An Auto-Tuning Algorithm for the IRBF Network of Brushless DC Motor

S. L. Ho, M. R. Fei, K. W. E. Cheng, and H. C. Wong

***Abstract*—The integrated radial basis function (IRBF) network has been reported as an efficient algorithm to study the performance of brushless dc motors. However, such an algorithm cannot be implemented readily since it is difficult to auto-tune or even to find the undetermined coefficients in the integrated RBF network. In this paper, a novel auto-tuning algorithm that can effectively guarantee the automatic implementation of the integrated RBF network of a brushless dc motor is reported.**

***Index Terms*—Auto-tuning, brushless dc motor, finite element, integrated radial basis function (IRBF) neural network.**

I. Introduction

A CIRCUIT-FIELD coupled model of the brushless dc motor is more accurate, albeit computationally less efficient than its circuit counterpart in studying the output characteristics of brushless dc motors. Hence, the circuit-field coupled model is generally limited to off-line simulations only. To develop an on-line algorithm, an efficient radial basis function (RBF) technique using artificial neural network (ANN) based techniques is proposed to simulate and control electrical machines [1].

The outputs of the stator phase currents i_A, i_B, i_C, and the motor torque T are dependent on the inputs of the normalized stator voltages u_A, u_B, u_C, and the rotor position θ in a highly nonlinear manner. However, changes of outputs against the impressed stator voltage u_m or the rotor speed ω can be regarded as virtually linear within a limited range. Hence, the integrated RBF (IRBF) network, which can be applied into the control and design optimization of brushless dc motor with the same computational efficiency and accuracy as that in a magnetic-field computation using finite-element modeling (FEM), has been proposed [2]. However, it is well known that the undetermined coefficients in the IRBF network cannot be found accurately and automatically. In this paper, a novel auto-tuning algorithm of the undetermined coefficients in the IRBF network of the brushless dc motor is reported.

II. IRBF Network Principle

The RBF network has a linear output layer and a simple structure with a nonlinear hidden layer to synthesis the local approximations corresponding to the nonlinear input–output mapping of the brushless dc motor as

$$\begin{cases} i_A = f_A(u_A, u_B, u_C, \theta, \omega, u_m) \\ i_B = f_B(u_A, u_B, u_C, \theta, \omega, u_m) \\ i_C = f_C(u_A, u_B, u_C, \theta, \omega, u_m) \end{cases} \quad (1)$$

$$T = f_T(i_A, i_B, i_C, \theta, \omega, u_m). \quad (2)$$

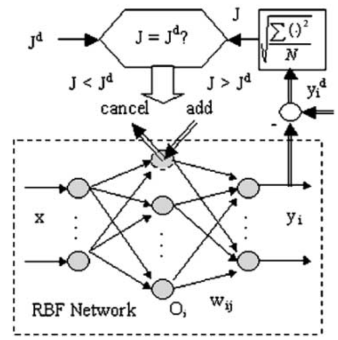

Fig. 1. Model of adaptive RBF network.

The basic idea of the IRBF network [2] is that, by using an approximate piecewise-linear assumption within a certain range, one should model the center point ($u_m = u_{mc}$ and $\omega = \omega_c$) by an adaptive RBF network. The adjacent and basis points ($u_m = u_{mb1}, u_{mb2}, \ldots$ and $\omega = \omega_{b1}, \omega_{b2}, \ldots$) could be modeled by the RBF network group with the same number of hidden layer nodes and parameters as the Gaussian based functions.

Both the adaptive RBF network (Fig. 1) used to model the center point and the RBF network group for modeling the basis points are dependent on the training data from the circuit-field coupled time stepping FEM computation [3].

In Fig. 1, y_i^d is the desired network output, J is the root-mean-square error function of $y_i^d - y_i$, and J^d is the allowable maximum value of J. If $J > J^d$, then, the number of nodes in the hidden layer is increased; If $J \leq J^d$, then, the number of nodes in the hidden layer is reduced, and the adaptive process of RBF is complete. The RBF network that comprises the input $x \in R^m$, the node output O_j in the hidden layer, the node output y_i in the network output layer, as well as the linking weight w_{ij} between the O_j hidden node and the y_i output node are governed by

$$O_j(x) = e^{-\|x-C_j\|^2/\sigma_j^2} \quad (3)$$

$$y_i = \sum_{j=1}^{n} w_{ij} O_j(x) \quad (4)$$

$$w_{ij}(k+1) = w_{ij}(k) + \lambda \left[y_i^d - y_i(k)\right] O_j(x) \Big/ \sum_{j=1}^{n} O_j^2(x) \quad (5)$$

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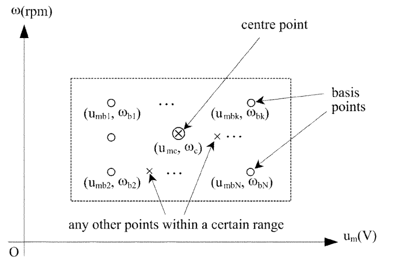

Fig. 2. Diagrammatic sketch of integrated RBF network mechanics.

where c_j is the center point of the j base function and σ_j is the selectable parameter controlling the width of the base function around the center point in the RBF network, and λ is the learning rate. In order to meet the monotonic attenuation condition of the changes in $e_i(k)(=y_i^d - y_i(k))$, one has $0 < \lambda < 1$.

According to the adaptive RBF network and the RBF network group, the IRBF network for modeling other points within a specific range (see Fig. 2) requires no additional training data for the same hidden layer.

To construct the IRBF network w_{ij}^{IR}, the integrating weights w_{ijk} of the RBF network group are used as the bases of the IRBF network. The fundamental equations of the IRBF network are

$$y_i^{\mathrm{IR}(I)} = \sum_{j=1}^{n} w_{ij}^{\mathrm{IR}(I)} O_j^{(I)}(x) \tag{6}$$

$$w_{ij}^{IR(I)} = \sum_{k=1}^{N_b} \mu_k^{\mathrm{IR}(I)} w_{ijk}^{(I)} \tag{7}$$

$$\mu_k^{\mathrm{IR}(I)} = f(\omega - \omega_{bk}, u_m - u_{mbk}) \tag{8}$$

where the superscript IR refers to the IRBF network, the superscript (I) refers to the network of current output or torque output, n is the total number of nodes in the hidden layer, and N_b is the total number of basis points. μ_k is an important coefficient of the integrating weights w_{ijk} that are also being used by the same hidden layer in the RBF network group.

The underlying principles governing μ_k are as follows.

1) The influence of the basis point becomes large, i.e., the μ_k value becomes large if the distance between the arbitrary point in a certain range and a basis point is short, i.e., the square of the rotor speed difference $(\omega - \omega_{bk})^2$ and the square of the voltage magnitude difference $(u_m - u_{mbk})^2$ are small.
2) The influence of the basis point becomes small, i.e., the μ_k value becomes small if the distance between the arbitrary point in a specific range and a basis point is not short, i.e., the $(\omega - \omega_{bk})^2$ and/or $(u_m - u_{mbk})^2$ values are large.

The μ_k relational expression upon $(\omega - \omega_{bk})^2$ and $(u_m - u_{mbk})^2$ is [2]

$$\mu_k = \left[\left(\frac{\omega_{bk} - \omega_c}{d_1}\right)^2 + \left(\frac{u_{mbk} - u_{mc}}{d_2}\right)^2\right] \cdot \exp\left(-\left[\left(\frac{\omega - \omega_{bk}}{\sigma_\omega}\right)^2 + \left(\frac{u_m - u_{mbk}}{\sigma_u}\right)^2\right]\right) \tag{9}$$

where d_1, d_2, and σ_ω, σ_u are the undetermined coefficients.

In order to reduce the number of undetermined coefficients and to propose the auto-tuning algorithm efficiently, (9) should be revised as follows:

$$\mu_k^{\mathrm{IR}(I)} = c_\omega^{(I)} \cdot g_1(\omega - \omega_{bk}, u_m - u_{mbk}) + c_{\mathrm{um}}^{(I)} \cdot g_2(\omega - \omega_{bk}, u_m - u_{mbk}) \tag{10}$$

where

$$g_1(\omega - \omega_{bk}, u_m - u_{mbk}) = \exp\left(-\left(\frac{\omega - \omega_{bk}}{\omega_c - \omega_{bk\,\max}}\right)^2\right) \Big/ \left(\left(\frac{u_m - u_{mbk}}{u_{mc} - u_{mbk\,\max}}\right)^2 + 1\right) \tag{11}$$

$$g_2(\omega - \omega_{bk}, u_m - u_{mbk}) = \exp\left(-\left(\frac{u_m - u_{mbk}}{u_{mc} - u_{mbk\,\max}}\right)^2\right) \Big/ \left(\left(\frac{\omega - \omega_{bk}}{\omega_c - \omega_{bk\,\max}}\right)^2 + 1\right). \tag{12}$$

c_ω and c_{um} in (10) are the undetermined coefficients. Usually, it is difficult to find suitable undetermined coefficients, i.e., the values of c_ω and c_{um} in the IRBF network.

III. AUTO-TUNING PRINCIPLE

In order to find the optimized values of the undetermined coefficients c_ω and c_{um} automatically, the quadratic object function of any specific point in an approximately linear range is defined as

$$J_{\mathrm{IR}} = \sum_{i=1}^{N} \left[y_i^{d(I)} - y_i^{\mathrm{IR}(I)}\right]^2 \tag{13}$$

according to (6) and (7) one obtains

$$J_{\mathrm{IR}} = \sum_{i=1}^{N} \left[y_i^{d(I)} - \sum_{k=1}^{N_b} \mu_k^{(I)} y_{ik}^{(I)}\right]^2 \to \min \tag{14}$$

where $y_i^{d(I)}$ is the output of the circuit-field-coupled time-stepping FEM computation, $y_{ik}^{(I)}$ is the output of the RBF network group, N is the total length of the output series.

A. Optimizing Calculation via a Specific Point

Assuming that $(\partial J_{\mathrm{IR}}/\partial \hat{c}_\omega) = 0, (\partial J_{\mathrm{IR}}/\partial \hat{c}_{\mathrm{um}}) = 0$, then, the optimized solutions c_ω and c_{um} are obtained as follows:

$$\begin{cases} \hat{c}_\omega^{(I)} = \dfrac{A_{uu}^{(I)} B_\omega^{(I)} - A_{u\omega}^{(I)} B_u^{(I)}}{A_{\omega\omega}^{(I)} A_{uu}^{(I)} - A_{\omega u}^{(I)} A_{u\omega}^{(I)}} \\ \hat{c}_{\mathrm{um}}^{(I)} = \dfrac{A_{\omega\omega}^{(I)} B_u^{(I)} - A_{\omega u}^{(I)} B_\omega^{(I)}}{A_{\omega\omega}^{(I)} A_{uu}^{(I)} - A_{\omega u}^{(I)} A_{u\omega}^{(I)}} \end{cases} \tag{15}$$

where $A_{uu}, A_{\omega u}, A_{u\omega}, A_{\omega\omega}$, and B_u, B_ω are measurable functions

$$A_{\omega\omega}^{(I)} = \sum_{i=1}^{N} \left[\sum_{k=1}^{N_b} g_1(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)}\right]^2 \tag{16}$$

$$A_{\omega u}^{(I)} = \sum_{i=1}^{N} \left\{\left[\sum_{k=1}^{N_b} g_1(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)}\right] \cdot \left[\sum_{k=1}^{N_b} g_2(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)}\right]\right\} = A_{u\omega}^{(I)} \tag{17}$$

$$A_{uu}^{(I)} = \sum_{i=1}^{N} \left[\sum_{k=1}^{N_b} g_2(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)} \right]^2 \tag{18}$$

$$B_{\omega}^{(I)} = \sum_{i=1}^{N} \left[y_i^{d(I)} \sum_{k=1}^{N_b} g_1(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)} \right] \tag{19}$$

$$B_{u}^{(I)} = \sum_{i=1}^{N} \left[y_i^{d(I)} \sum_{k=1}^{N_b} g_2(\omega - \omega_{bk}, u_m - u_{mbk}) \cdot y_{ik}^{(I)} \right] \tag{20}$$

The above equations are available if the auto-tuning algorithm is working at a limited range near the specific point at which a quadratic object function can be constructed.

B. Practicable Workflow in a Certain Range

The practicable programming steps to auto-tune the undetermined coefficients c_ω and c_{um} are

1) to confirm the basis points with equidistant spacing characteristics of the rotor speed ω_{bk} and the voltage magnitude u_{mbk} within a specific range (see Fig. 2);
2) to construct the IRBF network via one center point (included in the basis points, see Fig. 2), modeled by an adaptive RBF network, and the basis points are also modeled by a RBF network group;
3) to select one specific point in the specific range, which is far away from the basis points (see Fig. 2);
4) to calculate the undetermined coefficients c_ω and c_{um} using the data from the FEM computations of the specific point using (15) and (16)–(20);
5) to check the accuracy about the IRBF network output, which is calculated based on the values of c_ω and c_{um} at the specific point. Reduce the variation range of the rotor speed and voltage magnitude and/or increase the number of basis points, and return to 1) if it is not within the range of the target error;
6) to normalize the sum $\Sigma\mu_k$, the important coefficient μ_k can be used in the calculation of the IRBF network output about any other points in a specific range. The normalization is expressed as

$$\bar{\mu}_k = \frac{\mu_k}{\sum_{k=1}^{N_b}} \mu_k. \tag{21}$$

C. Fundamental Analysis About the Auto-Tuning Algorithm

Because of the convergence of the RBF network group for the basis points when $0 < \lambda < 1$ in (5), the nontrained IRBF network must be convergent by virtue of the Superposition Theorem. The analysis is given in [2].

However, the convergence precision of the nontrained IRBF network is determined by the coefficient μ_k of the integrating weights w_{ijk}, i.e., the undetermined coefficients c_ω and c_{um} in (10). Based on the analysis of (11) and (12), one has

μ_k is maximum, i.e., $g_{1\text{MAX}} = g_{2\text{MAX}} = 1$, when one specific point (ω, u_m) overlaps the basis point (ω_{bk}, u_{mbk}).

μ_k is minimum, i.e., $g_{1\text{MIN}} = g_{2\text{MIN}} \approx 0$ (the influence to the IRBF network output can be ignored for a given error tolerance) when one specific point (ω, u_m) overlaps the symmetrical basis point $(-\omega_{bk}, -u_{mbk})$.

μ_k is small, i.e., the influence to the IRBF network output is small, when one specific point (ω, u_m) is far away from the basis point (ω_{bk}, u_{mbk}).

Equations (11), (12), and (21) assure the relativity and the invariance of the sum of the influences upon the IRBF network output among one specific point and the basis points. So, the auto-tuning algorithm is convergent and sufficiently precise within a specific range.

D. Further Expansion About the Auto-Tuning Algorithm

Instead of (13), (14)–(20) can be modified with the quadratic object function about some specific points being constructed as follows:

$$J_{\text{IRS}} = \sum_{l=1}^{N_a} \sum_{i=1}^{N} \left[y_{li}^{d(I)} - y_{li}^{\text{IR}(I)} \right]^2 \tag{22}$$

where N_a is the number of specific points used in the auto-tuning algorithm.

Note that even though additional training data from some points are required by the IRBF algorithm when compared to that of the trial-and-error method, the IRBF network with the auto-tuning algorithm can effectively build as well as compute all the points accurately within a specific range without calling upon the time consuming FEM computation.

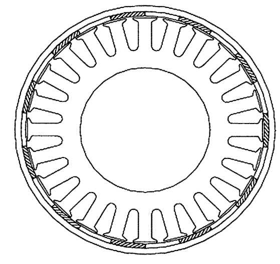

Fig. 3. Three-phase motor with 22 poles and 24 slots.

IV. Experiment on Brushless DC Motor

The auto-tuning algorithm has been used to automatically evaluate the undetermined coefficients c_ω and c_{um} of the IRBF network of a multipole PM brushless dc motor as shown in Fig. 3 (200 W/12 V, 22 poles, 24 stator slots, with Nd-Fe-B magnets). The training data used to build the frame of IRBF network as well as to optimize its undetermined coefficients are produced by FEM computations with a mesh of about 6000 nodes.

The experimental conditions being reported are within the range of $|\omega - 220| < 20$ rpm and $|u_m - 10| < 1$ V. The target error of an arbitrary point in the specific range satisfies

$$\sqrt{\frac{\sum_{i=1}^{N} \left[y_i^{d(I)} - y_i^{\text{IR}(I)} \right]^2}{\sum_{i=1}^{N} \left[y_i^{d(I)} \right]^2}} \le 0.05. \tag{23}$$

The nine basis points ($\omega = 220 \pm 20$ rpm, $u_m = 10 \pm 1$ V), including a center point ($\omega = 220$ rpm, $u_m = 10$ V), are being chosen in the study. The IRBF network can be constructed by these nine basis points. However, the IRBF network is not

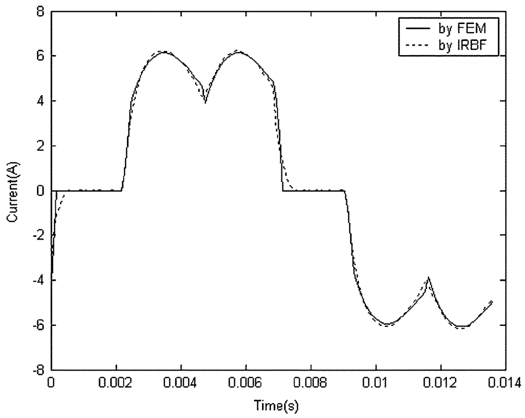

Fig. 4. Stator phase current obtained from the FEM computation and the IRBF network at one specific point, which is selected for calculating c_ω and $c_{\rm um}$.

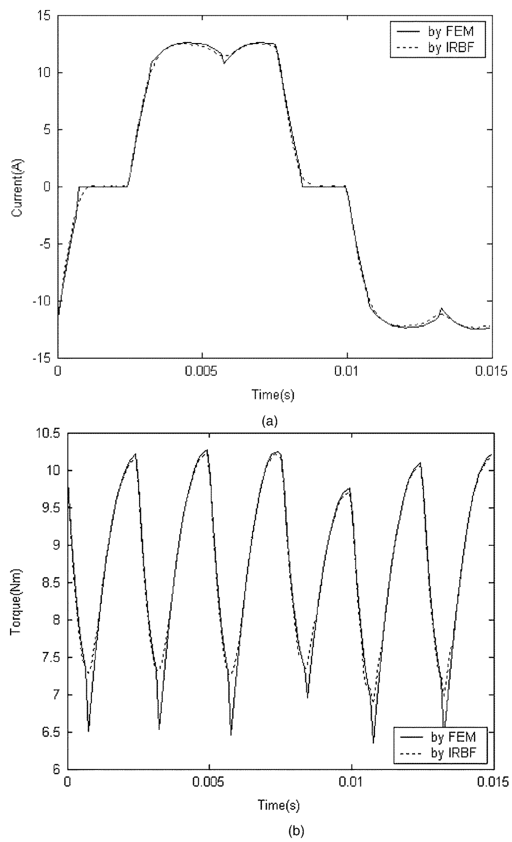

Fig. 5. Stator phase current and output torque obtained from the FEM computation and the IRBF network in those points within the specific range.

optimized because there are two undetermined coefficients c_ω and $c_{\rm um}$.

Therefore, the proposed auto-tuning algorithm to obtain the coefficients c_ω and $c_{\rm um}$ is applied. According to programming step 3), one specific point within a certain range will be selected, e.g., (230 rpm, 9.5 V), or (210 rpm, 10.5 V), etc.

It is found that instead of carrying out FEM computations, the IRBF network can be used to calculate the outputs of the stator phase currents i_A, i_B, i_C and the motor torque T based on the inputs of the normalized stator voltages u_A, u_B, u_C, the rotor position θ, the rotor speed ω, and the voltage magnitude u_m of an arbitrary point in a specific range.

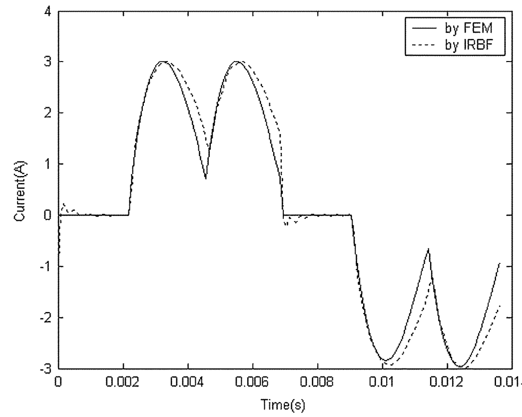

Fig. 6. Stator phase current obtained from the FEM computation and the IRBF network at one point outside the specific range.

Fig. 4 gives the correlation illustration of FEM computation results and IRBF network outputs at one specific point [see programming steps 3)–4)]. Fig. 5(a) and (b) gives a comparison of FEM and IRBF results in the specific range, whereas Fig. 6 is the comparison of FEM and IRBF results outside the specific range.

The experimental results show that the auto-tuning algorithm can effectively guarantee a fully automatic modeling of the IRBF network within an accuracy of 0.05 [see (23)] of target error in the specific range bounded by the nine basis points, although there are different amplitude variations in the stator phase current as can be seen between the contrast of Figs. 4 and 5(a). However, if the point is outside the specific range, the accuracy of the IRBF network output would not be guaranteed.

V. Conclusion

In order to ensure the undetermined coefficients c_ω and $c_{\rm um}$ are found quickly, accurately, and automatically, a novel auto-tuning algorithm in the IRBF network of the brushless dc motor is proposed. The fully automatic implementation of the IRBF network should be very valuable for practical and industrial applications. Based on the IRBF network of the brushless dc motor and its auto-tuning algorithm mentioned above, further investigations can also be carried out, in order to extend the range of the impressed stator voltage u_m and the rotor speed ω, for a given error tolerance from the simulation results that are obtained between the circuit-field coupled time stepping FEM computation and the IRBF network.

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