An Adaptive Optimal Strategy Based on the Combination of the Dynamic-Q Optimization Method and Response Surface Methodology

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The dynamic-Q optimization method is combined with an interpolating moving least-squares approximation-based response surface model to design an efficient adaptive strategy for solving computationally heavy design problems. The proposed optimal strategy is validated by comparing its performances in finding the solutions of other common optimal methods on two different kinds of problems.

***Index Terms*—Dynamic-Q method, global optimization, optimal design, response surface methodology.**

I. Introduction

FOR computationally heavy design problems, such as inverse electromagnetic problems, an efficient, but less strictly global, algorithm may work better than strictly global and, yet, expensive search ones due to limitations in the maximum number of iterations. In this regard, the dynamic-Q optimization method (DQOM) is worthy of investigation. The DQOM is a new deterministic algorithm that decomposes the original problem into a sequence of spherically quadratic optimization subproblems which are then solved by using the steepest descent method [1], [2]. The performance of such method is reliable and very competitive to the well-established conjugate gradient algorithm. In addition, DQOM has been shown to be highly linear, i.e., the number of iterations before the solution converges is independent of the number of design variables. Hence, DQOM is very promising for solving complex engineering design problems. Although DQOM does not require the use of second-order derivatives, it still requires the first-order ones in the decomposition process, and, hence, its application is rather limited. To address this problem, Craig and Stander propose to use the response surface gradients [3]. In this paper, an adaptive optimal strategy using the basic DQOM and the response surface methodology (RSM) is proposed. Compared with the corresponding work of [3], the first-order gradients in this paper are indirectly determined from the response surfaces of functions rather than from those of the gradients themselves. In other words, the proposed algorithm is used instead of those expensive numerical approaches that compute, directly, the gradients which are rich in noise due to numerical errors. Moreover, the performance of the DQOM is degraded when there are some noises in the gradient information, which is, however, very common in those cases where the derivatives are determined directly using simulation-based approaches such as finite-element analysis (FEA) [3]. Thus, it would be preferable if the corresponding RSM can filter away the numerical errors. In this regard, the moving least-squares approximation (MLS) based one is ideal. For the aforementioned reasons, the MLS is extended into an interpolating MLS (IMLS), which is then combined with the DQOM to develop an adaptive optimal strategy in this paper.

II. Dynamic-Q Method

For a general, nonlinear optimal problem expressed as

$$\min\ f(X)X = (x_1 \quad x_2 \quad \cdots \quad x_n) \in R^n$$
$$\text{s.t.}\ \ g_j(X) \le 0, h_k(X)=0 \quad (j=1,\dots,p; k=1,\dots,q) \quad (1)$$

its successive spherical quadratic approximation for step $i(i = 1,2,\cdots)$ in the dynamic-Q method (DQM) is [2]

$$\begin{aligned}\tilde{f}(X) &= f(X^i) + \nabla^T f(X^i)(X - X^i) + \frac{(X-X^i)^T A (X-X^i)}{2}\\ \tilde{g}_j(X) &= g_j(X^i) + \nabla^T g_j(X^i)(X - X^i) + \frac{(X-X^i)^T B_j (X-X^i)}{2}\\ \tilde{h}_k(X) &= h_k(X^i) + \nabla^T h_k(X^i)(X - X^i) + \frac{(X-X^i)^T C_k (X-X^i)}{2}\\ &(j=1,2,\dots,p; k=1,2,\dots,q) \quad (2)\end{aligned}$$

where the Hessian matrices A, B_j and C_k are taking the diagonal forms of

$$A = \text{diag}(a,a,\dots,a) = aI, \quad B_j = b_j I, \quad C_j = c_k I. \quad (3)$$

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The coefficient a, b_j and c_k are determined on the bases that the approximation interpolates its corresponding actual function at both points X^i and X^{i-1}, i.e.

$$a = \frac{2[f(X^{i-1}) - f(X^i) - \nabla^T f(X^i)(X^{i-1} - X^i)]}{\|X^{i-1} - X^i\|^2}$$
$$b_j = \frac{2[g_j(X^{i-1}) - g_j(X^i) - \nabla^T g_j(X^i)(X^{i-1} - X^i)]}{\|X^{i-1} - X^i\|^2}$$
$$c_k = \frac{2[h_k(X^{i-1}) - h_k(X^i) - \nabla^T h_k(X^i)(X^{i-1} - X^i)]}{\|X^{i-1} - X^i\|^2}. \quad (4)$$

The solution of the original optimization problem defined in (1) is transferred to those of its successive approximations as given by (2) and can then be solved successively using any optimal method until a termination criteria is satisfied. Obviously, the DQOM is a primitive form of RSM.

III. Determination of Gradients

To determine the gradients as required in (2), the first-order finite-difference approach was used in the original DQM [2]. Since the efficiency and robustness of the DQM are degraded in those cases in which there are noises in the computed gradient, Craig *et al.* propose an improved version of it using the linear Koshal response surface of the gradients to replace the finite-difference approach [3]. Their numerical results show that the improved algorithm is more robust in its ability to converge stably. However, it is still necessary to provide the first-order derivative information to construct the gradient responses. As it is well known, the determination of derivatives is very difficult, if not impossible, for some engineering design problems. Therefore, it would be preferable to build the response surface using only function values. To approximate the function (gradient) values with enough numerical accuracy, the sample points in the neighborhood of the approximation point, say X^i, should be more densely populated than those in other regions. Therefore, a local interpolation scheme with a zoom-in ability to handle the aforementioned problem is desirable. In this regard, the MLS approximation technique [4] may be a promising one. However, MLS is not necessarily interpolant, and a feasible solution may be taken as an infeasible one, or *vice versa*, if it is used to solve a constrained optimal problem. To overcome this shortcoming, the MLS approximation technique is firstly extended to be an IMLS, and it is then used to construct the objective and constraint functions. For the sake of completeness, a brief introduction about IMLS is given in the following paragraphs.

To reconstruct a function $f(x) : R^n \to R$ on the basis of its values f_i at a set of sample points $x_i \in R^D (i = 1, 2, \ldots, N)$ using the MLS approximation technique in terms of some basis $b = \{b^{(i)}\}_{i=1}^n \; (n \leq N)$, a local approximation $L_x f$ of it at each point $x \in \bar{R}^n \subset R^n$ is defined as

$$L_x f := \sum_{i=1}^{n} a_i(x) b^{(i)}. \quad (5)$$

One then defines a global projector Gf such that for any point $x \in \bar{R}^n$

$$Gf(x) = L_x f(x) = \sum_{i=1}^{n} a_i(x) b^{(i)}(x). \quad (6)$$

To determine the coefficient $a(x)$ in (5) and (6), one employs a discrete L^2 norm by an x-dependent inner product $(u, v)_x$ of vectors u and v, which is defined by

$$(u, v)_x = u^T w(x) v \quad (7)$$
$$\|u\|_2 = (u, u)_x^{1/2} \quad (8)$$

where $z = (z(x_1)\; z(x_2) \cdots z(x_N))^T (z = u, v)$ and $w(x)$ is a $N \times N$ diagonal matrix with $w^{(i)}(x)$ as its ith element, $w^{(i)}(x)$ is called the weight function of the MLS.

A characteristic of the MLS is that the weight function $w^{(i)}(x)$ is a compactly supported one centered at each sampling point, thereby making the MLS a local approximation of the function.

From the condition that Gf is the best approximation of f in the least-squares sense, one obtains

$$a(x) = A(x)^{-1} B(x) f \quad (9)$$

where $f = [f_1\; f_2\; \cdots\; f_N]^T$ and

$$A(x) = \sum_{i=1}^{N} w^{(i)}(x) b(x_i) b^T(x_i) \quad (10)$$
$$B(x) = [w^1(x) b(x_1)\; w^2(x) b(x_2)\; \cdots\; w^N(x) b(x_N)]. \quad (11)$$

Obviously, this approximation procedure imposes no specific requirement on the sample point pattern. The only condition for the procedure to work is that the coefficient matrix $A(x)$ must be invertible, and this can be guaranteed by automatically adjusting the support, which, in turn, refers to the size of the influence domain of a point of the weight functions so as to involve enough sampling points ($\geq n$) for each point whose influences are nonzero at that specific point. In other words, the MLS approximation has the self-adaptive regulating ability to take care of irregular sample point patterns. However, the MLS approximation described previously is not necessarily interpolant. To make it interpolant, singularities into the weight functions $w^{(k)}(x)$ at sampling points x_k is introduced. For this purpose, the weight function as proposed in this paper is modified to

$$\frac{w^i(x)}{|x - x^i|^\alpha} \quad (12)$$

where α is a positive even integer.

After the construction of the objective and constraint functions from their function values of the sampled points using the proposed response IMLS-based response surface model, the computation of the gradient information can be obtained from these response surfaces readily.

IV. Adaptive Optimal Strategy

In order to explain the proposed adaptive optimal strategy, its iterative procedure is firstly given step by step as follows.

1) Generate the initial sampling points. Compute their objective and constraint function values using a computationally expensive approach.
2) Reconstruct the original optimal problem using IMLS from these sampled points obtained so far, and solve it using a tabu search method to find a global optimal solution, say, X^i of the reconstructed problem.
3) Starting from X^i, solve the optimal problem using the DQM.
3.1) Intensify the sample points in the neighborhood of X^i and compute the objective and constraint function values of the newly added points. Construct the optimal problem again from all the points being sampled so far using IMLS;
3.2) Determine the required gradient information from the newly constructed response surfaces.
3.3) Termination test for DQM. If the test is passed, go to step 4; otherwise, go to step 3.4.
3.4) Build a local approximation $L(i)$ to the optimal problem at X^i using (2).
3.5) Solve the approximated problem $L(i)$ using a tabu search algorithm to yield an optimal solution $X^i_{\rm opt}$.
3.6) Set $\mathrm{i} := \mathrm{i} + 1, X^i := X^i_{\rm opt}$ and go to step 3.1.
4) Termination test. If the test is passed, stop; otherwise, go to step 2.

A. Generation of Initial Sampling Points

Generally speaking, the RSMs, including the proposed one, require an excessive number of function calls, which, in turn, requires a large number of sample points, in order to build a response surface with enough accuracy when the number of decision parameters is high. Due to the heavy computation burden in optimal problems such as in an inverse electromagnetic one, the number of sampling points cannot be more than several thousand points. Consequently, these points should be distributed in the feasible parameter space in an irregular pattern such that the point densities are higher in regions where the local optima are likely to exist. Thus, two problems arise, i.e., 1) how to generate the sample points with a desired distribution and 2) how to deal with the irregular distribution of the data in the corresponding RSM. To address the first problem, the simulated annealing (SA) algorithm is proposed to run first on the computationally heavy optimal problem to generate the sample points because it has some "intelligence" in generating new points, i.e., intensifying the points in regions where the local optima exist. Of course, when the number of the total sample points is only of the order of a few hundred, there is no need for a well engineered SA algorithm to generate the sampling points "intelligently." In such a case, one should deliberately design a "poorly" engineered SA algorithm by setting a relatively small control parameter (temperature). For the second problem, it is relatively easy to use the proposed IMLS-based RSM due to its power in reconstructing functions from irregularly distributed sample points as described and demonstrated previously.

TABLE I
Performance Comparison of Different Optimal Methods for Case 1

Algorithms	No. of averaged iterations	Relative error	Success rate
Proposed D-Q	831	1.8×10^{-10}	100
Original D-Q	788*	2.0×10^{-10}	100
GA+RSM	1024	1.7×10^{-10}	100

788 is the result of one run.

B. Enhancement of Global Search Ability

Although the DQM has found the global solution of most of the test problems [2] as explained in the introduction, there is no guarantee that it will never terminate on a local minimum. In other words, the global search ability of the DQM is not robust. To address this problem, the iterative procedure of the proposed strategy is designed as a multistart refinement searching procedure that includes an outer (global) and an inner (local) searching cycle. In the outer cycle, the tabu search is used to determine the initial point of the DQM to guarantee that the starting point of the inner cycle is the global optimal point of the approximated problem constructed from the points being sampled so far. In the inner one, i.e., the dynamic-Q searching process, the tabu search method is also used to ensure that the searched solution in this cycle is the global one of the local approximation. Therefore, a more robust algorithm in terms of global searching ability may be expected. The details about the tabu search method used in this paper are referred to [5].

V. Validation and Application

A. Validation and Performance Comparison

For performance comparison and validation purposes, the problem 13 with $n = 10$, as reported in [1], is selected as the test case. In the numerical experiment, three different optimal methods, i.e., the proposed, the original DQM, and a genetic algorithm combined with the same IMLS response surface methodology $(\mathrm{GA} + \mathrm{RSM})$ are used to solve this problem. For each method, it is run 100 times independently by starting from a randomly generated point, and the results are given in Table I. It is obvious that all three methods can find the exact optimal solution with a 100% success rate. In regard to the iterative number, the proposed algorithm is comparable to its predecessor, and is superior to a GA-based one. However, since the gradient information of the proposed strategy is obtained from the IMLS-based response surface, the proposed algorithm is more suitable for engineering design problems in which the gradient information of the objective and constraint functions is either unavailable or computationally too heavy to determine. Moreover, to demonstrate the performances of the proposed method for a problem with many decision parameters, it is used to solve this same problem with $n = 100$. The averaged iterative number for 100 runs with a 100% success rate is 2945, and this is comparable to the 2580 iterations of the original DQM.

To demonstrate the enhancement of global search abilities of the proposed algorithm for solving complex optimal problems, a very complicated function with 15^5 local optimums as detailed in [5] is solved using both the proposed and the original

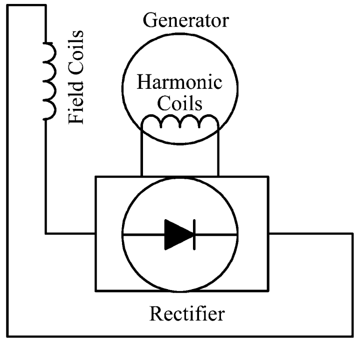

Fig. 1. Schematic diagram of the exciting system of a harmonic excited synchronous generator.

DQMs. In the numerical experiment, every algorithm is also independently run 100 times with a maximum of 1000 iterations for each function call. The success rates to find the global optimal solutions of such a complex problem for the proposed and the original DQM are, respectively, 41% and 32%. Evidently, the global searching ability of the proposed DQM has been enhanced significantly.

B. Application

In some small- and medium-scale salient synchronous generators, the harmonic currents (mainly the triple-frequency ones) induced in the harmonic coils mounted on the stator are usually converted into dc via a rectifier in the field circuit (Fig. 1). To obtain a maximum triple-frequency harmonic current, the five-sectional arc geometry of a pole arc rather than the traditional one-piece pole arc is studied. Therefore, one needs to optimize the parameters of the five-sectional arc. An efficient algorithm for achieving this goal is very important since the computation of the objective function is computationally expensive because FE analysis is involved. Therefore, this problem is an ideal example to demonstrate the feasibility of the proposed algorithm in solving realistic engineering design problems. The details about this problem are referred to [6]. Mathematically, the optimal problem can be formulated as

$$\begin{aligned} &\max\ w_1 e_1 + w_3 e_3 \\ \text{s.t.}\quad &g_{\min} \geq g_0 \\ &b_p \geq b_{p0} \end{aligned} \tag{13}$$

where w_1 and w_3 are two weighting factors; e_1 and e_3 are, respectively, the fundamental and the triple-frequency components waveforms of the open-circuit characteristics of the machine; $g_{\min}$ is the minimal length of the air gap; and b_p is the length of the pole arc.

The geometric parameters to be optimized are the position coordinates, arc radii, and the length of the five-sectional arc. In the optimizing process, the waveform of the open-circuit characteristics of the machine is determined using FE analysis. For the convenience of performance comparisons, this problem is solved using the proposed and the same GA combined with the IMLS-based RSM algorithms. The final solutions and the performance comparison results, together with those obtained using a SA algorithm [6], of a 6.5-kW harmonic excited generator, are summarized in Table II. From these numerical results, it can be seen that: 1) the iterative numbers using the two RSM-based optimal strategies are reduced significantly compared to that of a SA-based one; 2) the final solutions found by different methods are nearly the same; and 3) among all the three optimal strategies, the proposed method is the most efficient one.

TABLE II
PERFORMANCE COMPARISON OF DIFFERENT OPTIMAL METHODS FOR A 6.5-kW HARMONIC-EXCITED SYNCHRONOUS GENERATOR

Algorithm	No. of FE computations	e_1(pu)	e_3(pu)
Proposed	1284	1.09	1.20
GA+RSM	1876	1.10	1.20
SA	6258	1.08	1.21

VI. CONCLUSION

An adaptive optimal strategy based on the combination of DQOM and RSM is proposed to enhance its global searching ability. The proposed algorithm relaxes the requirement upon the direct gradient computations of the original DQMs without sacrificing its inherent advantages. To achieve these goals, the MLS approximation technique is extended to become an interpolant one, and a tabu search algorithm is embedded in the iterative procedure in twofold, i.e., one for the global search to give a starting point of the dynamic-Q search procedure, and the other is used for finding the global optimal solution of a local approximated subproblem. The numerical results as reported in this paper demonstrate fully the robustness and efficiency of the proposed algorithm in solving both mathematical and engineering design problems.

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