A Multislice Coupled Finite-Element Method With Uneven Slice Length Division for the Simulation Study of Electric Machines

W. N. Fu, S. L. Ho, H. L. Li, and H. C. Wong

***Abstract*—To consider the changes of the magnetic field along the axial direction when simulating the operations of electric machines, a multislice circuit-field coupled finite element method is presented. The formulations reported allow unequal division of the axial length of the motor in the multislice model. The proposed circuit-field coupled technique provides a simple, general, and yet systematic way to couple arbitrary circuits with the magnetic field.**

***Index Terms*—Coupled method, electric machine, finite element, multislice method.**

I. Introduction

In ORDER to consider skewed rotor slots in the simulation studies of induction motors using time stepping finite-element method (FEM), a two-dimensional (2-D) multislice model has been developed [1]. A set of nonskewed 2-D models, each corresponding to a section taken at different positions along the axis of the machine, is used to model the skewed rotor. The 2-D FEM equations of each slice are then coupled together and solved simultaneously. This model was further refined by the authors in [2]. To consider the interbar currents in the skewed rotor bars, a multislice-network-coupled FEM has also been proposed [3]. However, in all these formulations the axial length of each slice along the motor axis has to be equal.

Besides considering skewed slots, the model is also ideal to deal with motors with insignificant axial changes in the magnetic field.

This paper describes the necessary formulae to address the multislice FEM formulations with nonuniform slice lengths along the axial direction. The geometrical shape of the cross sections of each slice can also be different if the changes are small. The merit of the proposed method is that arbitrary connected stranded windings, solid conductors, and external circuits can be directly coupled readily.

II. Basic Equations in the Field Regions

In multislice FEM, the motor is divided axially into M slices, and the axial length of each slice is $l_1, l_2, \ldots, l_M$. In each slice, the magnetic vector potential has an axial component only. All the stranded windings and solid conductors in the magnetic field as well as the external circuits are connected by circuits which are governed by the Kirchhoff's laws.

A. In the Airgap and the Iron Cores

Using the aforementioned assumptions, the field equation expressed in the 2-D Cartesian coordinates in regions without source current and permanent magnets (PM) is

$$\sum_{m=1}^{M} l_m \left[\nabla \cdot (\nu \nabla A_m) + \sigma \frac{\partial A_m}{\partial t} \right] = 0 \tag{1}$$

where A_m is the axial component of the magnetic vector potential in the mth slice, ν is the reluctivity of the material, and σ is the electric conductivity.

B. In the Stranded Windings

In the armature winding where the current density is uniform, the field equation is

$$\sum_{m=1}^{M} \left[l_m \nabla \cdot (\nu \nabla A_m) - \frac{K_w l_m}{S_m} i_w \right] = 0 \tag{2}$$

where i_w is the winding current, S_m is the total cross-sectional area of one turn on one coil side of the mth slice (S_m between adjacent slices can differ slightly), and K_w is the polarity ($+1$ or -1) to represent either the forward paths or return paths. The winding circuit equation of one branch is

$$e + R_w i_w + L_\sigma \frac{d\, i_w}{d\, t} = u_w \tag{3}$$

where R_w is the total winding resistance, L_σ is the inductance of the end windings, and u_w is the branch voltage of the winding. The induced electromotive force (e.m.f.) in the winding is

$$e = \sum_{m=1}^{M} \frac{K_w l_m}{S_m} \iint_{\Omega_m} \frac{\partial A_m}{\partial t} d\Omega. \tag{4}$$

If (2) and (3) are directly coupled together, the coefficient matrix of the system equations will become asymmetrical in the nodal analysis. Consequently, an additional unknown in the stranded windings is proposed below in order to make the coefficient matrix symmetrical.

Using the backward Euler's method to discretize the time variable, one gets

$$\frac{di_w}{dt} = \frac{i_w - i_w^{k-1}}{\Delta t} \tag{5}$$

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where i_w^k is simply written as i_w, the superscript k is the step number in the time stepping process and the step size $\Delta t = t^k - t^{k^{-1}}$. Substituting the above equation into (3), one has

$$i_w = \frac{\left(-e + u_w + \frac{L_\sigma}{\Delta t} i_w^{k-1}\right)}{\left(R_w + \frac{L_\sigma}{\Delta t}\right)}. \quad (6)$$

Substituting (6) into (2), the field equation is

$$\sum_{m=1}^{M} \left[l_m \nabla \cdot (\nu \nabla A_m) + \frac{K_w l_m}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} e - \frac{K_w l_m}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} u_w \right] = \sum_{m=1}^{M} \frac{K_w l_m}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} \frac{L_\sigma}{\Delta t} i_w^{k-1}. \quad (7)$$

Substituting (4) and (6) into (3), the branch equation is

$$-\sum_{m=1}^{M} \frac{K_w l_m}{S_m (R_w + \frac{L_\sigma}{\Delta t})} \iint_\Omega \frac{\partial A_m}{\partial t} d\Omega + \frac{1}{\left(R_w + \frac{L_\sigma}{\Delta t}\right)} u_w = i_w - \frac{L_\sigma}{\Delta t (R_w + \frac{L_\sigma}{\Delta t})} i_w^{k-1}. \quad (8)$$

Equation (4) can also be written as

$$-\sum_{m=1}^{M} \frac{K_w l_m}{S_m (R_w + \frac{L_\sigma}{\Delta t})} \iint_\Omega \frac{\partial A_m}{\partial t} d\Omega + \frac{1}{\left(R_w + \frac{L_\sigma}{\Delta t}\right)} e = 0. \quad (9)$$

Equations (7)–(9) are the basic equations in the stranded winding region. The additional unknown e is introduced to make the coefficient matrix of the system equations symmetrical.

C. In the Solid Conductors

Eddy-current is inevitable in solid conductors. PM can be taken as solid conductors with the eddy-current effect being included in the FE solution . The Maxwell's equations for the nth conductor of the mth slice is

$$\sum_{m=1}^{M} \left[l_m \nabla \cdot (\nu \nabla A_m) + \sigma l_m \frac{\partial A_m}{\partial t} - \sigma u_{mn} \right] = \sum_{m=1}^{M} \nu \mu_0 l_m \left(\frac{\partial M_{my}}{\partial x} - \frac{\partial M_{mx}}{\partial y} \right) \quad (10)$$

where u_{mn} is the voltage between the two terminals of the nth conductor of the mth slice, M_{mx} and M_{my} are, respectively, the x and y components of the magnetization vector (amperes/meter) in the mth slice, ν is the equivalent reluctivity in the PM, and μ_o is the permeability of free space. The total current in the nth conductor of the mth slice is

$$i_{mn} = \sigma \iint_{\Omega_{mn}} \left(-\frac{\partial A_m}{\partial t} + \frac{u_{mn}}{l_m} \right) d\Omega. \quad (11)$$

Equation (11) can also be written as,

$$-\sigma \iint_{\Omega_{mn}} \frac{\partial A_m}{\partial t} d\Omega + \frac{1}{R_{mn}} u_{mn} = i_{mn} \quad (12)$$

where the dc resistance of the nth conductor of the mth slice is

$$R_{mn} = \frac{l_m}{\left(\sigma \iint_{\Omega_{mn}} d\Omega\right)}. \quad (13)$$

In the PM region, the total current i_{mn} in each piece of the PM should be zero. The field equation (10) and branch circuit equation (12) are the basic equations in the solid conductors. Since all conductors in each slice will be connected with the external circuit, the free divergence of the current density is satisfied by the Kirchhoff's law [3].

III. Subsystem Equations in the Field Regions

The potential can be expressed as the sum of the shape functions multiplied with the nodal potential

$$A_m = \sum_{i=1}^{M_e} N_i(x,y) A_{mi} \quad (14)$$

as there are M_e nodes in the element and N_i is the shape function. Using the Galerkin Method, one obtains the integral equations of the field problems. In the airgap and iron cores associated with (1), one has

$$\sum_{m=1}^{M} l_m \left[\iint_{\Omega_m} \frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \nu \sum_{j=1}^{M_e} N_j A_{mj} + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \nu \cdot \sum_{j=1}^{M_e} N_j A_{mj} + \sigma N_i \frac{\partial}{\partial t} \sum_{j=1}^{M_e} N_j A_{mj} \right] d\Omega = 0. \quad (15)$$

In the stranded windings associated with (7), one has

$$\sum_{m=1}^{M} l_m \iint_{\Omega_m} \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \nu \sum_{j=1}^{M_e} N_j A_{mj} + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \nu \sum_{j=1}^{M_e} N_j A_{mj} + \frac{N_i K_w}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} (e - u_w) \right] d\Omega = \sum_{m=1}^{M} \iint_{\Omega_m} \left[\frac{N_i K_w l_m}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} \frac{L_\sigma}{\Delta t} i_w^{k-1} \right] d\Omega. \quad (16)$$

In solid conductor or PM region associated with (10), one has

$$\sum_{m=1}^{M} l_m \iint_{\Omega_m} \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \nu \sum_{j=1}^{M_e} N_j A_{mj} + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \nu \sum_{j=1}^{M_e} N_j A_{mj} + \sigma N_i \frac{\partial}{\partial t} \sum_{j=1}^{M_e} N_j A_{mj} - N_i \left(\frac{\sigma}{l_m} u_{mn} \right) \right] d\Omega = \sum_{m=1}^{M} l_m \iint_{\Omega_m} \nu \mu_0 \left(M_{mx} \frac{\partial N_i}{\partial y} - M_{my} \frac{\partial N_i}{\partial x} \right) d\Omega. \quad (17)$$

Discretising (15)–(17), (8), (9), (12), and coupling them together, the global system matrix equation becomes

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ 0 & C_{22} & 0 & 0 \\ 0 & 0 & C_{33} & 0 \\ 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} A \\ e \\ u_w \\ u_{mn} \end{bmatrix} + \begin{bmatrix} D_{11} & 0 & 0 & 0 \\ C_{12}^T & 0 & 0 & 0 \\ C_{13}^T & 0 & 0 & 0 \\ C_{14}^T & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{dA}{dt} \\ \frac{de}{dt} \\ \frac{du_w}{dt} \\ \frac{du_{mn}}{dt} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_w \\ i_{mn} \end{bmatrix} + \begin{bmatrix} P_1 \\ 0 \\ P_3 \\ 0 \end{bmatrix} \quad (18)$$

where the first row of the submatrix is the magnetic field equations, the second row is the additional circuit equations in the stranded windings, the third row is the branch equations of the stranded windings, and the last row is the branch equations of the solid conductors. The unknowns are the magnetic vector potentials $[A]$, the e.m.f. $[e]$ in the stranded windings, the voltages

$[u_w]$ of the stranded windings, and the voltages $[u_{mn}]$ on each slice in the solid conductors.

The elements of the coefficient matrix are given in the following such that if the elements and nodes are numbered slice by slice, then

$$A = [\,A_1 \quad A_2 \quad \cdots \quad A_M\,]^T \tag{19}$$

C_{11} and D_{11} are associated with the field equations

$$C_{11} = \begin{bmatrix} C_{11(1)} & 0 & & 0 \\ 0 & C_{11(2)} & & 0 \\ & & \ddots & \\ 0 & 0 & & C_{11(M)} \end{bmatrix} \tag{20}$$

$$D_{11} = \begin{bmatrix} D_{11(1)} & 0 & & 0 \\ 0 & D_{11(2)} & & 0 \\ & & \ddots & \\ 0 & 0 & & D_{11(M)} \end{bmatrix} \tag{21}$$

where each element is

$$C_{11(m)ij} = l_m \iint_{\Omega_{me}} \nu \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \right) d\Omega \tag{22}$$

$$D_{11(m)ij} = l_m \iint_{\Omega_{me}} \sigma N_i N_j d\Omega. \tag{23}$$

In (22) and (23), $i = 1, 2, \ldots, N_e$ and $j = 1, 2, \ldots, N_e$.

C_{12} and C_{13} are associated with the coupling terms between the field and the stranded windings,

$$C_{12} = [\,C_{12(1)} \quad C_{12(2)} \quad \cdots \quad C_{12(M)}\,]^T = -C_{13} \tag{24}$$

$C_{12(m)}$ has W columns for the W stranded windings. The element of the ith node and the wth stranded winding is

$$C_{12(m)iw} = -C_{13(m)iw} = \frac{K_w l_m}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)} \iint_{\Omega_{me}} N_i dxdy. \tag{25}$$

C_{14} is associated with the coupling term between the field and the solid conductors

$$C_{14} = \begin{bmatrix} C_{14(1)} & 0 & & 0 \\ 0 & C_{14(2)} & & 0 \\ & & \ddots & \\ 0 & 0 & & C_{14(M)} \end{bmatrix} \tag{26}$$

$C_{14(m)}$ has N columns for the N solid conductors. The element of the ith node and the nth solid conductor is

$$C_{14(m)in} = -\sigma \iint_{\Omega_{me}} N_i dxdy. \tag{27}$$

The subcolumn matrix on the right-hand side

$$P_1 = [\,P_{1(1)} \quad P_{1(2)} \quad \cdots \quad P_{1(M)}\,]^T \tag{28}$$

can be computed as

$$\begin{aligned} P_{1(m)i} = & \frac{K_w l_m L_\sigma i_w^{k-1}}{S_m \left(R_w + \frac{L_\sigma}{\Delta t}\right)\Delta t} \iint_{\Omega_{me}} N_i dxdy \\ & + l_m \iint_{\Omega_{me}} \nu\mu_0 \left(M_{mx}\frac{\partial N_i}{\partial y} - M_{my}\frac{\partial N_i}{\partial x}\right) dxdy \\ & + l_m \nu \int_{C_{me}} N_i \frac{\partial A_m}{\partial n} dc \end{aligned} \tag{29}$$

where the first term is present only in the stranded winding region, the second term is present only in the PM, the last term is only found in the Neumann boundary C_{me}, and n in (29) is the outward normal unit vector.

In the stranded windings, C_{22} and C_{33} are $W \times W$ matrices

$$C_{22} = [\,C_{22(1)} \quad C_{22(2)} \quad \cdots \quad C_{22(W)}\,]^{\text{diagonal}} = C_{33} \tag{30}$$

and P_3 is a column matrix with W rows

$$P_3 = [\,P_{3(1)} \quad P_{3(2)} \quad \cdots \quad P_{3(W)}\,]^T \tag{31}$$

where

$$C_{22(w)} = C_{33(w)} = \frac{1}{R_w + \frac{L_\sigma}{\Delta t}} \tag{32}$$

$$P_{3(w)} = -\frac{L_\sigma}{\Delta t \left(R_w + \frac{L_\sigma}{\Delta t}\right)} i_w^{k-1}. \tag{33}$$

In the solid conductors

$$C_{44} = \begin{bmatrix} C_{44(1)} & 0 & & 0 \\ 0 & C_{44(2)} & & 0 \\ & & \ddots & \\ 0 & 0 & & C_{44(M)} \end{bmatrix} \tag{34}$$

where

$$C_{44(m)n} = \frac{1}{R_{mn}}. \tag{35}$$

Using the backward Euler's method to discretize the time variable, one obtains the recurrence formulas of the kth step for the time stepping process as follows:

$$\begin{bmatrix} C_{11} + \frac{D_{11}}{\Delta t} & C_{12} & C_{13} & C_{14} \\ \frac{C_{12}^T}{\Delta t} & C_{22} & 0 & 0 \\ \frac{C_{13}^T}{\Delta t} & 0 & C_{33} & 0 \\ \frac{C_{14}^T}{\Delta t} & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} A \\ e \\ u_w \\ u_{mn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_w \\ i_{mn} \end{bmatrix} + \begin{bmatrix} P_1 + \frac{D_{11}}{\Delta t} A^{k-1} \\ \frac{C_{12}^T}{\Delta t} A^{k-1} \\ P_3 + \frac{C_{13}^T}{\Delta t} A^{k-1} \\ \frac{C_{14}^T}{\Delta t} A^{k-1} \end{bmatrix}. \tag{36}$$

Multiplying Δt to the additional equation in the second row and the branch equations in the third and fourth rows, one obtains the following matrix equation with symmetrical coefficients:

$$\begin{bmatrix} C_{11} + \frac{D_{11}}{\Delta t} & C_{12} & C_{13} & C_{14} \\ C_{12}^T & C_{22}\Delta t & 0 & 0 \\ C_{13}^T & 0 & C_{33}\Delta t & 0 \\ C_{14}^T & 0 & 0 & C_{44}\Delta t \end{bmatrix} \begin{bmatrix} A \\ e \\ u_w \\ u_{mn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_w \Delta t \\ i_{mn}\Delta t \end{bmatrix} + \begin{bmatrix} P_1 + \frac{D_{11}}{\Delta t} A^{k-1} \\ C_{12}^T A^{k-1} \\ P_3 \Delta t + C_{13}^T A^{k-1} \\ C_{14}^T A^{k-1} \end{bmatrix}. \tag{37}$$

IV. Coupling With the External Circuits

In time domain, the branch equation of the external circuits can be simplified as

$$[G_e]\,[u_e] = [i_e] + [P_e] \tag{38}$$

where u_e and i_e are, respectively, the branch voltage and current, G_e is the matrix of the conductance, and P_e is the column matrix associated with voltage source, current source as well as the solution from the previous time step (for inductive and capacitive elements). Multiplying Δt to the two sides of the previous equation gives

$$[G_e \Delta t]\,[u_e] = [i_e \Delta t] + [P_e \Delta t]\,. \tag{39}$$

The addition of the external circuit equations into (37) gives

$$\begin{bmatrix} C_{11}+\frac{D_{11}}{\Delta t} & C_{12} & C_{13} & C_{14} & 0 \\ C_{12}^T & C_{22}\Delta t & 0 & 0 & 0 \\ C_{13}^T & 0 & C_{33}\Delta t & 0 & 0 \\ C_{14}^T & 0 & 0 & C_{44}\Delta t & 0 \\ 0 & 0 & 0 & 0 & G_e\Delta t \end{bmatrix} \begin{bmatrix} A \\ e \\ u_w \\ u_{mn} \\ u_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ i_w\Delta t \\ i_{mn}\Delta t \\ i_e\Delta t \end{bmatrix} + \begin{bmatrix} P_1+\frac{D_{11}}{\Delta t}A^{k-1} \\ C_{12}^T A^{k-1} \\ P_3\Delta t + C_{13}^T A^{k-1} \\ C_{14}^T A^{k-1} \\ P_e\Delta t \end{bmatrix}. \quad (40)$$

The branch voltage column matrix and the branch current column matrix are

$$[u_b] = [u_w \quad u_{mn} \quad u_e]^T \quad (41)$$

$$[i_b] = [i_w \quad i_{mn} \quad i_e]^T. \quad (42)$$

Using the nodal method, the relationship between the branch voltage u_b and the nodal voltage u_n is expressed as

$$[u_b] = \left[A_{nb}^T\right][u_n] \quad (43)$$

where $A_{nb} = (a_{ij})$ is the node-to-branch incidence matrix

$$a_{ij} = \begin{cases} +1 & \text{if branch } j \text{ is connected to node } i \\ & \text{and it is directed away from the node} \\ -1 & \text{if branch } j \text{ is connected to node } i \\ & \text{and it is directed toward the node} \\ 0 & \text{if branch } j \text{ is not connected to node } i \end{cases}.$$

The Kirchhoff's current law can be expressed as

$$[A_{nb}][i_b] = 0. \quad (44)$$

Substituting (43) into the system equations (40) and multiplying $[A_{nb}]$ to the two sides of the last three rows of (40) as well as including the relationships (41), (42) and (44), one obtains the final global equations

$$\begin{bmatrix} \begin{pmatrix} C_{11}+\frac{D_{11}}{\Delta t} & C_{12} \\ C_{12}^T & C_{22}\Delta t \end{pmatrix} & \begin{pmatrix} C_{13} & C_{14} & 0 \\ 0 & 0 & 0 \end{pmatrix} A_{nb}^T \\ A_{nb}\begin{pmatrix} C_{13}^T & 0 \\ C_{14}^T & 0 \\ 0 & 0 \end{pmatrix} & A_{nb}\begin{pmatrix} C_{33}\Delta t & 0 & 0 \\ 0 & C_{44}\Delta t & 0 \\ 0 & 0 & G_e\Delta t \end{pmatrix} A_{nb}^T \end{bmatrix} \cdot \left[\begin{pmatrix} A \\ e \\ u_n \end{pmatrix}\right] = \begin{bmatrix} \begin{pmatrix} P_1+\frac{D_{11}}{\Delta t}A^{k-1} \\ C_{12}^T A^{k-1} \end{pmatrix} \\ A_{nb}\begin{pmatrix} \Delta t P_3 + C_{13}^T A^{k-1} \\ C_{14}^T A^{k-1} \\ P_e\Delta t \end{pmatrix} \end{bmatrix} \quad (45)$$

where the coefficient matrices are still symmetrical.

V. Application Example

A brushless dc PM motor is used as an example to validate the proposed method. The main data of the motor is a 12-V power supply, 12 stator slots, 8 PM poles, a rated speed of 5000 rpm, and star-connected armature windings. The motor is driven by a converter with "two phase on at a time" excitation and the control circuits are directly coupled into the FEM in the simulation study. The motor has unevenly magnetized Nd–Fe–B along the axial length. The magnetization of the PM weakens gradually near the two ends of the PM bars along the axial direction. Consequently, one needs five slices near the end region of the motor in the FEM model. However, the total slice number M is only 6. The axial length of each slice is $l_1 = l_2 = l_3 = l_4 = l_5 = 0.04l_z, l_6 = 0.8l_z$, where l_z is the total axial length of the motor. Note that if the slice length is uniform, then the slice number would be 25. By using the proposed algorithm, the computing time can, thus, be reduced significantly. It can also be seen that the measured and computed phase currents as shown, respectively, in Fig. 1(a) and (b) are virtually identical to each other.

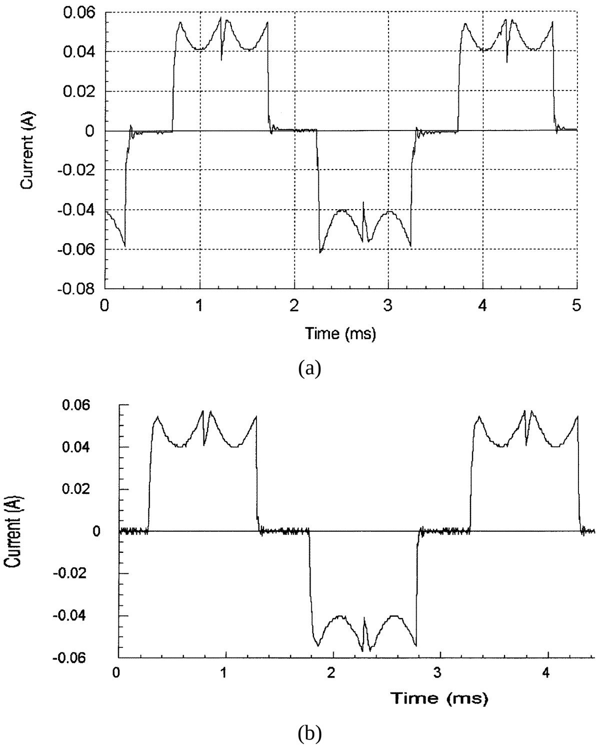

Fig. 1. Phase current of a PM motor. (a) Meaured current. (b) Computed current.

VI. Conclusion

The proposed multislice FEM for the analysis of the performance of electric machines allows nonuniform axial division of the motor in multislice models. The arbitrary connected stranded windings, solid conductors, and external circuits can be directly coupled with the FEM equations and the coefficient matrix of the system equations can be kept symmetrical. When the number of the multislices is equal to one, the deduced formulations become that of the normal 2-D FEM and circuit coupled method.

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