

# Trigonometry-Based Numerical Method to Compute Nonlinear Magnetic Characteristics in Switched Reluctance Motors

X.D. Xue, K.W.E. Cheng, Member IEEE, S.L. Ho, K.F.Kwok  
Department of Electrical Engineering, the Hong Kong Polytechnic University  
Hung Hom, Kowloon, Hong Kong, China

eexdxue@polyu.edu.hk; eecheng@polyu.edu.hk; eeslho@polyu.edu.hk; eekfwok@polyu.edu.hk

**Abstract**— Based on two-dimensional (2-D) trigonometry, a new numerical method is presented to describe nonlinear magnetic and electromagnet torque characteristics in switched reluctance motors (SRMs). The computed and experimental results demonstrate that the proposed numerical method can be used to precisely compute nonlinear magnetic and torque characteristics in SRMs.

## I. INTRODUCTION

An SRM usually runs in the region of magnetic saturation such that a large ratio of torque to mass is obtained. Thus, magnetic saturation is very important to the high-performance operation of SRMs. On the other hand, magnetic saturation results in that magnetic and torque characteristics in SRMs are highly nonlinear. Consequently, analysis and control of SRMs become complicated. Thus, accurately describing nonlinear magnetic characteristics is crucial for performance prediction, performance optimization, sensorless and torque controls of SRM drives.

## II. PROPOSED NUMERICAL METHOD

The flux linkage  $\psi$  is the nonlinear function of the rotor position  $\theta$  and current  $i$  in SRMs, which is defined as [1]

$$\psi(\theta, i) = \sum_{p_\theta=-M_\theta}^{M_\theta} \sum_{p_i=-M_i}^{M_i} c_{p_\theta, p_i} \exp(-j(p_\theta k_\theta \hat{\theta} + p_i k_i \hat{i})) \quad (1)$$

where  $M_\theta$  and  $M_i$  are two specified truncation levels,  $j$  is the imaginary unity,  $c_{p_\theta, p_i}$  are complex Fourier coefficients,  $k_\theta$  and  $k_i$  are constants,  $\hat{\theta} = \theta - \theta_{\min}$ , and  $\hat{i} = i - i_{\min}$ .

Based on the orthogonal properties of the trigonometric functions and the trapezoidal integration rule, the real and imaginary parts of  $c_{p_\theta, p_i}$  can be computed from, respectively,

$$\Re(c_{p_\theta, p_i}) = \frac{C_{N_\theta} C_{N_i}}{N_\theta N_i} \left\{ \frac{1}{4} (\psi_{1,1} + \psi_{1, N_\theta+1} + \psi_{N_\theta+1,1} + \psi_{N_\theta+1, N_\theta+1}) + \frac{1}{2} \sum_{r=2}^{N_\theta} (\psi_{1,r} + \psi_{N_\theta+1,r}) \cos[k_i h_i p_i (r-1)] + \frac{1}{2} \sum_{s=2}^{N_\theta} (\psi_{s,1} + \psi_{s, N_\theta+1}) \times \cos[k_\theta h_\theta p_\theta (s-1)] + \sum_{s=2}^{N_\theta} \sum_{r=2}^{N_i} \psi_{s,r} \cos[k_i h_i p_i (r-1) + k_\theta h_\theta p_\theta (s-1)] \right\} \quad (2)$$

and

$$\Im(c_{p_\theta, p_i}) = \frac{C_{N_\theta} C_{N_i}}{N_\theta N_i} \left\{ \frac{1}{2} \sum_{r=2}^{N_\theta} (\psi_{1,r} + \psi_{N_\theta+1,r}) \sin[k_i h_i p_i (r-1)] + \frac{1}{2} \sum_{s=2}^{N_\theta} (\psi_{s,1} + \psi_{s, N_\theta+1}) \sin[k_\theta h_\theta p_\theta (s-1)] + \sum_{s=2}^{N_\theta} \sum_{r=2}^{N_i} \psi_{s,r} \sin[k_i h_i p_i (r-1) + k_\theta h_\theta p_\theta (s-1)] \right\} \quad (3)$$

## III. COMPUTATION AND EXPERIMENT

The proposed numerical method is used to compute nonlinear magnetic characteristics of a four-phase SRM. Fig. 1 illustrates the computed and experimental results of the flux linkage versus the current at a set of the rotor positions. It can be observed that the computed results are well consistent with the experimental data.

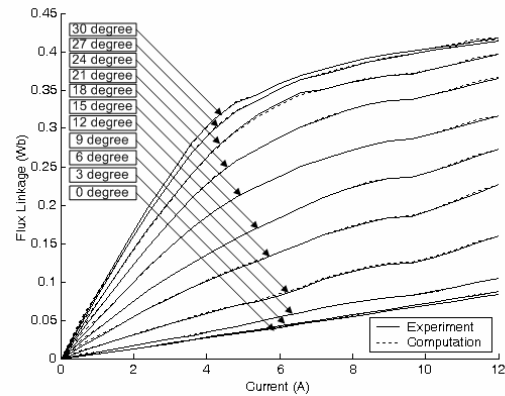


Fig. 1. Computed and experimental flux linkage characteristics

## IV. CONCLUSION

Based on 2-D trigonometry, this paper has presented a new numerical method to compute nonlinear magnetic characteristics in SRMs. The complex Fourier coefficients can be computed from the proposed expressions. The presented numerical method only depends on a limited number of given magnetic data acquired by experiment or FE analysis. The computed results and the experimental data have validated that the proposed numerical method can be used to precisely compute nonlinear magnetic characteristics. The salient advantage of the proposed method is the excellent accuracy. Therefore, this paper provides an effective and accurate numerical method for performance prediction, performance optimization, sensorless control, and torque control of SRM drives.

## V. ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of the Research Committee of the Hong Kong Polytechnic University (Project code: G-YX52).

## VI. REFERENCES

- [1] C. Pozrikidis, *Numerical Computation in Science and Engineering*, Oxford University Press, Inc., 1998.