An Improved Tabu-Based Vector Optimal Algorithm for Design Optimizations of Electromagnetic Devices

S. Y. Yang, J. R. Cardoso, S. L. Ho, P. H. Ni, J. M. Machado, and Edward W. C. Lo

***Abstract*—This paper introduces an improved tabu-based vector optimal algorithm for multiobjective optimal designs of electromagnetic devices. The improvements include a division of the entire search process, a new method for fitness assignment, a novel scheme for the generation and selection of neighborhood solutions, and so forth. Numerical results on a mathematical function and an engineering multiobjective design problem demonstrate that the proposed method can produce virtually the exact Pareto front, in both parameter and objective spaces, even though the iteration number used by it is only about 70% of that required by its ancestor.**

***Index Terms*—Multiobjective optimization, nondominated sorting, Pareto solution, tabu search, vector optimization.**

I. Introduction

RECENTLY, researches in multiobjective or vector optimizations have become very topical for both scientists and engineers because real world design problems are inevitably multiobjective in nature. However, despite the significant progresses in the development of multiobjective optimal algorithms, the robustness and efficiency of available vector optimal methods are still unsatisfactory; hence, there are still many open problems yet to be solved [1]. Moreover, most of the researches on multiobjective designs have focused on evolutionary algorithms only. Extensive researches reveal that the performances of evolutionary algorithms are often overshadowed by local search methods such as simulated annealing or tabu search methods [2]. Thus, an improved vector optimal method to enhance robustness and efficiency of a tabu-based algorithm [3] is proposed.

The most common concepts/terminologies used in this paper are already defined in [3] with the exception of weakly and strongly nondominated solutions. Consider the following minimization problem which is written in a shortened form as:

$$\min \bar{f}(\bar{x})(\bar{x} \in X) \tag{1}$$

$$\begin{aligned} &\bar{f}: X \rightarrow F \\ &X = \{\bar{x} \in E^n | \bar{g}(\bar{x}) \geq 0, \bar{h}(\bar{x}) = 0\} \\ &F = \{\bar{f} \in E^k | \bar{f}(\bar{x}), \bar{x} \in X\} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \bar{x} =& [x_1 \ x_2 \ \cdots \ x_n]^T \\ \bar{f}(\bar{x}) =& [f_1(\bar{x}) \ f_2(\bar{x}) \ \cdots \ f_k(\bar{x})]^T \\ \bar{g}(\bar{x}) =& [g_1(\bar{x}) \ g_2(\bar{x}) \ \cdots \ g_m(\bar{x})]^T \\ \bar{h}(\bar{x}) =& [h_1(\bar{x}) \ h_2(\bar{x}) \ \cdots \ h_p(\bar{x})]^T \end{aligned}$$

where $\bar{x}^*$ is a weakly nondominated solution if there is no $\bar{x} \in X(\bar{x} \neq \bar{x}^*)$, such that $f_i(\bar{x}) < f_i(\bar{x}^*)$ for $i = 1, 2, \ldots, k$; on the other hand, $\bar{x}^*$ is a strongly nondominated solution if there is no $\bar{x} \in X(\bar{x} \neq \bar{x}^*)$, such that $f_i(\bar{x}) \leq f_i(\bar{x}^*)$ for $i = 1, 2, \ldots, k$ and for at least one index of i, such that $f_i(\bar{x}) < f_i(\bar{x}^*)$. Obviously, if $\bar{x}^*$ is strongly nondominated, it is also weakly nondominated. The strongly and weakly nondominated solutions constitute the total Pareto front of a multiobjective optimization problem.

II. Improved Tabu-Based Vector Optimal Method

The tabu-based algorithm proposed by the authors in [3] is very robust in finding the Pareto solution of multiobjective optimal problems. To further enhance the robustness and the efficiency of the algorithm, some novel improvements are introduced in this paper. Due to space limitation, only those improvements which are not included in [3] are described.

A. Diversification and Intensification Phases

In general, an ideal solver for multiobjective optimization problems should have the following features: 1) to efficiently find the Pareto solutions, and 2) to uniformly sample the Pareto-optimal front, i.e., to maintain the diversity of the searched Pareto solutions. To achieve the first goal, the algorithm should reinforce the moves that incorporate the merits of the Pareto solutions found in the previous search process. To obtain the second objective, the search process should also drive the search into unexplored regions to sample the Pareto front uniformly. In other words, an ideal multiobjective method is the best compromise of the intensifying and diversifying searching processes. Accordingly, the search process of the proposed algorithm is divided into two phases, i.e., an intensification and a diversification phase. Once a solution is identified as a new Pareto solution, an intensifying search around the specified point using a gradient-based Newton method is activated to quickly search for better or new Pareto solutions. The algorithm will continue in this phase until a transition criterion is satisfied, and the algorithm will then switch into the diversification phase for the next iterative cycle. To evaluate the gradient information needed by the local search method which is not available for general optimization problems, the response surface model using the moving least-squares approximation as reported in

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[4] is used to reconstruct the Pareto front (surface) around the specific point.

B. Assigning Fitness Value for New States

It is well known that in the selection of new current points for a tabu search method, it is necessary to obtain the objective function values of their neighborhood solutions. As the objective function in a multiobjective optimization problem is a vector, some scalarization techniques must be used. Thus, in our earlier work [3], the ranking method is extended and used to evaluate the "fitness" of a solution. Extensive computer simulations show that after the introduction of the fitness sharing function, especially in both parameter and objective spaces, which are used to preserve the diversity of the searched Pareto solutions, there are cases that the total fitness value of a dominated solution is larger than that of a Pareto solution, if the point density of the dominated solution is much smaller than that of the Pareto solution. Hence, the dominated solution will be accepted as a new current one to begin a new iteration, thereby leading to an inefficient algorithm. To overcome this shortcoming of the ranking approach, the nondominated sorting technique is improved and used in the proposed algorithm to decide the "fitness" value of a neighborhood solution [5]. The general procedure for assigning the fitness value of a neighborhood solution in the proposed algorithm is described as:

Algorithm:

Assign fitness values using the nondominated sorting.
Choose a large dummy fitness value $v_{\rm fit}$. Find the nondominated individuals among the neighborhood solution using solutions in both the neighborhood and the Pareto-optimal archive [3], set the fitness value of the found solutions to $v_{\rm fit}$;
Repeat
  $v_{\rm fit} = \alpha\, v_{\rm fit}$;
  Find the nondominated individuals among the neighborhood solutions whose fitness values are not set;
  Set the fitness value of the solutions just found to $v_{\rm fit}$;
Until fitness values of all neighborhood solutions are set.

The initial value of the fitness value, $v_{\rm fit}$, and the decreasing rate, α, in the proposed algorithm are, respectively, set to 3, and $1/3$, so as to guarantee that the fitness value of a Pareto-optimal solution is always higher than that of a dominated individual despite the introduction of fitness sharing functions in both parameter and objective spaces.

C. Fitness Sharing Function

To produce a uniform distribution of the searched Pareto solutions not only in the objective but also in the parameter spaces, the fitness-sharing concept is introduced. In order to reduce the implementation complexity, a simple fitness-sharing function as defined below is proposed. Mathematically, the fitness sharing function is

$$f_{\rm share}(\bar{x}^{(i)}) = \frac{1/d_f(\bar{x}^{(i)})}{\sum_{j=1}^{N_h} 1/d_f(\bar{x}^{(j)})} + \frac{1/d_X(\bar{x}^{(i)})}{\sum_{j=1}^{N_h} 1/d_X(\bar{x}^{(j)})} \quad (3)$$

where $d_u(\bar{x}^{(k)})(u = f, X)$ is the point density of the Pareto optimal obtained around the specified point $\bar{x}^{(i)}$ in the u space, N_h is the number of the total neighborhood solutions of $\bar{x}^{(i)}$, and f and X are, respectively, the objective and parameter spaces.

To compute the density of the Pareto optimal for a specified point, a hyperbox with the point as the center is constructed, and the number of the Pareto solutions lying in this box is used as a measure of its fitness-sharing function. The fitness value of a neighborhood solution $\bar{x}^{(i)}$ is the sum of its fitness and fitness-sharing function values, i.e.,

$$f_{\rm fit}(\bar{x}^{(i)}) = v_{\rm fit}(\bar{x}^{(i)}) + f_{\rm share}(\bar{x}^{(i)}). \quad (4)$$

D. Generation and Selection of Neighborhood Solutions

Unlike the common procedure that generates the total number of N_h neighborhood solutions and then chooses the best one as the new current solution, the proposed algorithm will accept a new neighborhood solution if its total fitness value is not worse than that of the current one, irrespective of whether the number of neighborhood solutions generated so far during the neighborhood generating process has reached N_h or not. This will lead to a reduction in the number of total function evaluations. Moreover, to maintain the diversity of the searched Pareto solutions, the number of neighborhood solutions generated in the ith neighborhood of the current solution is proportional to the step length of its neighbor.

E. Transition Between Intensi- and Diversification Phases

The proposed algorithm will start from the diversification phase. Once a new Pareto solution is identified, the algorithm will automatically switch to the intensification phase to intensify the search around the specific point. The algorithm will continue in this phase until either there is no further possible improvement on the specific point, or there is no other Pareto solution found around the specific point.

F. Algorithm Description

Based on the previous description, it is now possible to give a schematic explanation of the proposed algorithm as follows.

1) Start the diversification phase—if a new Pareto solution has been found, switch to the intensification phase.
2) Start the intensification phase—the algorithm continues in this phase until no further improvement can be made on the specific point, or no other Pareto solutions can be found around the specific point.
3) Termination test—if the test is passed, stop; otherwise go to (1) to begin the next iteration.

III. Numerical Examples

A. Numerical Validation

To explore and demonstrate the applicability of the proposed algorithm to study standard multiobjective optimal problems, it is first used to solve a mathematical test function which is well designed and used by other researchers for evaluating the robustness and performance of vector optimizers by comparing the numerical solutions with their corresponding analytical ones

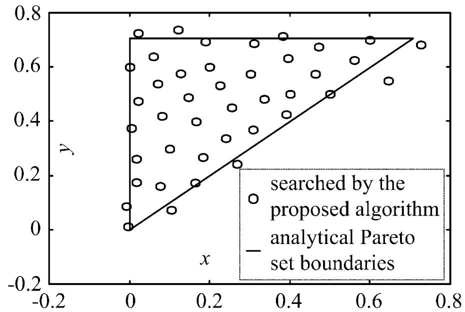

Fig. 1. Comparison of the searched Pareto solutions with the analytical ones in the parameter space for the mathematical function.

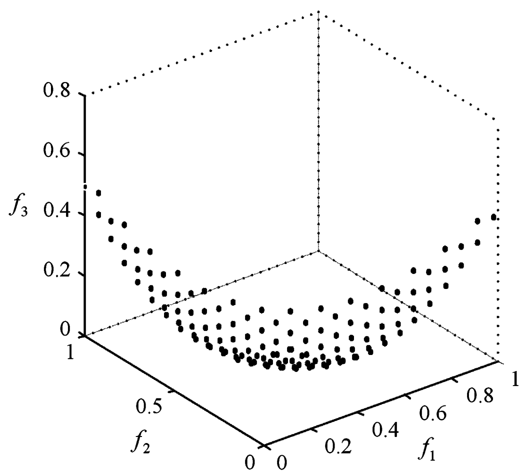

Fig. 2. Searched Pareto surfaces in the objective space for the mathematical function.

[6]. The test function is a three-objective two-decision variable problem, and is defined as

$$\begin{aligned} &\min_{(x,y)\in E^2} f_1(x,y) = x^2 + y^2 \\ &\min_{(x,y)\in E^2} f_2(x,y) = \left(x - \frac{\sqrt{2}}{2}\right)^2 + \left(y - \frac{\sqrt{2}}{2}\right)^2 \\ &\min_{(x,y)\in E^2} f_3(x,y) = x^2 + \left(y - \frac{\sqrt{2}}{2}\right)^2 \\ &\qquad\qquad E^2 = \{(x,y) | 0 \le x, y \le 2\}. \end{aligned} \tag{5}$$

The equations of the Pareto front for this mathematical function in the parameter spaces are

$$0 < x < \sqrt{2}/2, \quad x < y < \sqrt{2}/2. \tag{6}$$

The computed Pareto front of the proposed algorithm in the parameter and objective spaces are shown, respectively, in Figs. 1 and 2. It is obvious that the proposed algorithm has the ability to find the exact Pareto optimal. It could also sample the Pareto front uniformly in both the parameter and objective spaces. In order to demonstrate the necessity of the fitness sharing function in both parameter and objective spaces, other things being equal, the proposed algorithm is modified by excluding deliberately the fitness-sharing function in the parameter space, and the aforementioned mathematical function is solved again. The searched Pareto solutions obtained

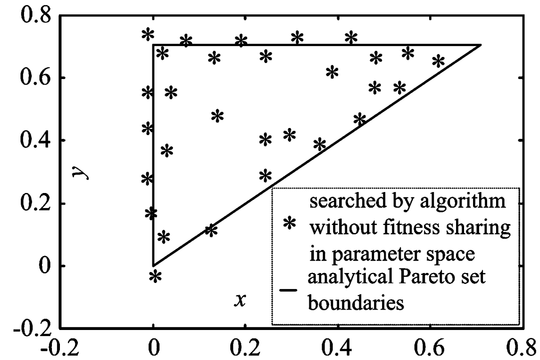

Fig. 3. Comparison of the searched Pareto solution with the analytical ones in the parameter space using the proposed algorithm without the fitness sharing in the parameter space for the mathematical function.

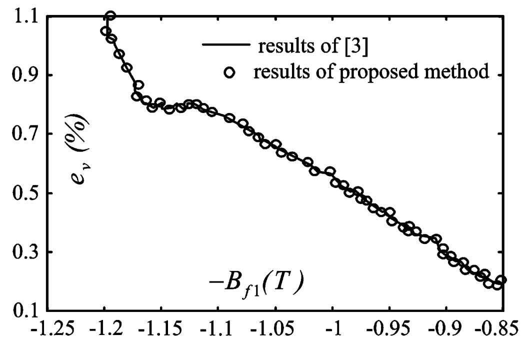

Fig. 4. Searched Pareto solutions of a 300-MW 20-pole hydrogenerator.

by using this modified algorithm in the parameter space are illustrated in Fig. 3. It can be seen that the use of a sharing procedure in only one of the two spaces cannot guarantee a satisfactory approximation of the Pareto-optimal front in the other space.

B. Case Study

The geometrical optimal design of the multisectional pole arcs of large hydrogenerators as reported in [3] is solved by using the proposed method to demonstrate its efficiency and robustness in solving a real word multiobjective optimal design problem. Mathematically, the problem is formulated as

$$\begin{array}{cc} \max & B_{f1}(X) \\ \min & e_v \\ \text{s.t} & \mathrm{SCR} - \mathrm{SCR}_0 \ge 0 \\ & X'_d - X'_{d0} \le 0 \\ & \mathrm{THF} - \mathrm{THF}_0 \le 0 \end{array} \tag{7}$$

where B_{f1} is the amplitude of the fundamental component of the flux density in the air gap, e_v is the distortion factor of a sinusoidal voltage of the machine under no-load condition, THF is the abbreviation for telephone harmonic factor, $X_d^{'}$ is the direct axis transient reactance of the generator, and SCR is the abbreviation for short circuit ratio.

The corresponding geometrical parameters to be optimized are the center positions and radii of the multisectional arcs of the pole shoes. The searched Pareto solutions for the optimal design of multisectional pole arcs of a 300-MW 20-pole hydrogenerator using the proposed algorithm, together with those of [3], are shown in Fig. 4. The corresponding iterative numbers used by

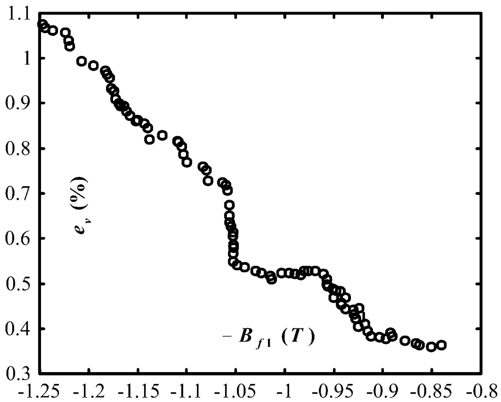

Fig. 5. Searched Pareto solutions of a 300-MW 44-pole hydrogenerator.

TABLE I
ITERATIVE NUMBERS OF DIFFERENT METHODS

Method	The Proposed one	Results of [3]
300 MW 20 pole machine	4864	6500
300 MW 44 pole machine	4958	6426

the two methods are given in Table I. To further validate the proposed algorithm, the proposed algorithm is employed to study the multisectional pole arc design of another 300-MW 44-pole hydro-generator. The computed Pareto solutions are shown in Fig. 5. Also, the iterative numbers of the two different methods for this example are outlined in Table I. From these two numerical examples, one can see that for the geometrical optimal designs of the multisectional pole arcs of large hydro-generators, the Pareto fronts searched by the two algorithms, i.e., one with and another without the improvements proposed in this paper, are virtually the same and are equally uniformly distributed in the objective spaces. However, the iterative number used by the proposed method is significantly less than that required by the original one. It should also be noted that, for a general optimal design problem, the reduction in computational time is not as significant as that in the number of iterations when considering the complication in the algorithm procedure, since the searching process of the proposed algorithm is divided into two phases. However, one should also keep in mind that, for solutions of electromagnetic inverse problems, almost all the computational time is consumed by the computational procedure of the electromagnetic fields. For example, the ratio between the reduction in iteration numbers and that for the computational time for the proposed algorithm for the case being studied is 1.05.

IV. Conclusion

As a continuation of earlier research by the authors, some new improvements to enhance both the robustness and the efficiency of a tabu-based algorithm for searching the Pareto solutions of vector optimal design problems are proposed. Two numerical examples are solved to demonstrate the applicability of the proposed method on standard and practical multiobjective optimal design problems. The computational results reveal that: 1) the computed Pareto solutions of the proposed algorithm are nearly identical to that obtained analytically, and the solutions from the proposed algorithm are distributed uniformly in both parameter and objective spaces; 2) the Pareto fronts searched by the two algorithms, i.e., one with and one without the improvements as reported in this paper, are almost the same, while the iterative number used by the proposed method is significantly less than that required by the original one; 3) a sharing procedure in only one of the two spaces cannot guarantee a satisfactory approximation of the Pareto-optimal front in the other space; 4) the proposed algorithm is however more complicated in computer implementations when compared with its ancestor.

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