

Analysis of postponement strategy for perishable items by EOQ-based models

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Abstract

This paper develops EOQ-based models with perishable items to evaluate the impact of a form postponement strategy on the retailer in a supply chain. We formulate models for a postponement system and an independent system to minimize the total average cost function per unit time for ordering and keeping n perishable end-products. An algorithm is given to derive the optimal solutions of the proposed models. The impact of the deterioration rate on the inventory replenishment policies is studied with the help of theoretical analysis and numerical examples. Our theoretical analysis and computational results show that a postponement strategy for perishable items can give a lower total average cost under certain circumstances.

Keywords: Postponement strategy; Economic-order-quantity model (EOQ); Perishable items; Inventory management

1. Introduction

Postponement, also known as late customization or delayed product differentiation, refers to delaying some product differentiation processes in a supply chain as late as possible until the supply chain is cost effective (Garg and Lee, 1998). Postponement is one of central features of mass customization (van Hoek, 2001). It has been reported that postponement strategy is highly successful in a wide range of industries that require high differentiation, e.g., high-tech

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industry, food industry, and fashion industry, etc. One practical example is Hewlett-Packard Development Company. HP produces generic printers in its factories and distributes them to local distribution centers, where power plugs with appropriate voltage and user manuals in the right language are packed. They have saved a lot of money every year by adopting the postponement strategy (Lee, 1998).

However, postponement is not an omnipotent strategy. It has both advantages and disadvantages. The advantages include following the JIT principles of production, reducing end-product inventory (Brown et al., 2000), making forecasting easier (Ernst and Kamrad, 2000), and pooling risks (Garg and Tang, 1997). The high cost of redesigning and manufacturing generic components is the main drawback of postponement (Lee, 1998). Thus, evaluation of postponement structures is an important issue. Many qualitative and quantitative models have been developed to evaluate the cost-effectiveness of postponement strategy under different scenarios. Details can be found in the review articles by van Hoek (2001), and Wan et al. (2003a). Recent quantitative models include, but are not limited to, those by Lee (1996), Garg and Tang (1997), Garg and Lee (1998), Ernst and Kamrad (2000), Aviv and Federgruen (2001), Ma et al. (2002), Su (2005), and Reiner (2005). They evaluated the cost and benefits of applying postponement in a large variety of stochastic settings. If demand is deterministic, e.g., because there is a long-term supply contract between a manufacturer and its customers, the benefits due to economies of scope from risk pooling do not exist. It is necessary to develop deterministic models to evaluate postponement. Recent deterministic models include, among others, those by Wan et al. (2003b, 2004), and Li et al. (2005). Wan et al. (2003b, 2004) analyzed pull postponement using EOQ-based models and EPQ-based models. They showed that the postponed customization of end-products will result in a lower total average cost under certain circumstances. Li et al. (2005) considered EPQ-based models with planned backorders. They showed that a postponement system can outperform an independent system under certain circumstances and identified the key factors for postponement decisions.

All of the above models assumed that the inventoried items can be stored indefinitely to meet future demands. However, certain types of products either deteriorate or become obsolete in the course of time. Perishable products are commonly found in commerce and industry, for example, fruits, fresh fish, perfumes, alcohol, gasoline, photographic films, etc. For these kinds of products, traditional inventory models are no longer applicable. An early study of perishable inventory systems was carried out by Whitin (1957). Since then, considerable effort has been expended on this line of research. Comprehensive surveys of related research can be found in Nahmias (1982), Raafat (1991), and Goyal et al. (2001), where relevant literature published

before the 1980s, in the 1980s, and in the 1990s was reviewed, respectively. Recent studies before 2004 can be found in Song et al. (2005).

One of the focuses of the research on perishable products is interaction and coordination in supply chains (Song et al., 2005). For example, Goyal and Gunasekaran (1995) developed an integrated production-inventory-marketing model for determining the economic production quantity and economic order quantity for raw materials in a multi-stage production system. Yan and Cheng (1998) studied a production-inventory model for perishable products, where they assumed that the production, demand and deterioration rate are all time-dependent. They gave the conditions for a feasible point to be optimal. Arcelus et al. (2003) modeled a profit-maximizing retail promotion strategy for a retailer confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable products. Kanchanasuntorn and Techanitisawad (2006) investigated the effect of product deterioration and retailers' stockout policies on system total cost, net profit, service level, and average inventory level in a two-echelon inventory–distribution system, and developed an approximate inventory model to evaluate system performance. There are many papers addressing the interaction and coordination between inventory and marketing, financing, distribution, and production. To the best of our knowledge, there exists no paper studying the interaction between inventory and postponement in a supply chain with perishable items. In this paper we will use the EOQ-based model with perishable items to analyze postponement to fill this gap in the literature.

In this paper we study a supply chain involving a retailer and n customers. The retailer orders n different products in response to the demands of the customers. It is assumed that the n perishable end-products are manufactured from the same raw material or semi-manufactured products. The end-products belong to the same product category, but they have slight differences and the customization process can be delayed after ordering. The retailer can order the n perishable end-products independently in an independent system. However, the retailer may order the material or semi-manufactured product and finish the customization itself, i.e., customization is postponed after ordering. The ordering decisions can be combined. This can be viewed as a postponement system. For example, retailers of a soft drink supplier can order concentrated syrup and mix it with carbonated water in-house to make different soda products for sale at their retail stores. In this case, the retailers make only one decision to acquire the concentrated syrup rather than making many different decisions to acquire different products marketed by the soft drink supplier.

The objective of this paper is to investigate whether or not the postponement system can outperform the independent system with perishable items. We formulate two models to describe the supply chain, and give an algorithm to derive the optimal ordering strategies. We also investigate the effect of product deterioration on the total cost of the retailer and on inventory replenishment policies. Some numerical examples are provided to illustrate the theoretical results. We show that postponement strategy can give a lower total average cost under certain circumstances with perishable items. The results presented in this paper provide insights for managers that guide them to find a proper tradeoff between postponement and non-postponement.

The rest of this paper is organized as follows. In the next section we describe the notation and assumptions used throughout this paper. In Section 3 we establish a mathematical model to evaluate the impacts of inventory deterioration rate on inventory replenishment policies. We then provide a simple algorithm to find the optimal replenishment schedule. In Section 4 we show that a postponement system can outperform an independent system under certain circumstances. In Section 5 some numerical examples are provided to illustrate the theoretical results. Finally, we conclude the paper, and suggest some directions for future research in Section 6.

2. Notation and Assumptions

We assume that the demand rates of the end-products are independent and constant. The unsatisfied demands (due to shortage) are completely backlogged. So we formulate two EOQ-based models to describe the supply chain. In the first model, the retailer orders the n perishable end-products independently with different schedules, so there are n EOQ decisions. However, in the second model customization is postponed after ordering, and the ordering decisions can be combined so that a single EOQ decision is made. This practice can be viewed as a form postponement strategy.

Definitions of the notation of this paper are presented below.

- i = end-product, $i = 1, 2, \dots, n$,
- I_i = demand rate of end-product i , $I_i > 0$,
- q = deterioration rate of end-products and raw materials, $q \geq 0$,
- c = common variable production cost, $c > 0$,

- k = common fixed ordering cost, $k > 0$,
- h = common unit holding cost per unit time, $h > 0$,
- b = unit backorder cost for end-products, $b > 0$,
- p = common extra unit customization cost, $p > 0$,
- $I(t)$ = inventory level at time t ,
- T_i = total cycle time for end-product i , $T_i > 0$,
- t_i = the time up to which the inventory of end-product i is positive in a cycle,
- $C(T_i, t_i)$ = total average cost per unit time for ordering and keeping end-product i ,
- TC = total average cost per unit time for ordering and keeping n end-products in an independent system,
- TCP = total average cost per unit time for ordering and keeping n end-products in a postponement system (excluding the customization cost).

In addition, the following assumptions are imposed on the models:

1. The replenishment rate is infinite and the lead time is zero.
2. The end-product demand rates I_i are deterministic and constant.
3. Shortages are allowed and completely backlogged.
4. All the end-products are produced from the same type of raw materials and the ratio of raw material to end-product is 1:1.
5. An extra customization process cost per end-product p is incurred if the customization process is delayed. The lead-time for customization is negligible.
6. The distribution of the deterioration time of the items follows the exponential distribution with parameter q , i.e., a constant rate of deterioration.
7. Deterioration of the raw materials and end-product is considered only after they have been received into inventory, and there is no replacement of deteriorated inventory.

3. Model formulation

Based on the above assumptions, the inventory level of an end-product at time t , $I(t)$, is governed by the following differential equation:

$$\frac{dI(t)}{dt} = \begin{cases} -\mathbf{q}I(t) - \mathbf{1}, & 0 \leq t \leq t_0, \\ -\mathbf{1}, & t_0 \leq t \leq T, \end{cases} \quad (1)$$

with the boundary condition $I(t_0) = 0$, where t_0 is the time up to which the inventory level is positive in a cycle, and T is the cycle length. The solution of (1) is

$$I(t) = \begin{cases} \mathbf{1}(e^{\mathbf{q}(t_0-t)} - 1)/\mathbf{q}, & 0 \leq t \leq t_0, \\ -\mathbf{1}(t - t_0), & t_0 \leq t \leq T. \end{cases} \quad (2)$$

$I(t)$ follows the pattern depicted in Fig. 1.

Based on (2), we obtain the total average cost per unit time for ordering and keeping the end-product as follows

$$C(t_0, T | \mathbf{q}) = \frac{k}{T} + c\mathbf{1} + \frac{\mathbf{1}(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - \mathbf{q}t_0 - 1)}{T\mathbf{q}^2} + \frac{\mathbf{1}b(T - t_0)^2}{2T}. \quad (3)$$

The necessary conditions for the minimum value of $C(t_0, T | \mathbf{q})$ are

$$\frac{\partial C(t_0, T | \mathbf{q})}{\partial t_0} = \frac{\mathbf{1}(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)}{T\mathbf{q}} + \frac{\mathbf{1}b(t_0 - T)}{T} = 0, \quad (4)$$

$$\frac{\partial C(t_0, T | \mathbf{q})}{\partial T} = -\frac{k}{T^2} - \frac{\mathbf{1}(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - \mathbf{q}t_0 - 1)}{T^2\mathbf{q}^2} + \frac{\mathbf{1}b(T^2 - t_0^2)}{2T^2} = 0. \quad (5)$$

After rearranging the terms in (4) and (5), we get

$$-\frac{k}{\mathbf{1}(c\mathbf{q} + h)} - \frac{e^{\mathbf{q}t_0} - \mathbf{q}t_0 - 1}{\mathbf{q}^2} + \frac{(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)^2}{2b\mathbf{q}^2} + \frac{t_0(e^{\mathbf{q}t_0} - 1)}{\mathbf{q}} = 0, \quad (6)$$

$$T - t_0 - \frac{(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)}{b\mathbf{q}} = 0. \quad (7)$$

Lemma 3.1. *If $c\mathbf{q} + h > 0$, then the point $(t_0^* > 0, T^* > 0)$ that solves (6) and (7) simultaneously exists and is unique. The point (t_0^*, T^*) is also the unique global optimum for the problem $\min\{C(t_0, T | \mathbf{q}) : 0 < t_0 < T < \infty\}$.*

Proof. Our lemma is a special case of Propositions 2 and 3 of Dye et al. (2005).

Thus t_0^* can be uniquely determined as a function of \mathbf{q} , say $t_0^* = t(\mathbf{q})$, and T^* can be uniquely determined as a function of \mathbf{q} , say $T^* = T(\mathbf{q})$. This also implies that $C(t_0^*, T^* | \mathbf{q})$ can be uniquely determined as a function of \mathbf{q} , say $C(t_0^*, T^* | \mathbf{q}) = C(t(\mathbf{q}), T(\mathbf{q}) | \mathbf{q})$.

Theorem 3.1. $\widehat{C}(\mathbf{q}) \triangleq C(t(\mathbf{q}), T(\mathbf{q}) | \mathbf{q}) = \min\{C(t_0, T | \mathbf{q}) : 0 < t_0 < T < \infty\}$ is an increasing and continuous function of \mathbf{q} in $[0, +\infty)$, and $\lim_{\mathbf{q} \rightarrow 0} \widehat{C}(\mathbf{q}) = c\mathbf{1} + \sqrt{2k\mathbf{1}hb/(b+h)}$.

Proof. Recalling that the power series for e^x is $\sum_{n=0}^{\infty} (x^n/n!)$, we have

$$\begin{aligned} C(t_0, T | \mathbf{q}) &= \frac{k}{T} + c\mathbf{1} + \frac{\mathbf{1}(c\mathbf{q} + h) \left(\sum_{n=0}^{\infty} \frac{(\mathbf{q}t_0)^n}{n!} - \mathbf{q}t_0 - 1 \right)}{T\mathbf{q}^2} + \frac{\mathbf{1}b(T - t_0)^2}{2T} \\ &= \frac{k}{T} + c\mathbf{1} + \frac{t_0^2 \mathbf{1}(c\mathbf{q} + h) \sum_{n=2}^{\infty} \frac{(\mathbf{q}t_0)^{n-2}}{n!}}{T} + \frac{\mathbf{1}b(T - t_0)^2}{2T}. \end{aligned} \quad (8)$$

For $\mathbf{q} \geq 0$, it is obvious that $C(t_0, T | \mathbf{q})$ is an increasing function of \mathbf{q} for each fixed value of $t_0 > 0$ and $T > 0$. If $\mathbf{q}_1 < \mathbf{q}_2$, we have

$$\widehat{C}(\mathbf{q}_2) = C(t(\mathbf{q}_2), T(\mathbf{q}_2) | \mathbf{q}_2) > C(t(\mathbf{q}_2), T(\mathbf{q}_2) | \mathbf{q}_1) \geq C(t(\mathbf{q}_1), T(\mathbf{q}_1) | \mathbf{q}_1) = \widehat{C}(\mathbf{q}_1).$$

Thus, $\widehat{C}(\mathbf{q})$ is an increasing function of \mathbf{q} in $[0, +\infty)$. Let

$$\begin{aligned} f_1(t_0, T, \mathbf{q}) &\triangleq -\frac{k}{\mathbf{1}(c\mathbf{q} + h)} - \frac{e^{\mathbf{q}t_0} - \mathbf{q}t_0 - 1}{\mathbf{q}^2} + \frac{(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)^2}{2b\mathbf{q}^2} + \frac{t_0(e^{\mathbf{q}t_0} - 1)}{\mathbf{q}}, \\ f_2(t_0, T, \mathbf{q}) &\triangleq T - t_0 - \frac{(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)}{b\mathbf{q}}. \end{aligned}$$

For $\mathbf{q} > -\frac{h}{c}$, we have

$$\begin{aligned} \frac{\partial f_1}{\partial t_0} &= \frac{\mathbf{1}(c\mathbf{q} + h)(e^{\mathbf{q}t_0} - 1)e^{\mathbf{q}t_0}}{b\mathbf{q}} + \mathbf{1}t_0e^{\mathbf{q}t_0}, \quad \frac{\partial f_1}{\partial T} = 0, \\ \frac{\partial f_2}{\partial t_0} &= -1 - \frac{(c\mathbf{q} + h)e^{\mathbf{q}t_0}}{b}, \quad \frac{\partial f_2}{\partial T} = 1, \end{aligned}$$

from where we deduce that

$$\left| \begin{array}{cc} \frac{\partial f_1}{\partial t_0} & \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial t_0} & \frac{\partial f_2}{\partial T} \end{array} \right| = \frac{\partial f_1}{\partial t_0} > 0.$$

From the implicit function theorem, we know that $t(\mathbf{q})$ and $T(\mathbf{q})$ are continuous functions of \mathbf{q} in $[0, +\infty)$, respectively. Moreover, $C(t_0, T | \mathbf{q})$ is a continuously differentiable real

function for $0 < t_0 < T$, and $\mathbf{q} > -h/c$. Thus, $\widehat{C}(\mathbf{q})$ is also a continuous function of \mathbf{q} in $[0, +\infty)$.

Because $\widehat{C}(\mathbf{q})$ is continuous in $[0, +\infty)$, we have

$$\lim_{\mathbf{q} \rightarrow 0} \widehat{C}(\mathbf{q}) = \widehat{C}(0) = \min_{0 < t_0 < T < \infty} \left\{ \frac{k}{T} + c\mathbf{I} + \frac{\mathbf{I}ht_0^2}{2T} + \frac{\mathbf{I}b(T-t_0)^2}{2T} \right\} = c\mathbf{I} + \sqrt{\frac{2k\mathbf{I}hb}{b+h}}.$$

Theorem 3.2. $t(\mathbf{q})$ is a decreasing function of \mathbf{q} in $[0, +\infty)$, and $t(\mathbf{q}) \leq \sqrt{2kb/(\mathbf{I}h(b+h))}$.

Proof. $t(\mathbf{q})$ is the unique solution of equation (6). After rearranging the terms in (6), we get

$$\begin{aligned} \frac{k}{\mathbf{I}(c\mathbf{q}+h)} &= -\frac{e^{q_0} - \mathbf{q}t_0 - 1}{\mathbf{q}^2} + \frac{t_0(e^{q_0} - 1)}{\mathbf{q}} + \frac{(c\mathbf{q}+h)(e^{q_0} - 1)^2}{2b\mathbf{q}^2} \\ &= t_0^2 \sum_{i=1}^{+\infty} \left(\frac{1}{i!} - \frac{1}{(i+1)!} \right) (\mathbf{q}t_0)^{i-1} + \frac{(c\mathbf{q}+h)(e^{q_0} - 1)^2}{2b\mathbf{q}^2}. \end{aligned} \quad (9)$$

The left side of (9) is a decreasing function of \mathbf{q} , and the right side of (9) is an increasing function of \mathbf{q} for each fixed value of $t_0 > 0$. When \mathbf{q} increases, $t(\mathbf{q})$ must decrease in order to satisfy equation (9). So $t(\mathbf{q})$ is a decreasing function of \mathbf{q} in $[0, +\infty)$, and $t(\mathbf{q}) \leq t(0) = \sqrt{2kb/(\mathbf{I}h(b+h))}$.

Because t_0^* and T^* cannot be determined in a closed form from (6) and (7), we have to determine them numerically using the following algorithm.

Algorithm 3.1.

Step1. Obtain the value of t_0^* by solving the nonlinear equation (6) with the help of some mathematical software such as MatLab or Mathematica.

Step2. Compute T^* by using (7).

Step3. The corresponding optimal cost per unit time $C(t_0^*, T^* | \mathbf{q})$ is obtained by (3).

Remark 3.1. If $\mathbf{q}\sqrt{2kb/(\mathbf{I}h(b+h))}$ is small enough, we can give an approximate optimal solution of (3). We can approximate e^{q_0} by the first three terms in its power series. Then, we have $C(t_0, T | \mathbf{q}) \approx k/T + c\mathbf{I} + \mathbf{I}(c\mathbf{q}+h)t_0^2/(2T) + \mathbf{I}b(T-t_0)^2/(2T)$. This is the classical EOQ model. By the EOQ formula, we can obtain that the approximate optimal cost is $c\mathbf{I} + \sqrt{2k\mathbf{I}(h+c\mathbf{q})b/(b+h+c\mathbf{q})}$. From the approximate optimal cost, we can find that

deterioration effectively adds an additional component to the holding cost, i.e., from h to $h + cq$.

4. The postponement and independent systems

Now we discuss the postponement system and the independent system. In the independent system the raw materials are ordered independently (i.e., without postponement). The total average cost for ordering and keeping the n end-products is

$$TC(\mathbf{q}) = \sum_{i=1}^n C(t_i, T_i | \mathbf{q}) = \sum_{i=1}^n \left\{ \frac{k}{T_i} + cI_i + \frac{I_i(cq + h)(e^{qt_i} - qt_i - 1)}{T_i q^2} + \frac{I_i b (T_i - t_i)^2}{2T_i} \right\}. \quad (10)$$

In the form postponement system, all the raw materials are ordered together (i.e., postponing the customization process) and the demand rate is $\hat{I} = I_1 + I_2 + \dots + I_n$. The total average cost for ordering and keeping the n end-products is given by (excluding the customization cost)

$$TCP(\hat{t}, \hat{T} | \mathbf{q}) = \frac{k}{\hat{T}} + c\hat{I} + \frac{\hat{I}(cq + h)(e^{q\hat{t}} - q\hat{t} - 1)}{\hat{T}q^2} + \frac{\hat{I}b(\hat{T} - \hat{t})^2}{2\hat{T}}. \quad (11)$$

The difference in the optimal total average cost per unit time of the two systems is defined as $z^* = TCP^*(\mathbf{q}) - TC^*(\mathbf{q})$.

Theorem 4.1 *There exists a $\bar{q} > 0$ such that for any $0 \leq q \leq \bar{q}$, $TCP^*(\mathbf{q}) < TC^*(\mathbf{q})$, i.e., the postponement system can give a lower total average cost than the independent system.*

Proof. Because $\hat{C}(\mathbf{q})$ is continuous in $[0, +\infty)$, we have

$$\lim_{q \rightarrow 0} TCP^*(\mathbf{q}) = TCP^*(0) = c\hat{I} + \sqrt{\frac{2k\hat{I}hb}{b+h}}, \quad (12)$$

$$\lim_{q \rightarrow 0} TC^*(\mathbf{q}) = TC^*(0) = c\hat{I} + \sum_{i=1}^n \sqrt{\frac{2kI_i hb}{b+h}}. \quad (13)$$

Wan et al. (2003) have proved that (12)–(13) < 0 . So there exists a $\bar{q} > 0$ such that for any $0 \leq q \leq \bar{q}$, $TCP^*(\mathbf{q}) < TC^*(\mathbf{q})$.

Remark 4.1. If q is small, we can obtain an approximate optimal cost as Remark 3.1. Then

$$z^* \approx \sqrt{2k(h+cq)b/(b+h+cq)} \left(\sqrt{\sum_{i=1}^n I_i} - \sum_{i=1}^n \sqrt{I_i} \right).$$

From this equation, we see that the postponement system can outperform the independent system, and the absolute value of z^* becomes larger when q becomes larger.

5. Numerical examples and sensitivity analysis

We give some numerical examples to illustrate how the deterioration rate impacts on the minimum total average cost and postponement. To illustrate the results, we consider the example in Padmanabhan and Vrat (1995).

Example 1. In order to study how various deterioration rates affect the optimal cost of the EOQ model, deterioration sensitivity analysis is performed. The value of the deterioration rate varies as follows: (0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60). The demand rate is $I = 600$, the common variable ordering cost c is 5, the common fixed ordering cost k is 250, the common unit holding cost h per unit time is 1.75, and the unit backorder cost b is 3 (all in appropriate units).

Applying the solution procedure in Section 3, we derive the results shown in Table 1 and Fig. 2, from which the following observations can be made.

- 1 $\hat{C}(q)$ is an increasing and concave function of q in $[0, +\infty)$.
- 2 $t(q)$ is a decreasing function of q in $[0, +\infty)$.
- 3 qt^* is less sensitive to q . The reason is that $t(q)$ is a decreasing function of q .

Example 2. In order to study how various deterioration rates affect the difference in cost between the postponement system and the independent system, we assume that there are eleven end-products. For the eleven products, we assume that $I_1 = 550$, $I_2 = 560$, $I_3 = 570$, $I_4 = 580$, $I_5 = 590$, $I_6 = 600$, $I_7 = 610$, $I_8 = 620$, $I_9 = 630$, $I_{10} = 640$, $I_{11} = 650$. The other related data are the same as the data of Example 1. Applying the solution procedure in Section 3, we obtain the results of the sensitivity analysis with these parameters, which are shown in Table 2 and Fig. 3, from which the following observations can be made.

- 4 The postponement system yields savings in the total average cost.
- 5 The absolute value of z^* becomes larger when the deterioration rate becomes larger.
- 6 The absolute value of z^*/TC^* becomes larger when the deterioration rate becomes larger.

Observations 5 and 6 imply that the larger the deterioration rate is, the more cost-effective the postponement strategy is.

6. Conclusions

In this paper an EOQ model for deteriorating items with a constant deterioration rate q was developed. We showed that the postponement strategy outperforms the independent strategy when q is small. Our numerical experiments showed that the difference between the two strategies will become larger when q becomes larger. We assumed that the deterioration rate of the raw materials is the same as that of the end-products. But the raw materials, such as IC chips, are easy to be used for other products by design changes, the deterioration rate of the raw materials is often smaller than that of the end-products. So postponement can yield more savings in total average cost in practice. Now we consider the extra customization cost in a postponement system. It is obvious that the average customization cost per unit time is $p \sum_{i=1}^n I_i$. The difference in the optimal total average cost per unit time between the two systems is $z^* + p \sum_{i=1}^n I_i$. So postponement is more cost-effective if $z^* + p \sum_{i=1}^n I_i < 0$. One potential future research direction is to study the impact of postponement on stochastic models with perishable products.

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Table 1

The impact of deterioration rate on inventory replenishment policies

q	0	0.02	0.04	0.06	0.08	0.10	0.20	0.30	0.40	0.50	0.60
t^*	0.548	0.526	0.505	0.486	0.468	0.453	0.385	0.336	0.299	0.269	0.244
qt^*	0	0.011	0.020	0.029	0.037	0.045	0.077	0.100	0.119	0.134	0.147
$\widehat{C}(q)$	3576	3587	3597	3606	3515	3625	3661	3690	3713	3733	3750

Table 2

The impact of deterioration rate on the difference in cost between the two systems

q	0	0.02	0.04	0.06	0.08	0.10	0.20	0.30	0.40	0.50	0.60
$-z^*$	4422	4506	4585	4659	4730	4796	5083	5309	5493	5646	5775
$\left \frac{z^*}{TC^*} \right $	0.112	0.114	0.116	0.118	0.119	0.120	0.126	0.131	0.135	0.138	0.140

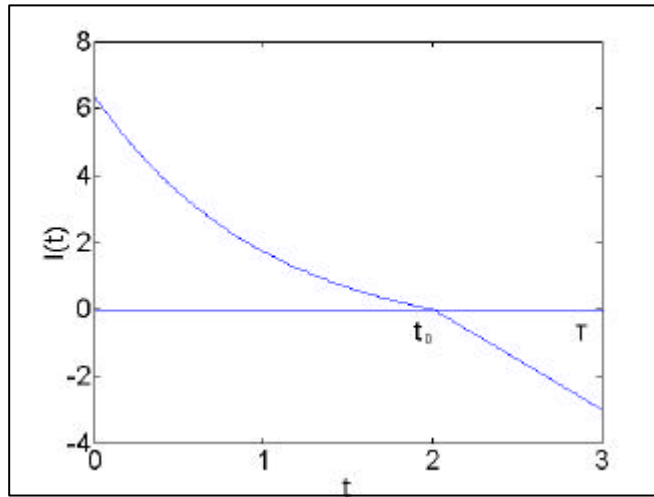


Fig. 1. Graphical representation of inventory level

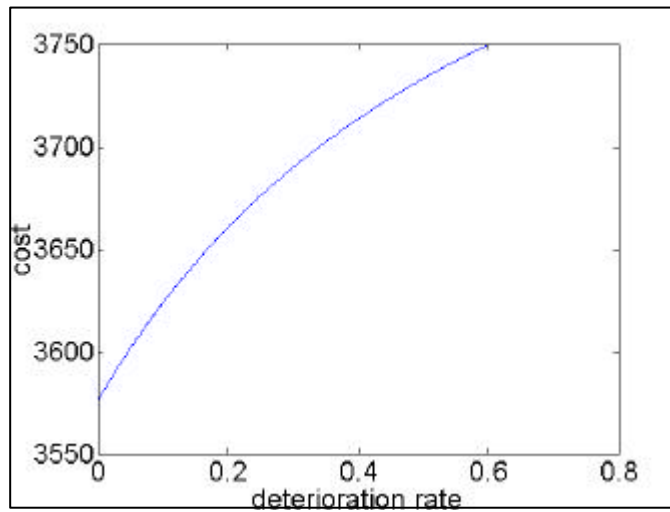


Fig. 2. The impact of deterioration rate on the total cost

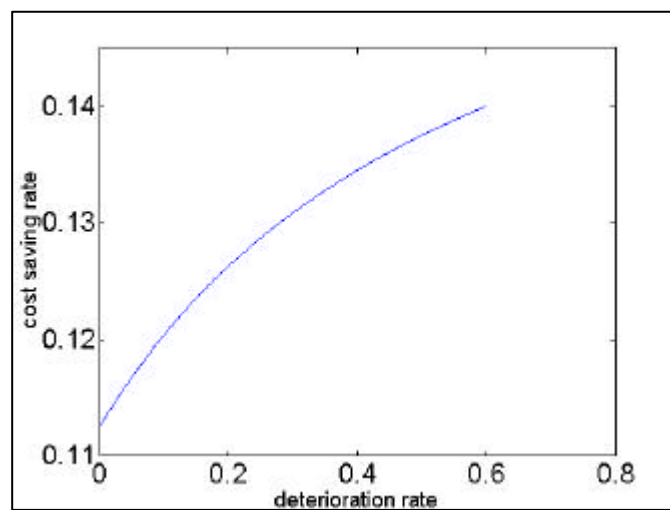


Fig. 3. The impact of deterioration rate on the difference in cost between the two systems