# Modeling and Analysis of Piezoelectric Transformer using Multi-mesh Loop Matrix Circuit under Square-wave Excitation Conditions

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**Abstract— A multi-mesh loop matrix model for a piezoelectric transformer operating with switched circuit excitation is presented. The modeling requires the inclusion of harmonics and the coupling of vibration in different directions of the piezoelectric transformer. Simulation and experimental results are used to validate the proposed model using matrix circuits. The proposed method has also compared with other conventional modeling method, a good agreement is obtained.**

***Keywords*—Modeling, Nonlinear magnetics, Reluctance motors, Simulation**

## I. INTRODUCTION

Piezoelectric transformer is considered as one of the most important power transfer device in the next decade because of its low profile, no EMI problem and high efficiency. It can also achieve voltage conversion ratio using frequency control easily whereas thee magnetic transformer cannot. Therefore piezoelectric transformer can be used to construct a regulated power converter easily with a frequency control as the parameters. Today, piezoelectric transformer has found applications in many power conversions in consumer products, medical electronics and lighting system [1-2], rectifier [3], miniature power supplier [4] and low power source [5].

Recently the use of multilayer piezoelectric transformer is getting more attention because of the requirement of larger voltage conversion ratio. All the layers can be connected in either parallel or series internally in order to follow similar method in magnetic transformers connection. The resultant multilayer can give very high output voltage. The typical applications are high voltage power supply, electrostatic precipitator, and ionizer.

Common modeling of the piezoelectric transformer is based on a simple LRC circuit [6] which can only describe the vibrating structure in only one direction, such as the longitudinal direction. However, the coupling between different vibration directions will have significant influence upon the energy transmission and operating efficiency along the axes in many cases [7].

Finite element model (FEM) is a powerful tool to study the operation of piezoelectric transformers [8]. However, modeling of the piezoelectric transformer using FEM requires a complete set of the electromechanical properties for each constituent material. Such properties include the dielectric and coupling constants. The loss mechanics including the attenuation is also very complex [9]. Moreover, an accurate calculation of the voltage conversion ratio is of paramount importance in the circuit design for power conversion applications. Conventional, the modeling of piezoelectric transformer is based on circuit model which the resistance, capacitance and inductance are used. The modeling of multi-layer type is especially important because the model exhibits small higher order effect and the coupling between the layers make the simple model of the RLS not sufficient to represent the whole piezoelectric transformer. Hence it is necessary to develop a model that can give simple and accurate results on the coupling, energy loss and conversion ratio. A multilayer type is used as an example to demonstrate the model of the piezoelectric transformer. In this paper, a mutual coupling is introduced in the circuit model so that the parasitic effect inn different layers of the piezoelectric transformer can be modeled accurately.

## II. PROPOSED MODEL

The model is based on a matrix resonant circuit having similar conventional LRC resonant elements together with the inclusion of different coupling mechanics in different directions. Fig. 1 shows the proposed model that represents the three directional couplings in a realistic manner. The mesh current as shown in Fig. 2 in each facet of the matrix model is $I_i$, where $1 \le i \le 6$ is the number of each facet. The element in

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each of the edge gives rise to the more physical meaning of the piezoelectric transformer. Each edge gives the connection between the longitudinal vibration and the coupling between the longitudinal vibrations. They are therefore contributes to the resonant tank in each edge and the coupling between the components will be added in the later session of the paper. The impedance of each edge of the matrix can be represented by $Z_{uvw\text{-}xyz}$, where *uvw* and *xyz* are the coordinates of the $Z_{uvw\text{-}xyz}$ to which it is connected. The matrix model of the Fig 1 can be expressed as:

$$V_k = C_{ij} A_{ij} I_k \tag{1}$$

where
the subscript has the range of

$$1<k<6;\ 1<i<6;\ 1<j<6. \tag{2}$$

$k$ is the number of the mesh loops. $i$ and $j$ are also equal to the number of mesh loop. For the present model, $A_{ij}$ is the 6×6 matrix. $V_k$ is the voltage source in the circuit. It includes the input and output voltages. $C_{ij}$ is a connection matrix which is 1 when there is an element in the loop and 0 when there is no element in the loop. The element $A_{ij}$ is the impedance matrix. $I_k$ is the loop current. Also,

$$A_{ij} \in Z_{uvw\text{-}xyz} \tag{3}$$

The impedance $Z_{uvw\text{-}xyz}$ is a series connection of R, L and C.

$$Z_{uvw-xyz} = R_{uvw-xyz} + sL_{uvw-xyz} + \frac{1}{sC_{uvw-xyz}} \tag{4}$$

The voltage conversion ratio is then:

$$\frac{V_o}{V_{in}} = \frac{(I_1 + I_5) Z_{100-000}}{(I_1 + I_4) Z_{010-000}} \tag{5}$$

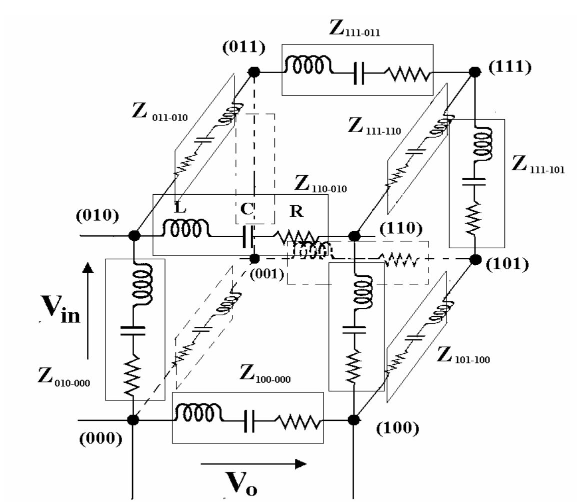

Fig. 1. Matrix model for the piezoelectric transformer

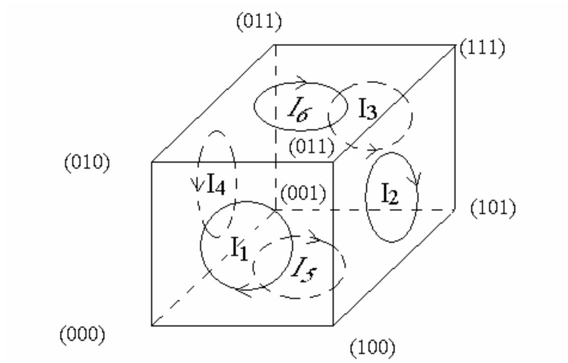

Fig. 2. Mesh current of each facet of the matrix model

## II. OLD MODEL

Conventional model [10] for modeling the piezoelectric transformer can be revisited in Fig 3. The model only gives the fundamental resonance of the transformer and a 2nd order resonant frequency. However, for multilayer case, the coupling between the layers cannot be realized.

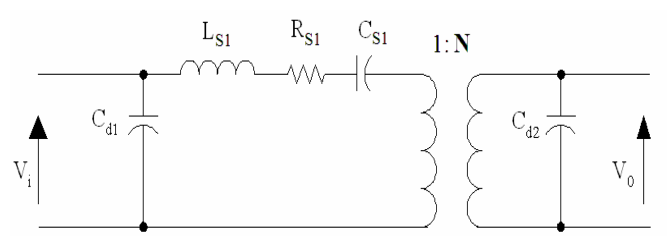

Fig 3: Conventional model of piezoelectric transformer

Fig 4 shows a modified model which some dielectric loss components has been added to facilitate some of the loss and coupling. The loss and the higher order frequencies are expressed by use of the capacitance $C_{d1}$, $C_{d2}$ and $C_{p1}$. It now uses three models presented here to examine which model is more accurate for the captioned applications.

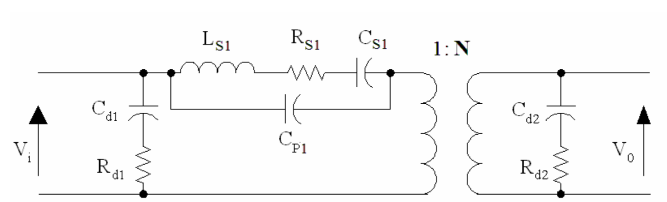

Fig 4: Modified Piezoelectric transformer model

## III. EXPERIMENT RESULT OF THE MULT-LAYER

A multilayer piezoelectric transformer is used for the analysis. The dimension of the multilayer piezoelectric transformer is 8mm x 3mm x 36mm. The primary side is multilayer but the secondary side is single layer. The load resistance for this test is 101kΩ.

 

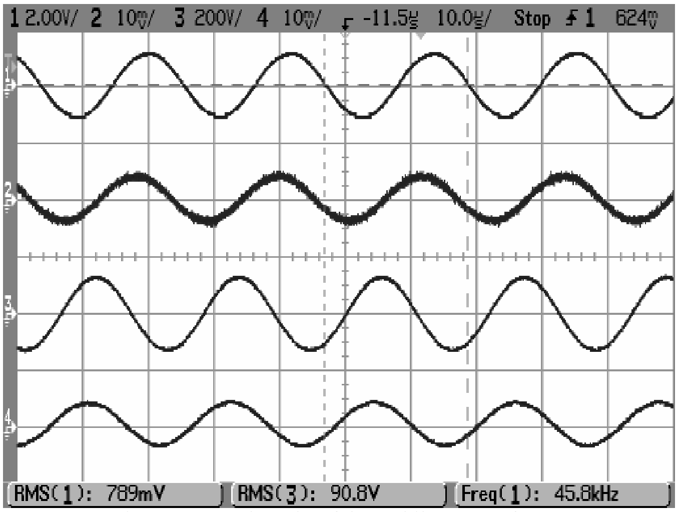


Fig 5: Measured results of the multilayer transformer
Ch1= input voltage (2V/div) & Ch2=input current (0.5A/div),
Ch3=output voltage (200V/div) & Ch4=output current (10mA/div)

The voltage conversion characteristics versus frequency is measured and shown in Fig 6. It can be seen that there are 4 resonance frequencies which are at: 46kHz, 85kHz, 135.5kHz and 172.8kHz. It can also be seen that there is no direct relationship between each resonant frequencies such as the multiple relation. The resonant frequency is the self-resonant of its energy storage components and therefore using circuit model, usually a connection of the R-L-C is used to model the piezoelectric transformer. To be accurate, a more systemic approach is needed to express the frequency,

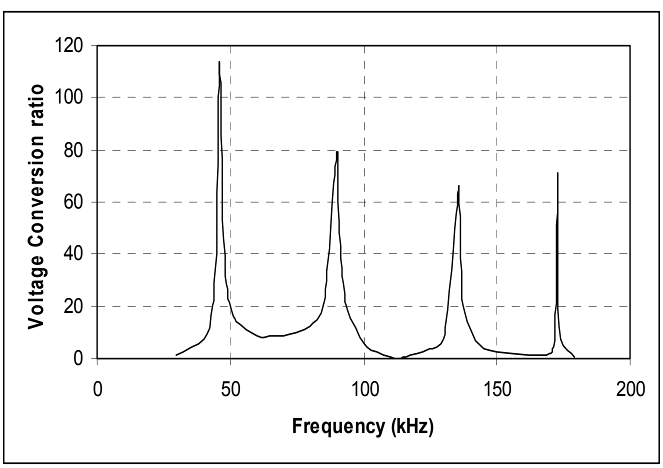


Fig 6: Measured voltage conversion ratio under sinusoidal input and load = 101kΩ

## IV. MODELLING RESULTS

Three modeling methods are used to model the piezoelectric transformer. They are the proposed matrix model, conventional and modified methods. The experiment was conducted using a high frequency amplifier with an accurate signal generator and the resonant peak is recorded. The frequency of interest is from 0Hz to 200kHz. Experimental measurement of multilayer transformer with the proposed matrix model was compared and present in Fig 7. It can be seen that a reasonable agreement can be found in the model. However, the 3rd and 4th resonant frequencies cannot be predicted very well with 10-30% error.

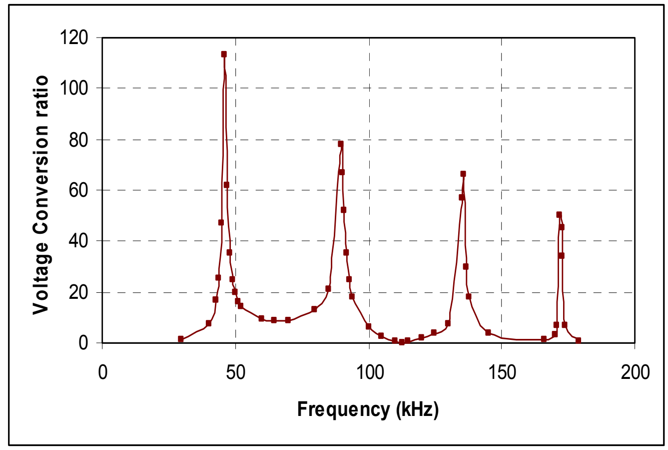


Fig 7: Comparison between the calculated and measured result.

Fig 8 and 9 shows the results of the ratio for the modified model and the conventional model. It can be seen that the modified model can describe the voltage conversion ratio very well and is different from the measurement by 10% for the resonant peaks. However, the 4th resonant frequency cannot be predicted very well and there is 80% error. The conventional model give up to 20% error in the resonant peak and the 4th resonant frequency at 172kHz cannot be retrieved from the model. It is quite obvious because the conventional model does not implemented with the 4th order self resonant frequency and therefore it is out of score for the capability. The thee resonance frequencies are generated from the combination of the frequency resonant tank from the $C_{d1}$, $C_{d2}$ and $C_{sS1}$.

Therefore it is necessary that a simple circuit model is unable to predict the complicate self-resonant frequency of the piezoelectric transformer and a model with higher order of resonant element as proposed is needed to order to give higher order of resonance frequency for the proper modeling.

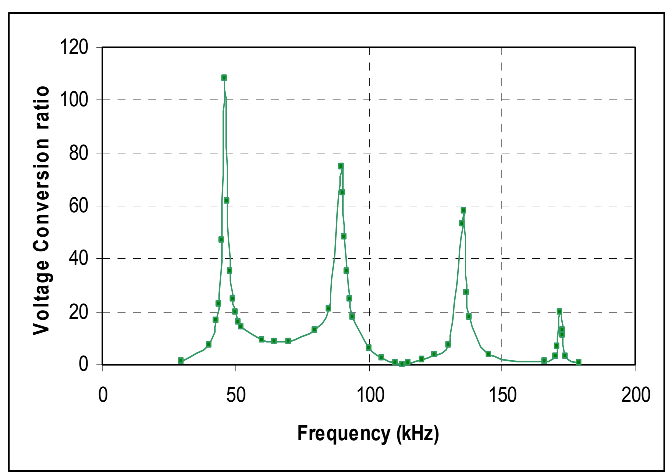


Fig.8. Calculated results using modified model

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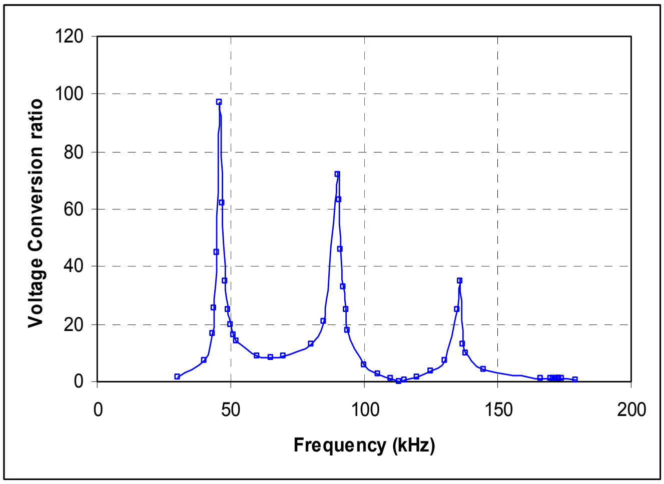

Fig 9: Calculated Piezoelectric transformer model using conventional model.

## V. COUPLED MODEL

The above matrix method is suffered from the high frequency accuracy because of the coupled between the each branch has not been activated. Equations (1-4) are modified as follows:

$$V_k = C_{ij} A_{ij} I_k + \sum C_{xy} M_{ij} I_k \quad (6)$$

where $1 \le x \le 6$; and $1 \le y \le 6$. $C_{xy}$ is the connection matrix to connect the $M_{ij}$ to the voltage equation. $C_{xy}$ defines the switching function is either 0 when no coupling between the current and the inductance and equal to 1 when there is a mutual couple between $M_{ij}$ and $I_k$. As the coupling between the inductance and current are small when the layer are far away and large when the layer are close. Therefore the modified equations give a more realistic representation of the multilayer piezoelectric transformer. The model is closer to the realistic situation because the coupling is the same effect as the mechanical coupling between the layers of the piezoelectric transformer.

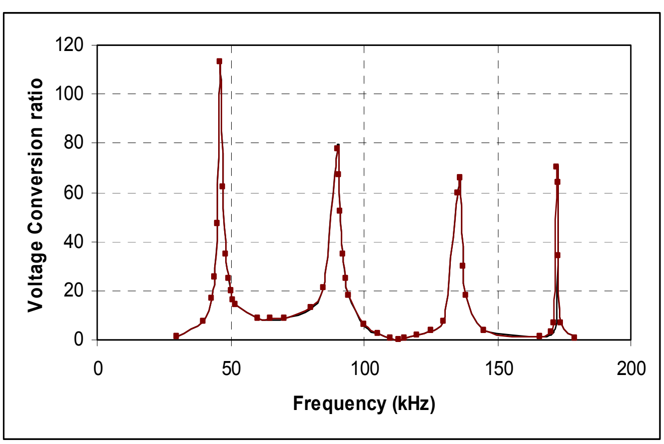

Fig 10: Matrix modeling with mutual coupling

## VI. ERROR OF THE MODELS AND DISCUSSION

The conventional model can model the 1st and 2nd resonant frequencies very well. For high values of resonance, the model cannot meet the requirement because the inter-coupled and parasitic capacitance cannot be revealed in the model. The modified one is constructed with additional parasitic capacitance. The modeling of the higher order resonance is good, but still the 4th order resonant frequency is not modeled very well.

By introduction the proposed matrix model, it allows a more network connection of each of the layers of the piezoelectric transformer. All the R-L-C networks are now connected to give coupling between each. However, the matrix model does not give mutual coupling between the layers that are needed for the multi-layer piezoelectric transformer. Finally, the matrix model with mutual coupling is used. It can now be seen that the new model give a better modeling method for the transformer.

Under square wave operation, eqn (6) can be used to represent square wave accordingly. Table 1 shows the error of the model as compared with other model and the experimental results. It is clearly seen that the proposed modeling method give a good agreement in both sinusoidal and switching square wave applications. The switching square wave is produced by a full-bridge converter which gives the same rms voltage as the sine waveform as in Section III and IV. The rms error of 1st 4 resonant peaks is calculated. It is clearly seen that again the matrix method with mutual coupling give the best solution to model the multi-layer piezoelectric transformer.

Table 1: RMS Error compared with experiment

| | Sinusoidal | Square wave |
|---|---|---|
| Conventional | 32% | 35% |
| Modified conventional | 20% | 25% |
| Matrix | 10% | 14% |
| Matrix with mutual coupling | 2% | 3% |

The method has also been tested for another piezoelectric transformer. It can also model the proposed 4 peaks of resonance frequencies. The error of the measurement is for the matrix with mutual coupling as compared with the theoretical calculation is also 2%.

## VII. CONCLUSION

This paper proposes an improved model of the piezoelectric transformer. The model is first expressed as impedance of each R-L-C network and it is then using mesh calculation to compute the characteristics. Four modeling method have been presented. They are the conventional, modified version which have layer capacitances, matrix model and coupled matrix model. The conventional model cannot predict the high resonant frequencies. The modified model cannot predict the peak value of the resonant peak. Therefore both models have the difficulty to express the coupling effect between layer and the non-longituitual energy transfer.

The matrix model is based on the intra-connection of the reactive components and the agreement has been improved. The model is further extended to cover the mutual coupling among the reactive component and the completed model namely matrix model using mutual coupling is developed. It can give very good agreement to the experimental results. It is clearly shown that after the coupling between the impedance edges are considered, the resonant peaks can be accurately calculated.

This paper demonstrates an alternative approach to model the piezoelectric transformer such that only R, L and C are used. Using the mutual coupling, the model gives improved results as compared with conventional method. The method also give a remarkable result as compared to the conventional modeling method that is only based on simple circuit mode.

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