only 1% of total volume $2000\times10^4m^3$ in Dishui Lake. Therefore, it is concluded that 90% frequency rainfall amount can meet normal water demand in maintaining 2.5~2.7m water level. It is also suggested that the inner circle is adopted for wastewater treatment plant operation. Fig.4 describes the daily water level process of Dishui Lake with rainwater complement.

ii) Determination of the size of wastewater treatment plant

The result of pure precipitation computation in 1984 demonstrates that rainwater resources are very abundant. So precipitation should be fully considered in order to optimize the size of wastewater treatment plant.

The primary objective of the water quality rehabilitation is to meliorate WQI from Ⅴ to Ⅵ，and to rehabilitate WQI from Ⅵ to Ⅲ. During the construction period, rainwater utilization is not considered since stormwater pipe networks are not completed. However, after the completion rainwater pipe networks, both rainwater and outflow from wastewater treatment plant are utilized to rehabilitate the water quality of Dishui Lake. In the present study, the time span T in the water quality rehabilitation model is 10 days, which comprises the first 5-day drainage and the second 5-day inflow for Dishui Lake.

In this paper, three wastewater treatment plants with different sizes (100,000m^3, 150,000 m^3, and 200,000 m^3 per day) are computed in the analytical water quality model. Results in Table 2 show that rehabilitating WQI from Ⅵ to Ⅲ with the size of 150,000 m^3 per day requires 133 days with rainwater utilization, whereas the size of 182,000 m^3 per day is required in order to achieve the same goal in the same days without rainwater utilization. It is demonstrated that utilizing rainwater can significantly reduce size and operation cost of the plant. In this case study, after having compared sizes, construction investment, operation cost, and water exchange efficiency of different plants, the capacity of 150,000 m^3 per day is adopted.

4.2 Determination of sizes of pump and floodgate

In order to determine the size of the pump, the amount of water exchange used to dilute initial rainwater has to be determined. In fact, it is the amount of water discharged from river networks before water level goes up to 3.3m. Since this city is of a planed new city, no initial rainwater water quality data in Haigang city are available. Based on study results on urban rainwater from different regions including America (EPA, 1983), France (Gromarie-Mertz et al., 1999), Canada (Davek et al., 1990), and Beijing of China (Wu et al., 2003), the initial rainwater WQI in Haigang city is determined and shown in Table 3. The computed WQI after mixing based on an analytical water quality model is also shown in Table 3.

In order to meet the stipulated standard or maintain WQI of river networks prior to rainfall, the polluted water has to be successively pumped by pump station or released by floodgate Based on the analytical water quality model, the required discharge amount is 1,150,000m^3 before WQI meets the stipulated standard. Hence, sizes of pump and floodgate A must comply with the condition that the maximum water level in river networks should be lower than 3.3m before 1,150,000m^3 is released.

Based on the iteration algorithm aforementioned in determining sizes of pump and floodgate A, results indicate that the designed discharge of the pump is 50m^3/s and that of floodgate A is 140m^3/s with the size of 3×5m. As shown in Fig. 5, the left graph describes the process of rainwater inflow and drainage outflow whilst the right graph depicts the process of water levels in the sea and lake. It is found that the maximum water level during drainage is 3.15m and operation times of the pump and floodgate A are about 6 hours and 12 hours, respectively.

5. Conclusions

Rainwater utilization is of significant value in city planning. It not only reduces urban sewage amount, but also reduces water supply for urban environment demand, which obviously decreases maintenance cost for urban environment. In this paper we accomplish the following conclusions and recommendations through studying water quality rehabilitation and drainage system planning of Haigang city in China.

1) For regions abundant in precipitation, the size of wastewater treatment plant and operation cost can be significantly reduced through reasonably utilizing later precipitation. In this case study, based on a 90% rainfall frequency, the size of the wastewater treatment plant can be reduced from 182,000m^3 per day to 150,000m^3 via utilizing rainwater when WQI is required from Ⅳ to Ⅲ within 133 days.

2) The study shows that a reasonable waterlock system planning not only guarantees effectual operation of inner circle but also facilitates water quality rehabilitation of Dishui Lake.

3) The drainage system and rainwater utilization are effectively and reasonably integrated in this study. The initial polluted rainwater is discharged through the pump station and floodgate A, mainly by pumping, whereas the major objective of later pure rainwater is to meliorate water quality of Dishui Lake and is discharged by floodgate A when rainwater is in excess.

4) The drainage computational results show that about 1,150,000m^3 of rainwater is required to dilute the initial rainwater under the designed rainfall condition. Therefore part of the later pure rainwater is wasted. It is suggested to control the initial rainwater flows into sewage pipes by designing a kind of servo-mechanism. In that case, the pump station

can be deleted and the control mechanism can improve the utilization efficiency of later rainwater.

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Table 1 Relationships between water levels, water areas, and reservoir volumes in Haigang city

Water levels (m)	Water areas (km^2)			Relative reservoir volumes (10^4 m^3)		
	Dishui Lake	River networks	Total	Dishui Lake	River networks	Total
2.2	3.80	4.33	8.1	V_D	V_R	V
2.4	3.88	4.42	8.3	V_D+76.7	V_R+87.5	V+164.2
2.5	3.92	4.47	8.4	V_D +115.7	V_R+131.9	V+247.6
2.7	4.00	4.56	8.56	V_D+194.8	V_R+222.2	V+417.0
2.9	4.08	4.65	8.7	V_D+275.6	V_R+314.3	V+589.9
3.1	4.16	4.74	8.9	V_D+358.0	V_R+408.2	V+766.2
3.3	4.24	4.83	9.1	V_D+442.0	V_R+504.0	V+946.0

Note： V_D, and V_R denote absolute reservoir volumes of Dishui Lake and river networks at the water level of 2.2m , respectively ($V = V_D + V_R$).

Table 2 Comparison of required days for water quality rehabilitation with different sizes of wastewater treatment plant (unit: days)

Sizes (m^3 per day)	From Ⅴ to Ⅳ	From Ⅳ to Ⅲ	
		Without rainwater	With rainwater
100,000	94	211	180
150,000	78	173	133
200,000	43	115	100
182,000		133	

Table 3 Water quality indices of initial rainwater and river water before and after mixing

Pollutant (mg/L)	COD	BOD	NH_3	Total nitrogen	Total phosphorus
WQ of initial rainwater	180	11	0.6	1.5	0.4
WQ of river water before mixing	30	6	1.50	1.50	0.10
WQ of river water after mixing	34.8	6.16	1.47	1.79	0.11

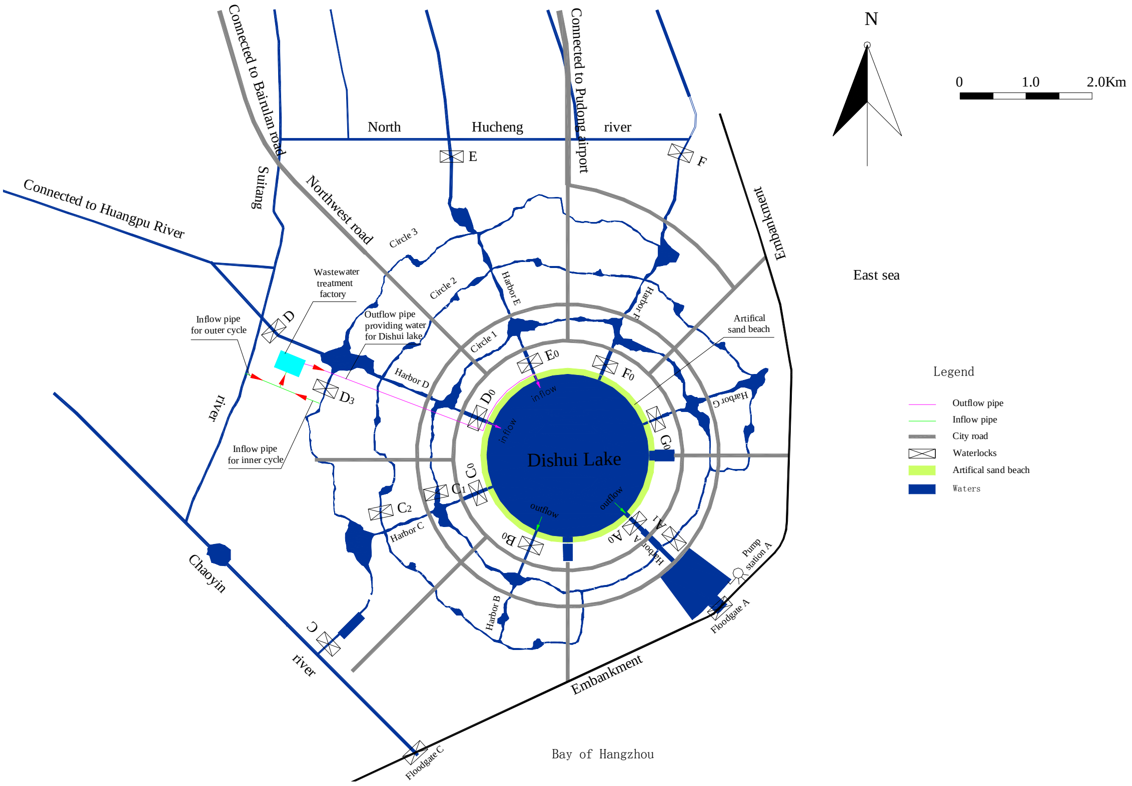

Fig.1 Schematic map of water networks and project layout in Haigang city

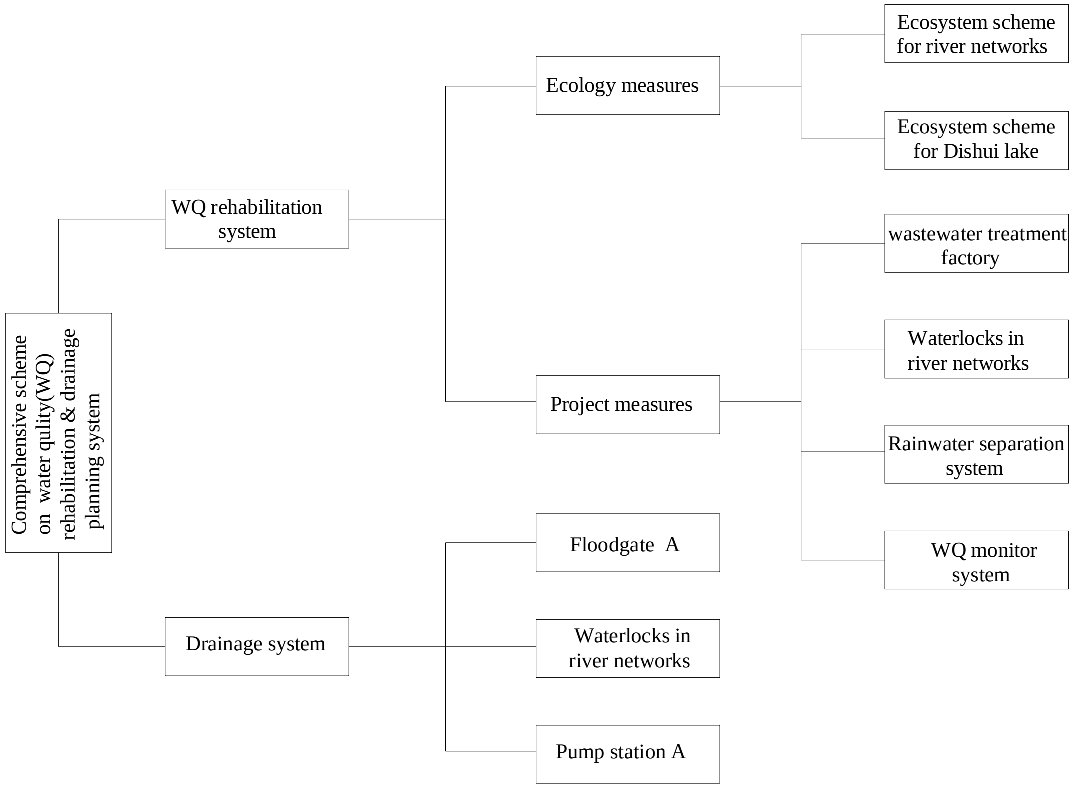

Fig.2 Flowchart of water quality rehabilitation scheme on Haigang city

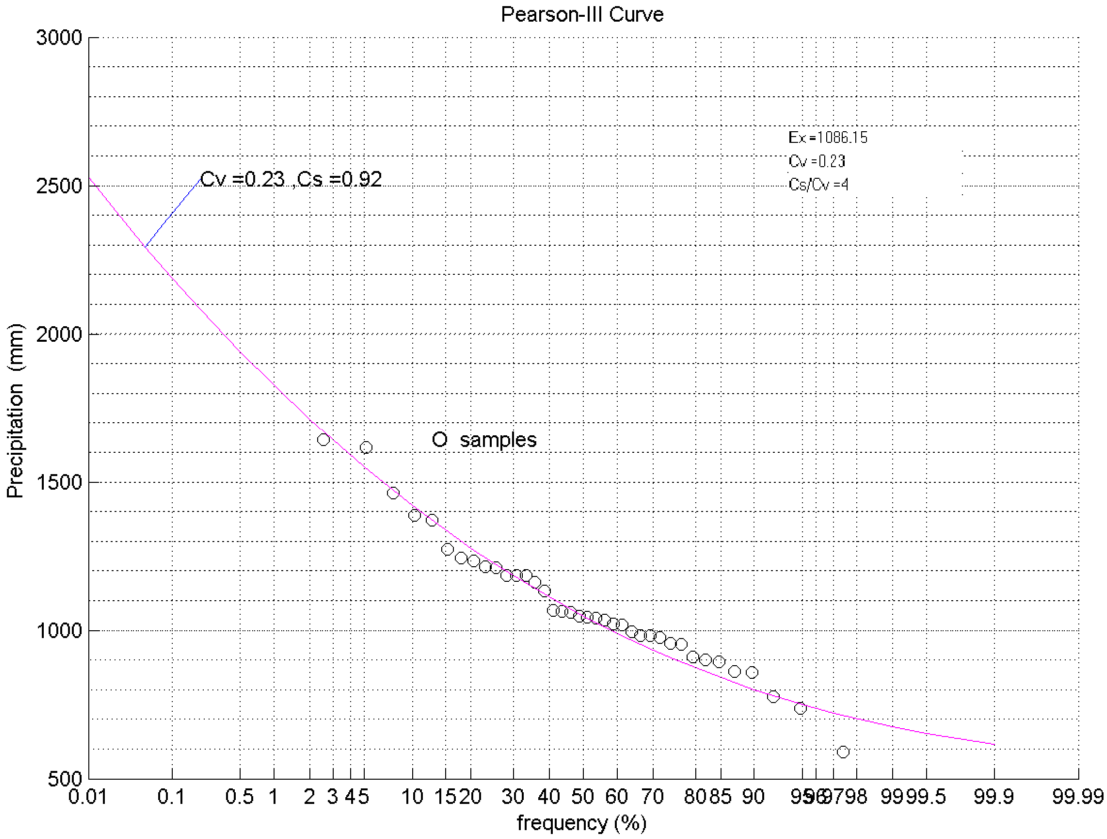

Fig.3 The result of frequency analysis on rainfall data in 1966~2003

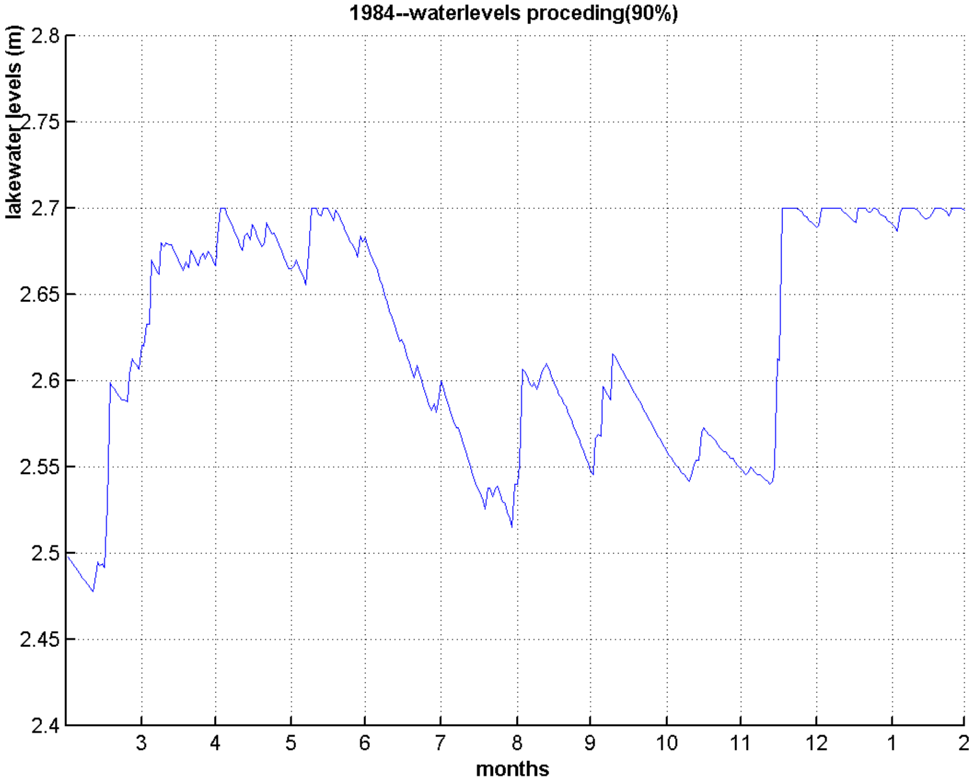

Fig.4 Daily water level variation of Dishui Lake with pure rainfall inflow in 1984

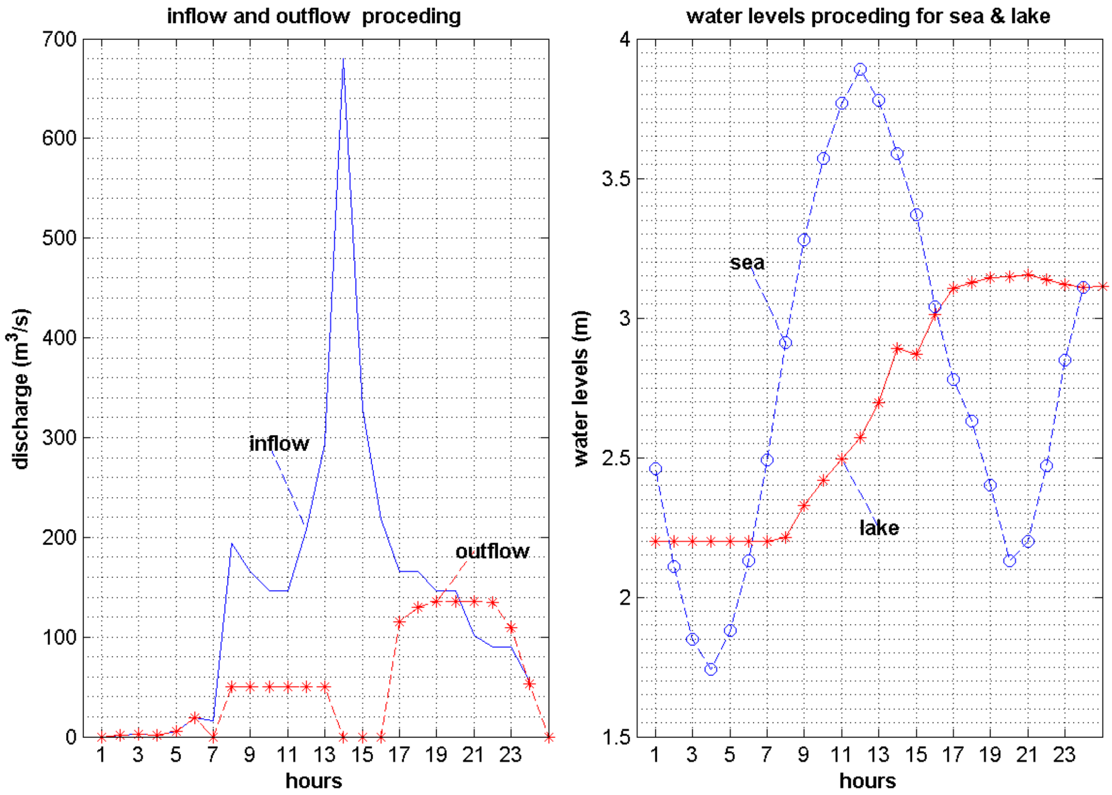

Fig.5 Drainage characteristic curves under the designed rainstorm condition